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Observatoire de la CÔTE d'AZUR

Astronomy in the radio band

Introduction to Radioastronomy

First Part

January 2011

- **Fundamentals of radio observations:** first part
- Physical mechanisms responsible for emission at radio frequencies: second part
- Galactic and extra-galactic radio sources: third part

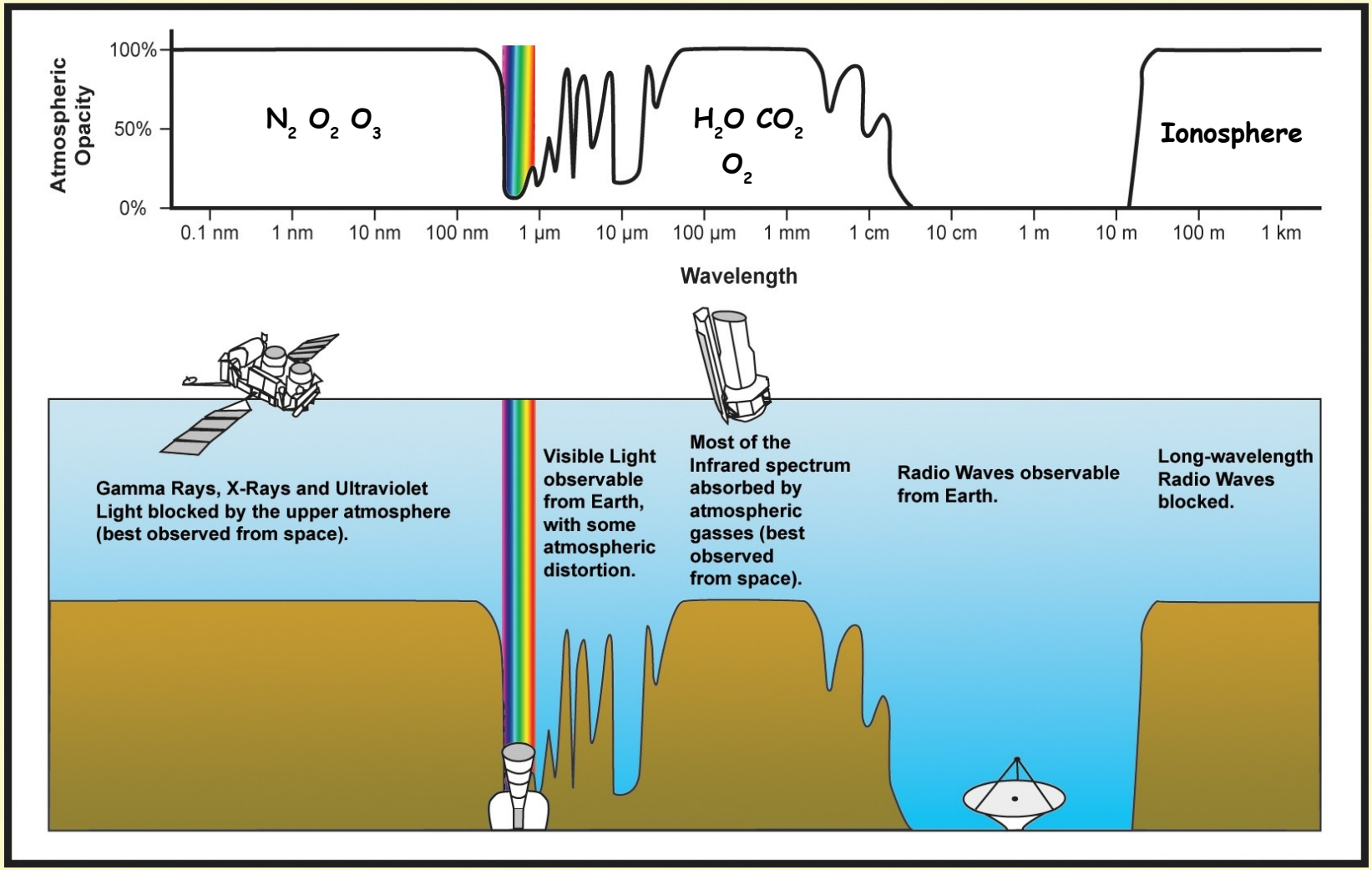
Lecture Outline

- Why radioastronomy?
- Electromagnetic waves
- Some basic definitions: brightness & flux density
- Fraunhofer diffraction and its applications
- The radio telescope: hints to its basic properties
- The need of resolution: interferometry
- Data reduction: imaging & deconvolution

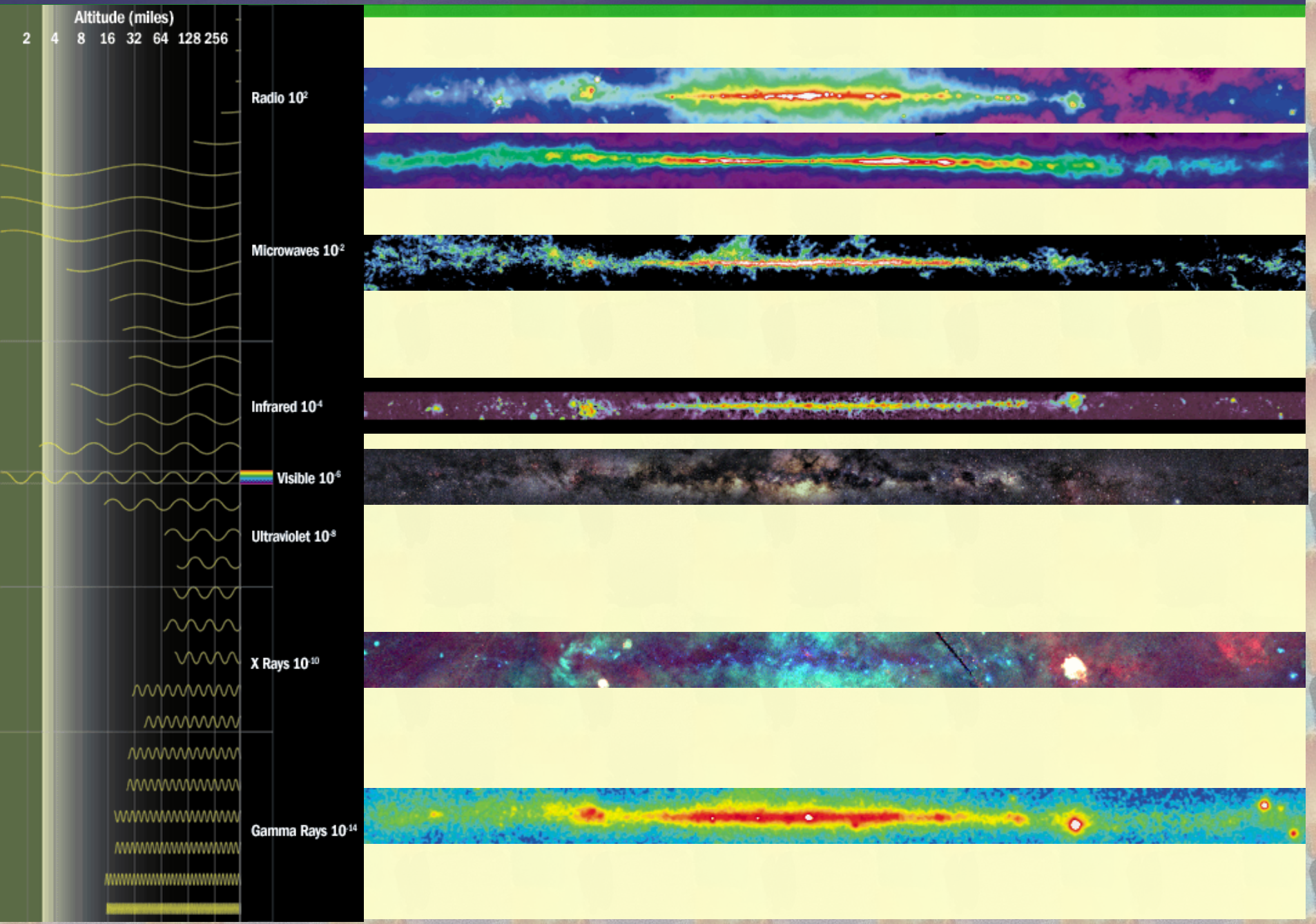
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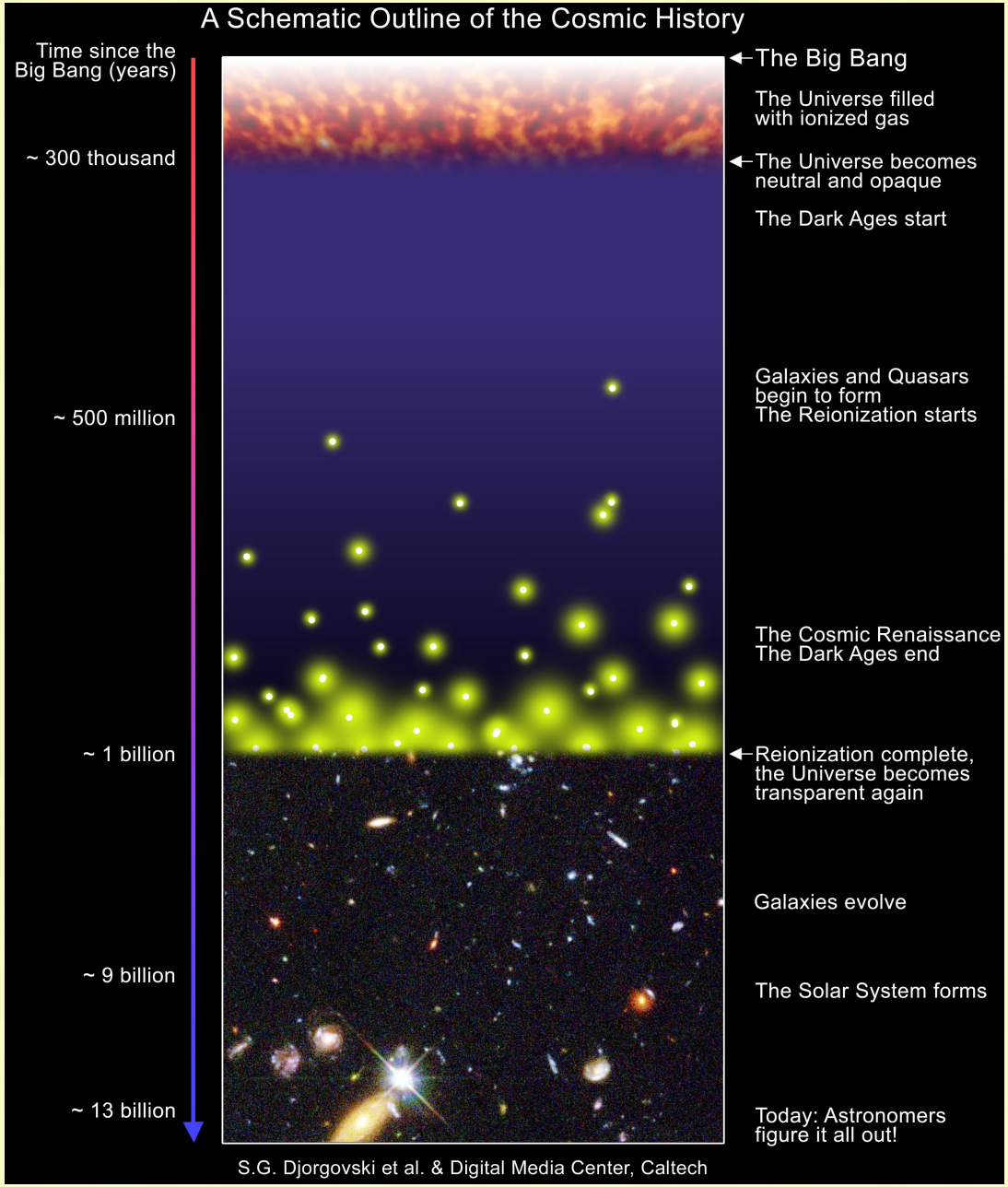
The electromagnetic spectrum



A multi-wavelength view of our Galaxy



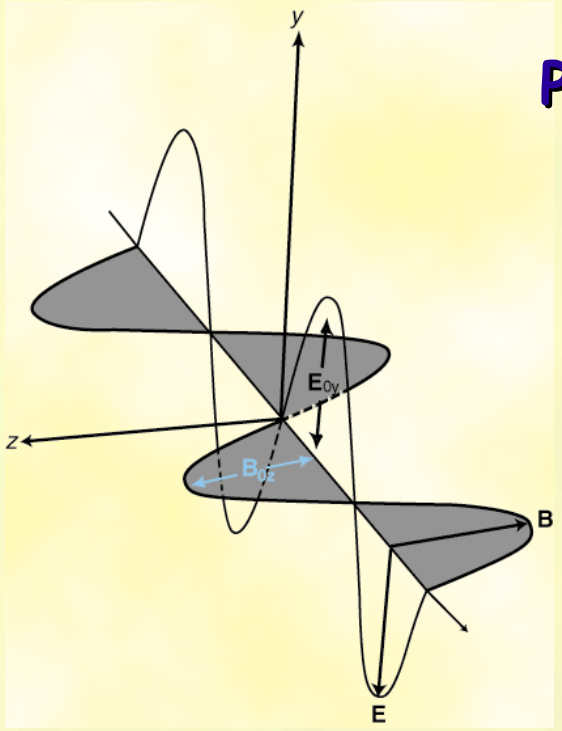
One of the main discovery of (radio)astronomy: the Cosmic Microwave Background (CMB)



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$\vec{E} \times \vec{B}$

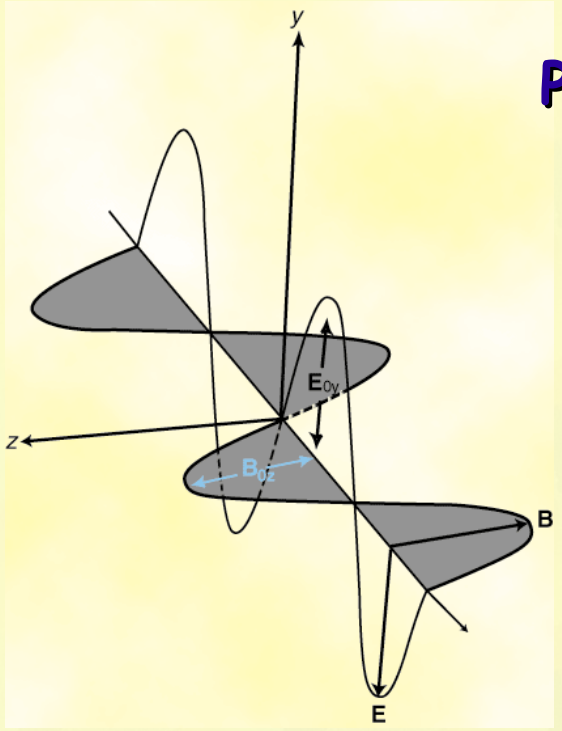


Propagation of e.m. waves: wave equation

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

→ spherical waves

$\vec{E} \times \vec{B}$



Propagation of e.m. waves: wave equation

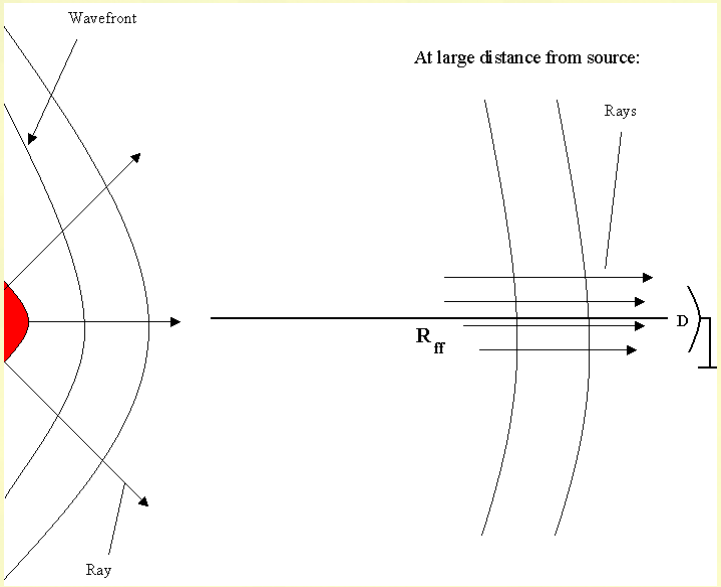
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→ spherical waves

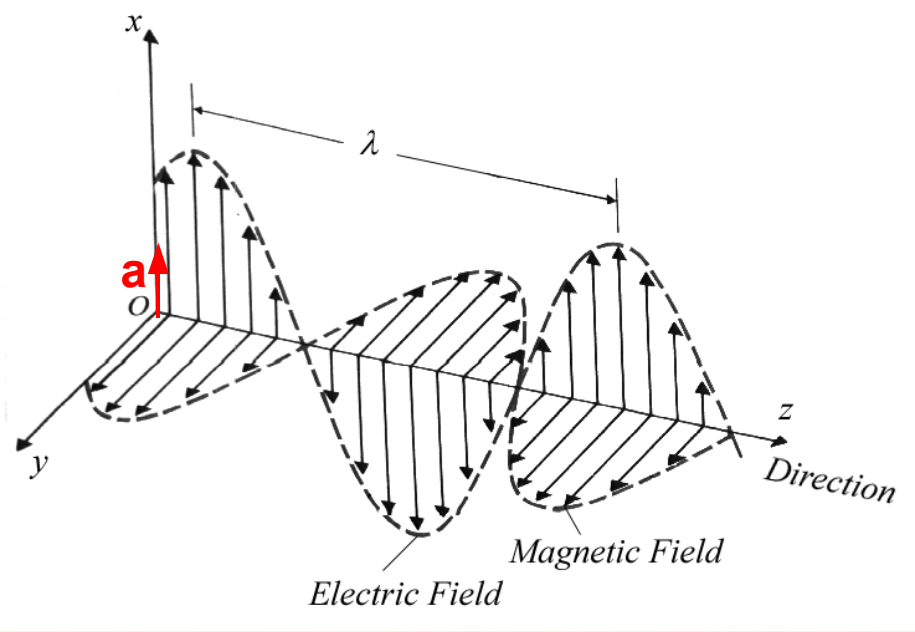
For astronomical sources the **far field condition** is always satisfied:

$$R_{ff} \gg \frac{D^2}{\lambda}$$

→ **plane waves** sufficiently good approximation



Electromagnetic waves & polarization



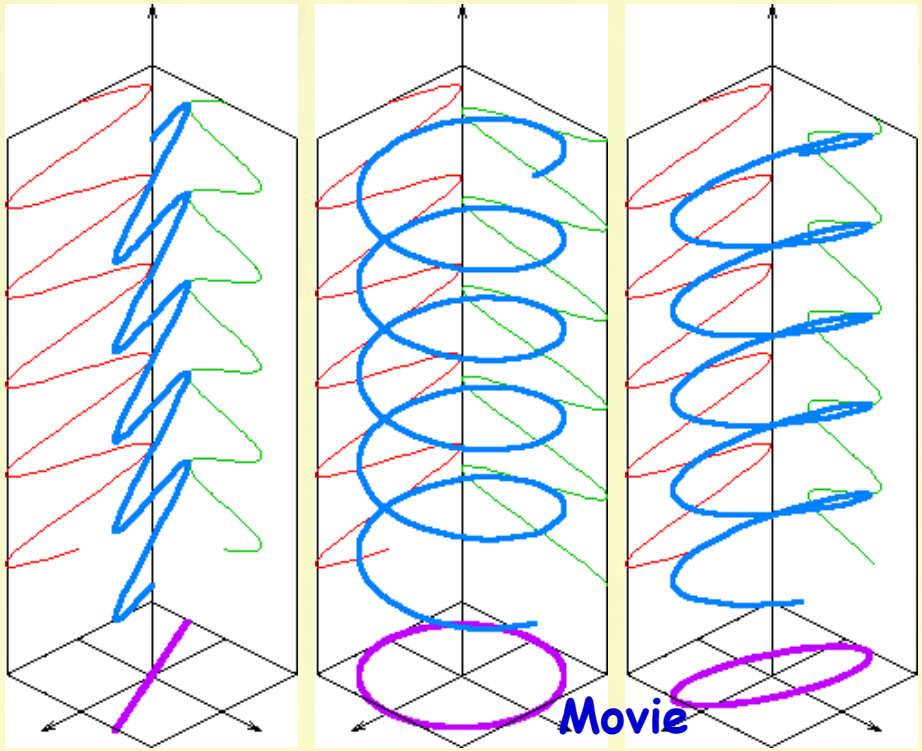
Monochromatic transverse wave:

$$\vec{E}(x,y) = \hat{a}\, E_0 \cos(2\pi(\nu t - z/\lambda) + \Phi)$$

- direction of propagation : $\vec{z}$
- wavelength : λ
- frequency : $\nu = \lambda / c$
- phase : Φ

From left to right:

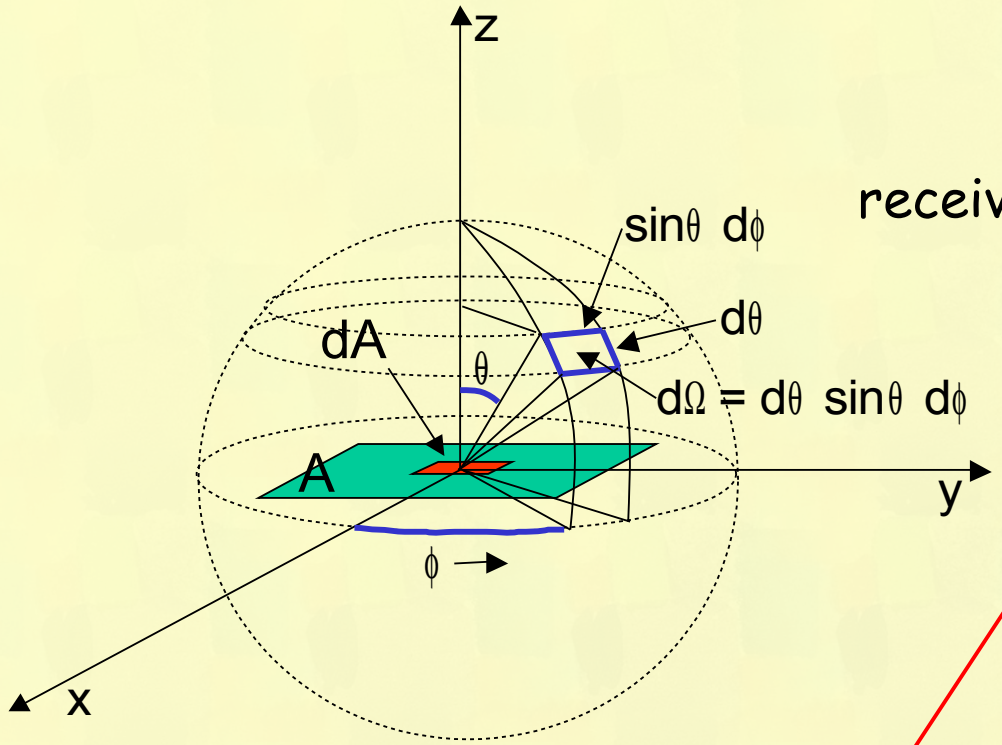
- linear polarization
- circular polarization
- elliptical polarization



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Surface brightness



Infinitesimal power *dW*
(watts)
received by an infinitesimal surface *dA* :

$$dW = B_\nu (dA \cos\theta)\, d\Omega\, d\nu$$

Angle between the normal to *dA* and the direction to *dΩ*

Infinitesimal solid angle

Infinitesimal bandwidth

Surface brightness
(watts m^{-2} Hz^{-1} sr^{-2})

= power received per unit area, per unit bandwidth, per unit solid angle

Total and specific power

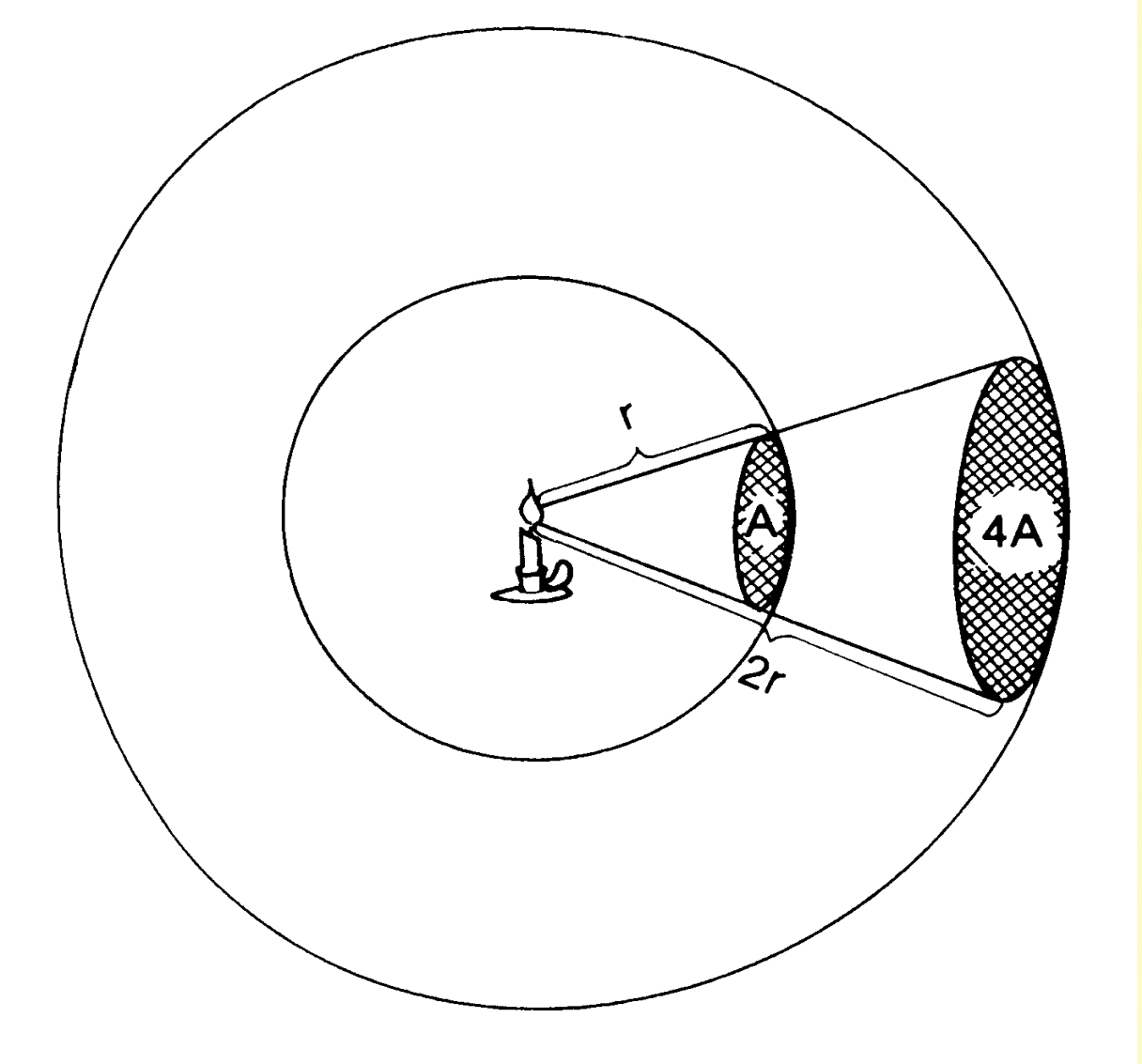
$$W = \int_{\Delta\nu} \iint_{\Omega_S} B(\nu)\, dA \cos\theta\, d\,\Omega\, d\,\nu$$

Total power received in a frequency range $\Delta\nu$

$$dw = B(\nu)\, d\,\Omega\, dA \cos\theta$$

Specific power or **power density** (power per unit bandwidth)

Flux of a source measured at Earth: it depends on its distance from us



Total and specific power & flux

$$S(\nu) = W(\nu)/4\pi d^2$$

Distance of the radio source

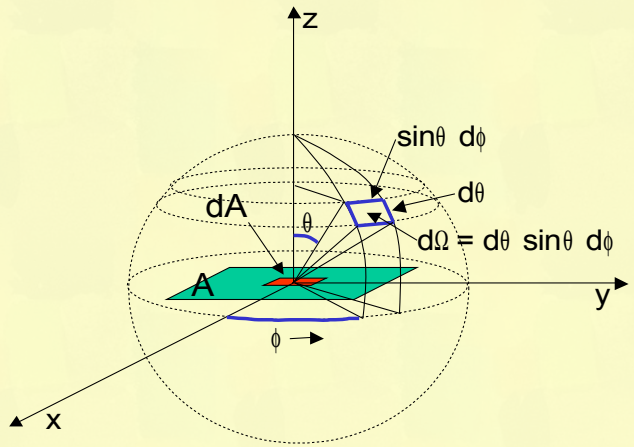
Spectral flux density or simply **flux** of a source

It is obtained by integrating the surface brightness over the total solid angle subtended by the source Ω_S:

$$S_\nu = \int_{\Omega_S} B_\nu(\theta,\phi)\, d\Omega$$

[Sν] = watts m^{-2} Hz^{-1}

1 Jy = 10^{-26} watts m^{-2} Hz^{-1} (~ flux received on Earth from a 100 watt lamp on the Moon)



Brightness temperature T_b

T_b: temperature at which a black body would have to be in order to radiate the observed brightness B_ν

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

Spectral distribution of the radiation of a black body (**Planck law**)

$$B_{RJ}(\nu, T) = \frac{2\nu^2}{c^2}kT$$

Planck law in the **Rayleigh-Jeans** regime ($h\nu \ll kT$)

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$$B_{R.}(\nu, T) = \frac{2\nu^2}{c^2}kT$$

Planck law in the **Rayleigh-Jeans** regime ($h\nu \ll kT$)

$$T_b = \frac{c^2}{2k}\frac{1}{\nu^2}B_\nu$$

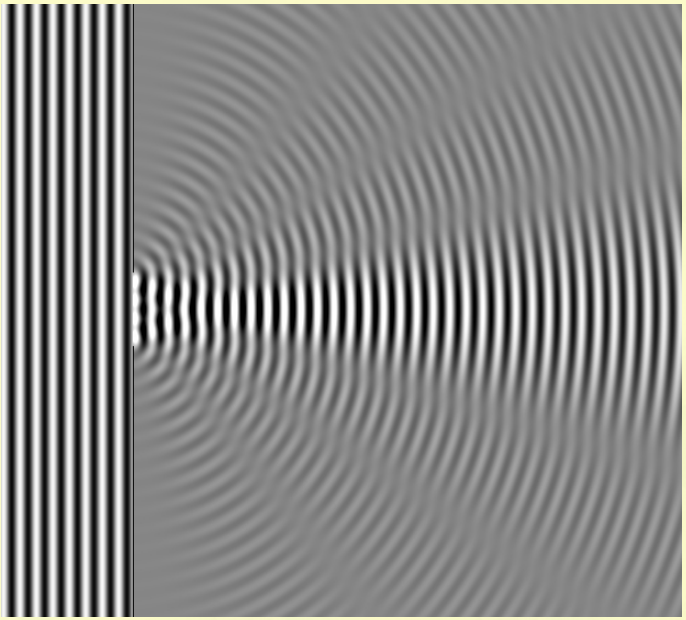
Brightness temperature

N.B.: If B_ν is emitted by a black body and $h\nu \ll kT$ it gives the thermodynamic temperature of the source. But if other processes are responsible for the emission of the radiation, T_b does not correspond to a real physical temperature!!!

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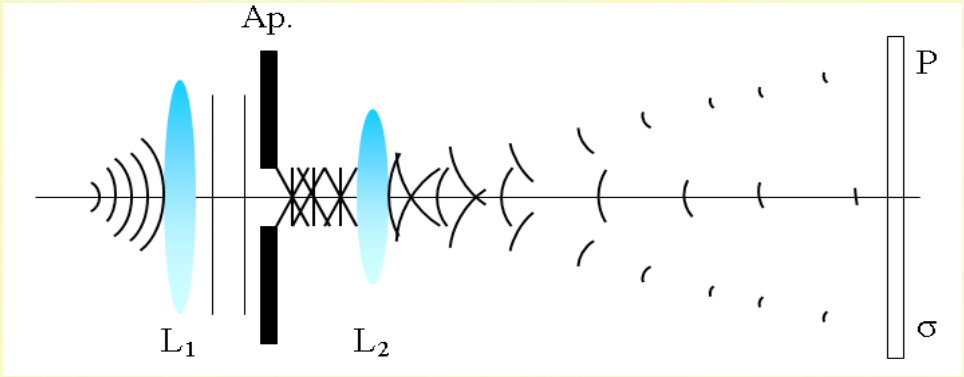
Fraunhofer diffraction



Diffraction occurs when waves pass through small openings, around obstacles, etc.

Fraunhofer diffraction:

incident and diffracted waves are planes
→ the distance from the light source to the diffracting obstacle to the observation point is large enough to neglect the curvature of incident and diffracted light
→ far field condition



Diffraction pattern in astronomical observations

The **diffraction pattern** describes how the energy of an incoming plane wave distributes on the focal plane of the instrument

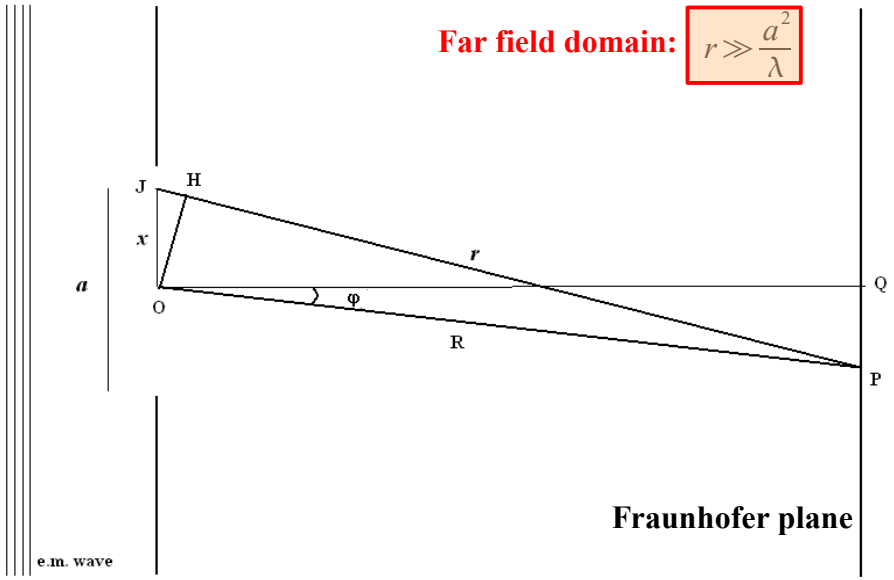
It assumes different names in the different bands of the electro-magnetic spectrum, e.g.:

Optical: Point Spread Function (PSF)

Radio: antenna pattern, or power pattern, or beam

- ➔ Determined empirically
- ➔ Determined based on the instrument properties in the approximation of Fraunhofer diffraction

Grading or aperture distribution



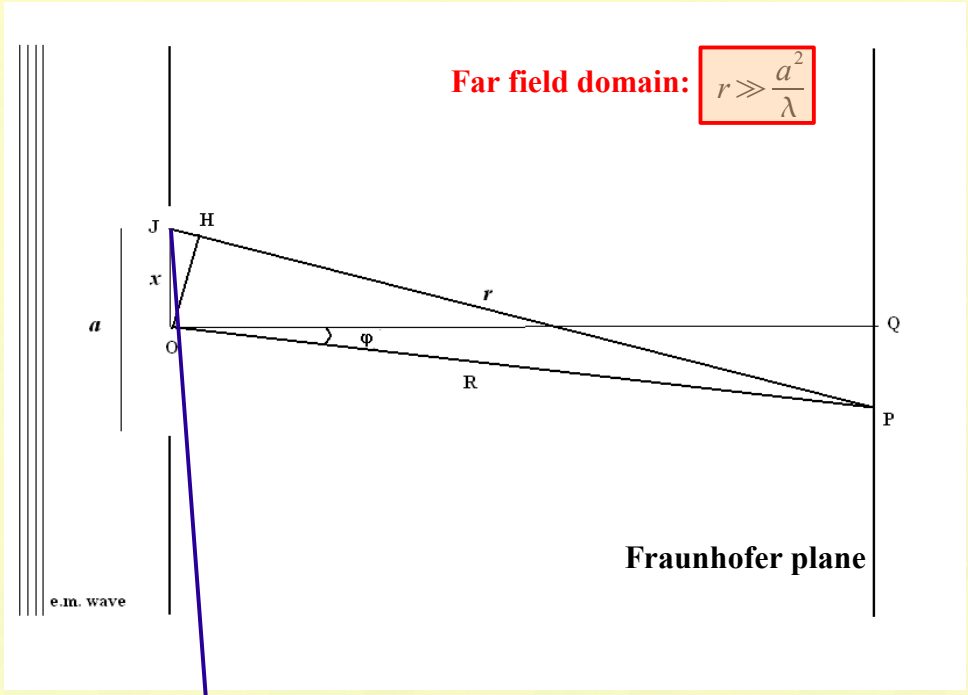
$$\overline{OP} = R \simeq \overline{HP}$$

$$\overline{JP} = r$$

$$\overline{JO} = x$$

$$\overline{JH} = x \sin \varphi$$

Grading or aperture distribution



$$\overline{OP} = R \simeq \overline{HP}$$

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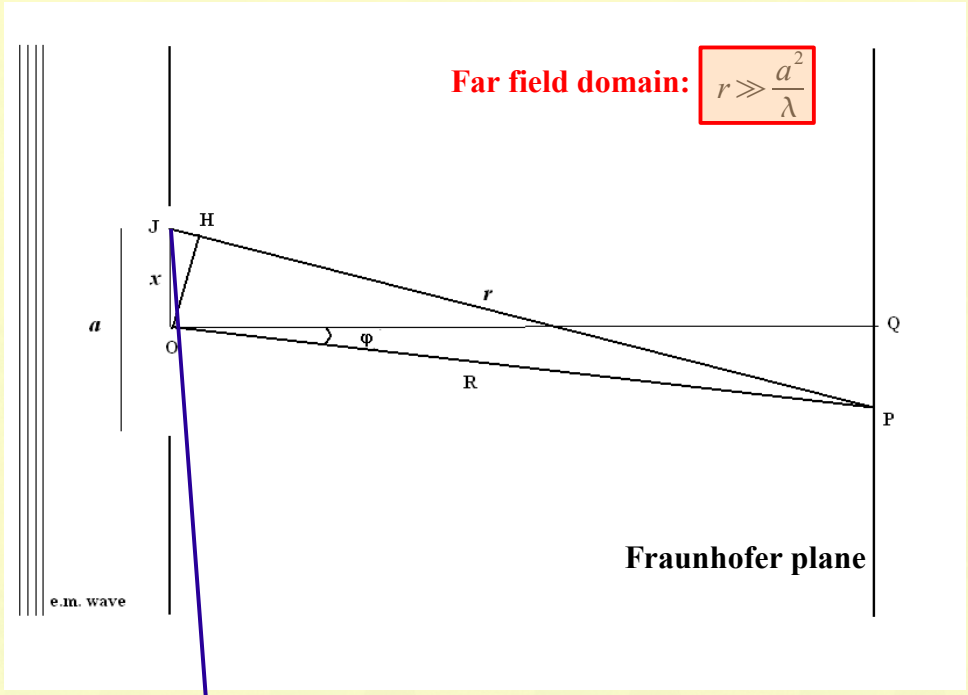
$$\overline{JH} = x \sin\varphi$$

Electric field: $F(x)\cos(\nu t - \phi(t))$

$$g(x) = F(x)\mathrm{e}^{-i\phi(t)}\mathrm{e}^{i\nu t}$$

Grading or aperture distribution

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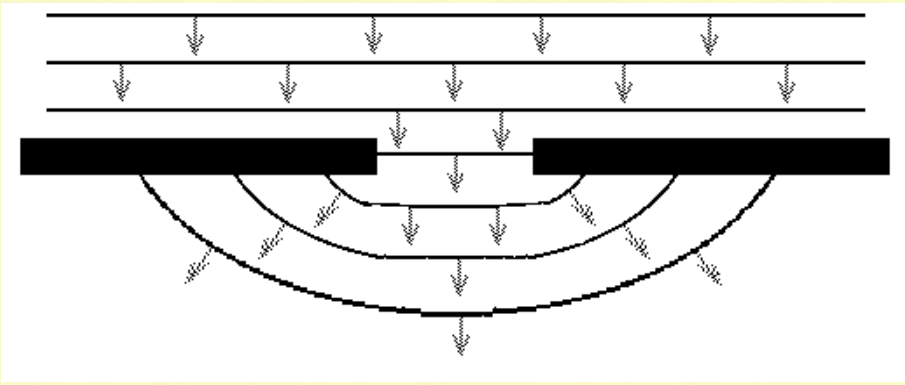
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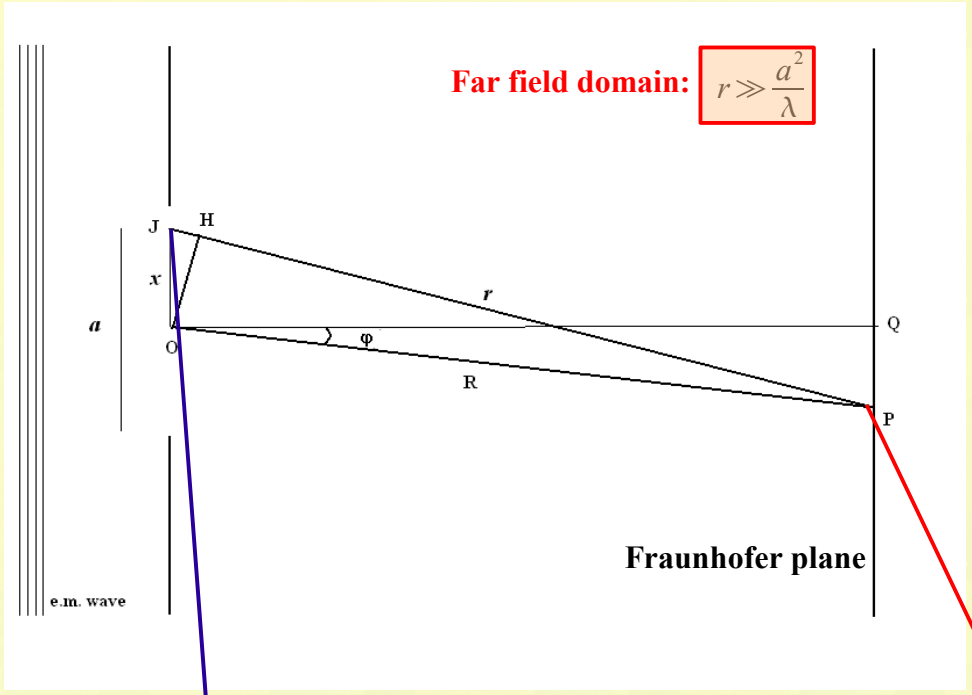
$$g(x) = F(x)\mathrm{e}^{-i\phi(t)}\mathrm{e}^{i\nu t}$$

Grading or aperture distribution

Huygens' principle:
every wavefront point on the aperture is a source of a secondary wavelet



Grading or aperture distribution



$$\overline{OP} = R \simeq \overline{HP}$$

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Electric field: $F(x)\cos(\nu t - \phi(t))$

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Grading or aperture distribution

Electric field: $dE = g(x)\mathrm{e}^{-2\pi i r/\lambda}dx$

- $r = R + x\sin\varphi \simeq R + x\varphi$

$$dE = g(x)\mathrm{e}^{-2\pi i R/\lambda}e^{-2\pi i(x/\lambda)\sin\varphi}dx$$

- Integral over the whole aperture

$$E = e^{-2\pi i R/\lambda}\int g(x)\mathrm{e}^{-2\pi i(x/\lambda)\sin\varphi}dx$$

$$\left|e^{-2\pi i R/\lambda}\right| = 1$$

= 0 out of the aperture

Field radiation pattern

$$E = e^{-2\pi i R/\lambda} \int g(x) \mathrm{e}^{-2\pi i (x/\lambda)\sin\varphi}\, dx$$

$|e^{-2\pi i R/\lambda}| = 1$

= 0 out of the aperture

$$u = x/\lambda$$
$$s = \sin\varphi$$

$$E(\varphi) = e^{-2\pi i R/\lambda} \int_{-\infty}^{+\infty} g(x)\, \mathrm{e}^{-2\pi i (x/\lambda)\sin\varphi}\, dx \propto \int_{-\infty}^{+\infty} g(u)\, \mathrm{e}^{-2\pi i u s}\, du$$

Field radiation pattern
Antenna pattern
Beam

Grading & field radiation pattern

$$g(x)=F(x)\mathrm{e}^{-i\phi(t)}\mathrm{e}^{i\nu t}$$

Grading or aperture distribution

+

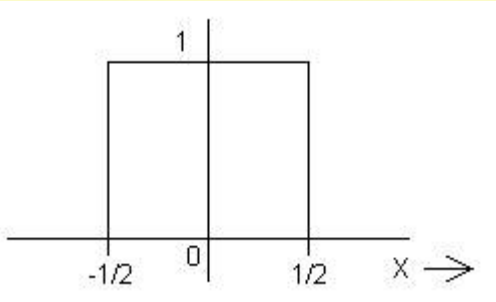
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Field radiation pattern
Antenna pattern
Beam

=

The **field radiation pattern**
(electric field in a generic point P of the image screen)
is given by the **Fourier Transform (F.T.)** of the **grading**
(electric field on the aperture)

Single slit



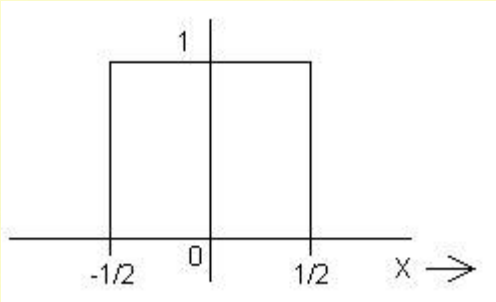
$$x(m)/\lambda(m)$$

$$g(x)\neq 0 \Leftrightarrow |x|<u/2$$

Grading

$$g(x)\propto \prod(x/u)$$

Single slit



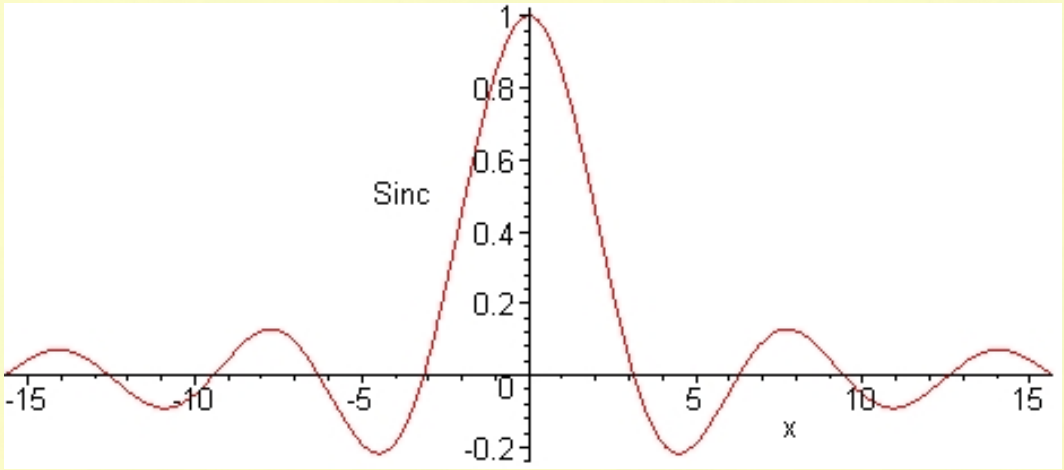
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F.T.



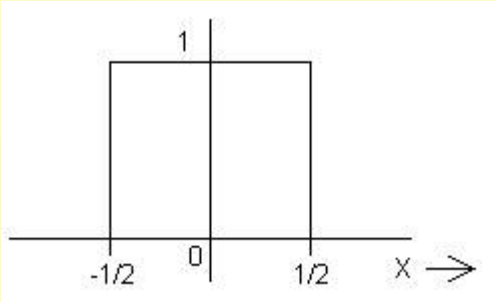
Field radiation pattern

$$E(\varphi)\propto sinc(u\sin\varphi)=\frac{\sin(\pi u\sin\varphi)}{\pi u\sin\varphi}$$

= sum of infinite sinusoids with frequencies between 0 and *u*/2

$$\psi=2\pi u\sin\varphi \rightarrow E(\varphi)\propto\frac{\sin\psi/2}{\psi/2}$$

Single slit



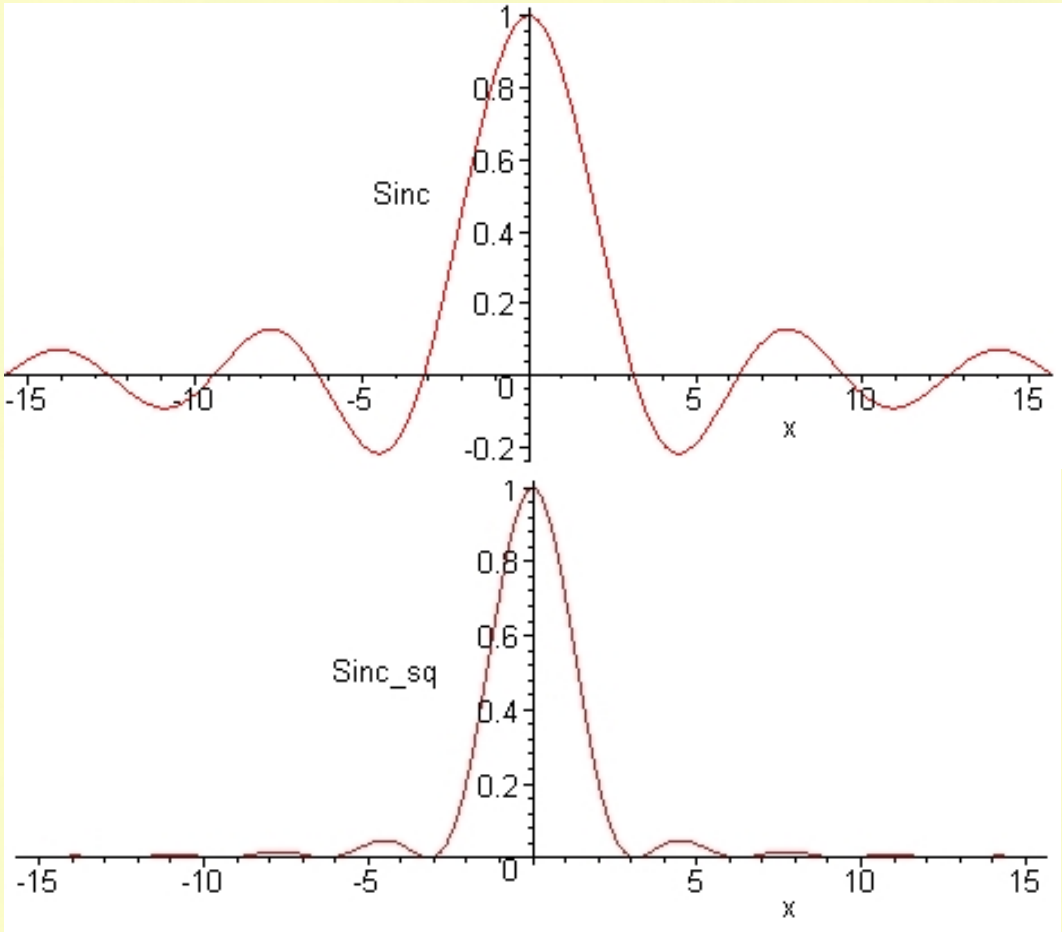
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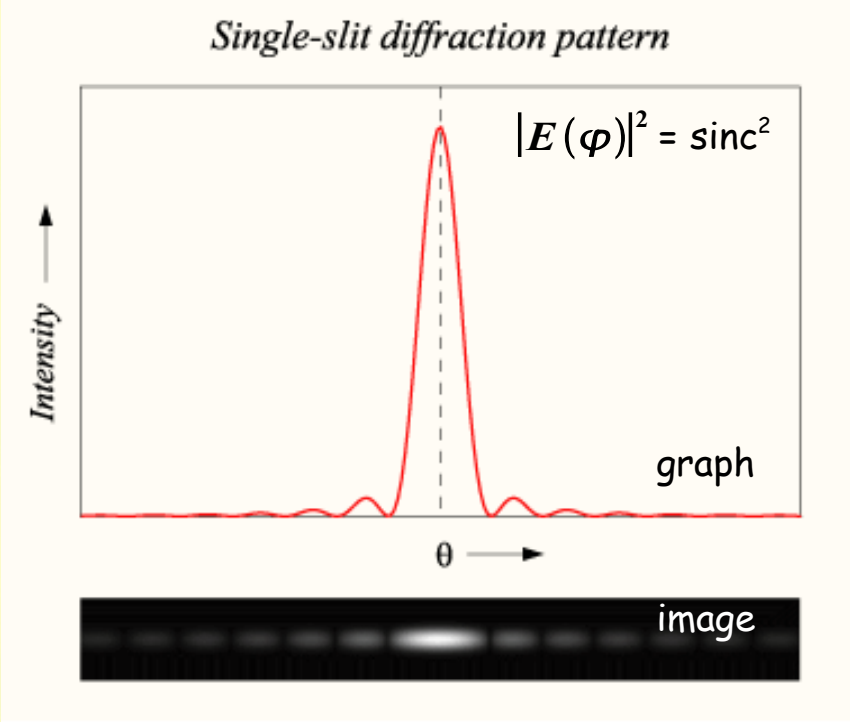
2

$$\boxed{|E(\varphi)|^2}$$

= image at infinite distance

= power measured by the receiver

Single slit diffraction pattern

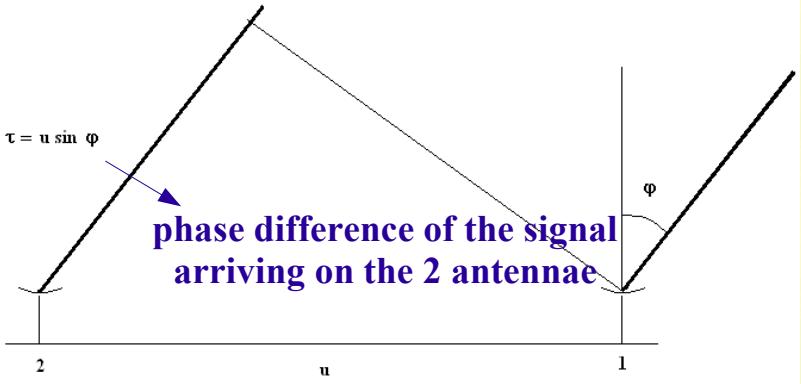


If the incoming wave is plane, such as the wave emitted by a **point source at infinite distance**, the **image** gives directly the **diffraction pattern of the instrument**

sinc²:

- Strong principal maximum
- Secondary maxima with decreasing intensity
- First zero of the electric field: $\pi u \sin\varphi = \pi \rightarrow \varphi \approx \sin\varphi = 1/u$ $\left[E(\varphi) \propto sinc(u\sin\varphi) = \frac{\sin(\pi u \sin\varphi)}{\pi u \sin\varphi}\right]$
- The width of the principal maximum therefore decreases with increasing *u* (the width of the aperture in units of wavelengths)

Double aperture



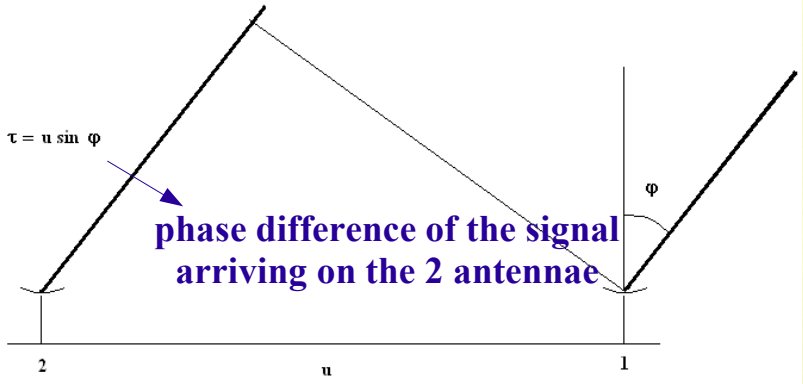
$$g(x) = g_0(x) \star [\delta(x - u/2) + \delta(x + u/2)]$$

Dirac delta function

convolution

grading of each antenna

Double aperture



$$g(x) = g_0(x) * [\delta(x - u/2) + \delta(x + u/2)]$$

Dirac delta function

convolution

grading of each antenna

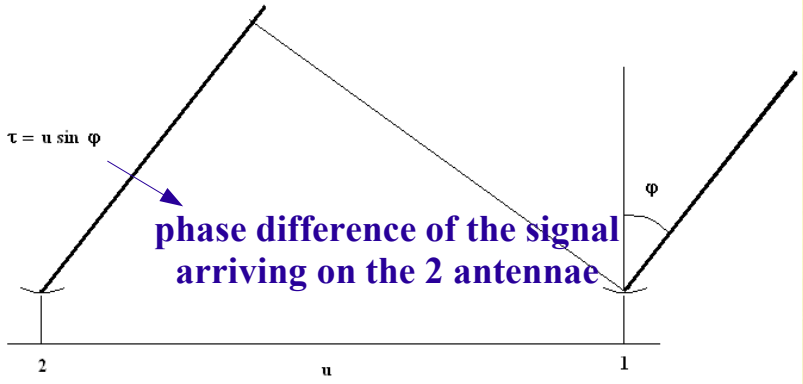
$$F.T.(f(x) * g(x)) = F(u) \times G(u)$$

$$F.T.[g_0(x)] = E_0(\varphi)$$

$$F.T.[\delta(x - u/2) + \delta(x + u/2)] = 2\cos(\pi u \sin\varphi)$$

F.T.

Double aperture



$$g(x) = g_0(x) * [\delta(x - u/2) + \delta(x + u/2)]$$

Dirac delta function

convolution

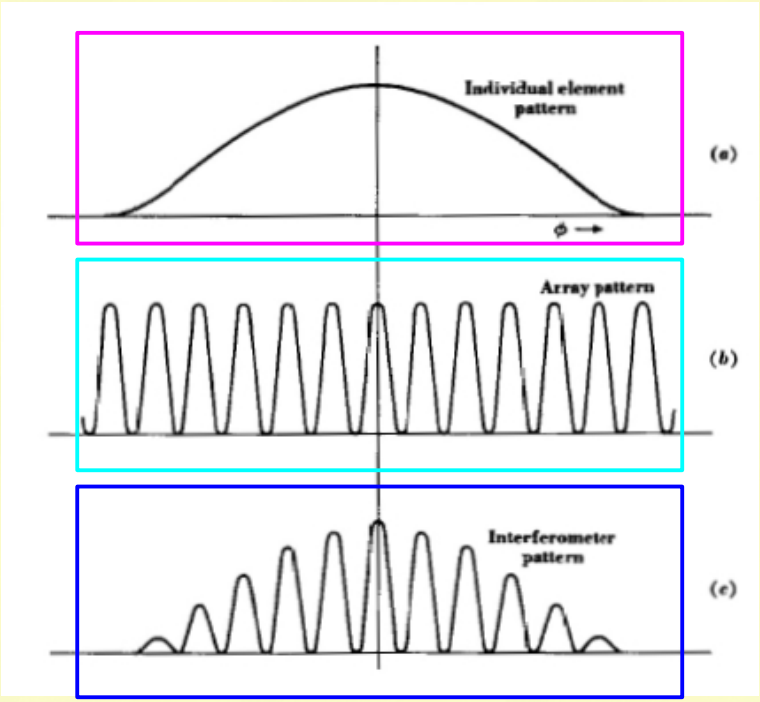
grading of each antenna

$$F.T.(f(x) * g(x)) = F(u) \times G(u)$$

$$F.T.[g_0(x)] = E_0(\varphi)$$

$$F.T.[\delta(x - u/2) + \delta(x + u/2)] = 2\cos(\pi u \sin\varphi)$$

F.T.

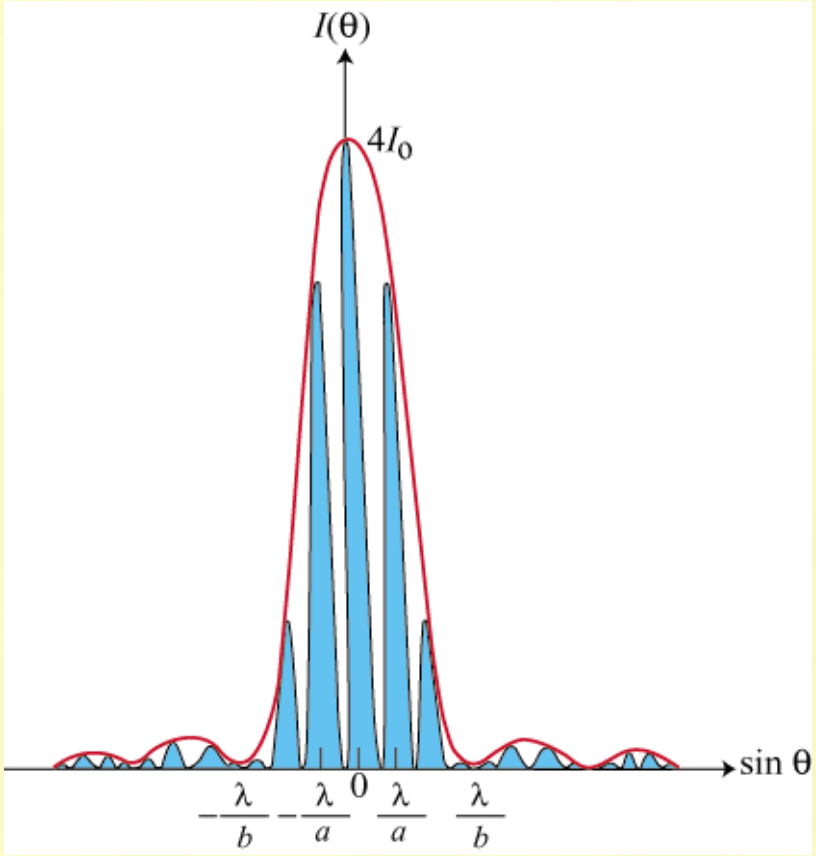


$$E(\varphi) = 2E_0(\varphi)\cos(\pi u \sin\varphi) = 2E_0(\varphi)\cos\psi/2$$

= cosinusoid characterized by a single spatial frequency ($u/2$ in units of λ)

$$E(\varphi)^2 = 4E_0(\varphi)^2\cos^2(\pi u \sin\varphi)$$

Double aperture diffraction pattern

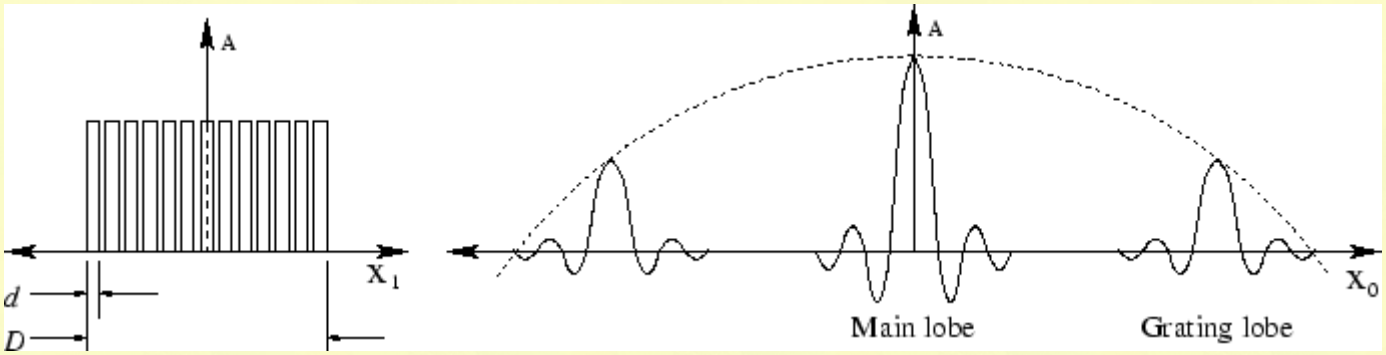


$$E(\varphi)^2 = 4\,E_0(\varphi)^2 \cos^2(\pi u \sin\varphi)$$

In such a case, the **rapid oscillations** of the diffraction pattern are due to the **interference coming from the two antennae**, while the **modulation** is due to the **diffraction pattern of each single antenna**

- First zero of the electric field: $\pi u \sin\varphi = \pi/2 \rightarrow \varphi \approx \sin\varphi = 1/2u$
- Zero's for: $\pi u \sin\varphi = (2k+1)\pi/2, k = 0,1,2,\ldots$
- Maxima for: $[E(\varphi) = 2E_0(\varphi)\cos(\pi u \sin\varphi)]$ $\pi u \sin\varphi = k\pi$, $k = 0,1,2,\ldots$
- Distance between two successive maxima: $\Delta(\sin\varphi) = 1/u$

Multiple interferometer with *N* antennae (grating)



In a similar way we can show that, in the case of a **multiple interferometer made up by N equidistant antennae** the **field radiation pattern** is given by (d being the distance in units of λ):

$$E(\varphi)=E_0(\varphi)\frac{\sin N\psi/2}{\sin \psi/2}$$

$$\psi=2\pi d\sin\varphi$$

$E_0(\varphi)$= radiation pattern of each single antenna

- First zero of the electric field: $N\psi/2=\pi\rightarrow N\frac{2\pi d\sin\varphi}{2}=\pi\rightarrow\varphi\sim\sin\varphi=\frac{1}{Nd}$
- Distance between two successive maxima: $1/d\cos\bar{\varphi}$

Resolving power or angular resolution



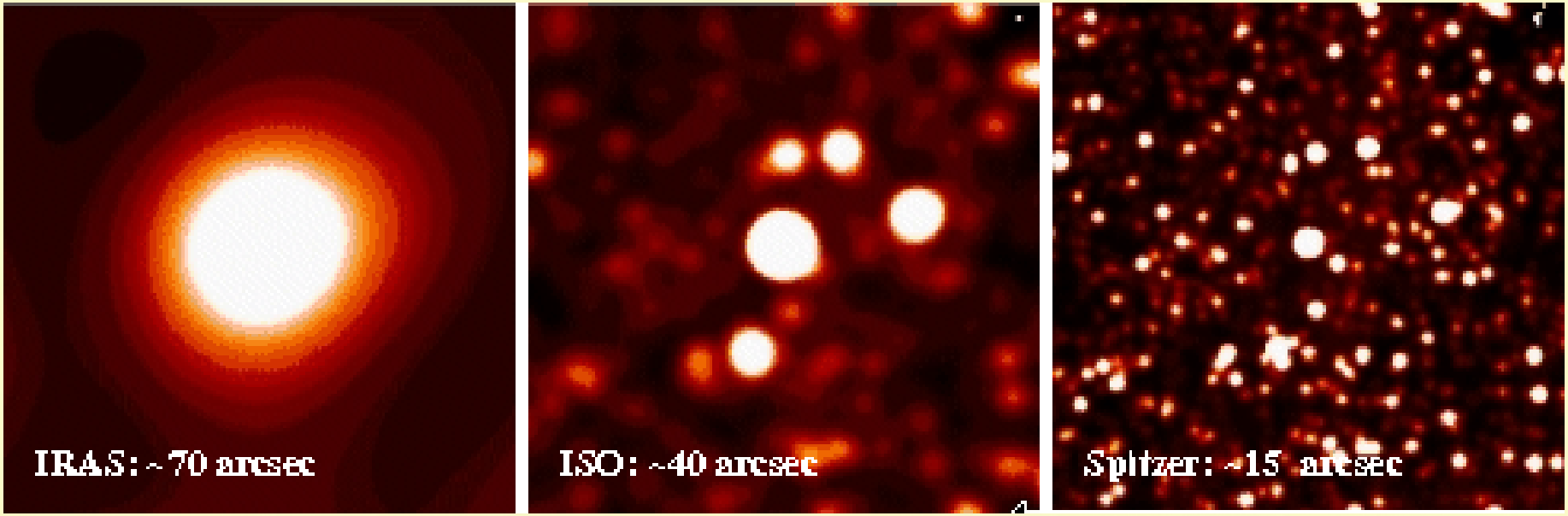
The spiral galaxy M31 observed in optical



Instrument's angular resolution:

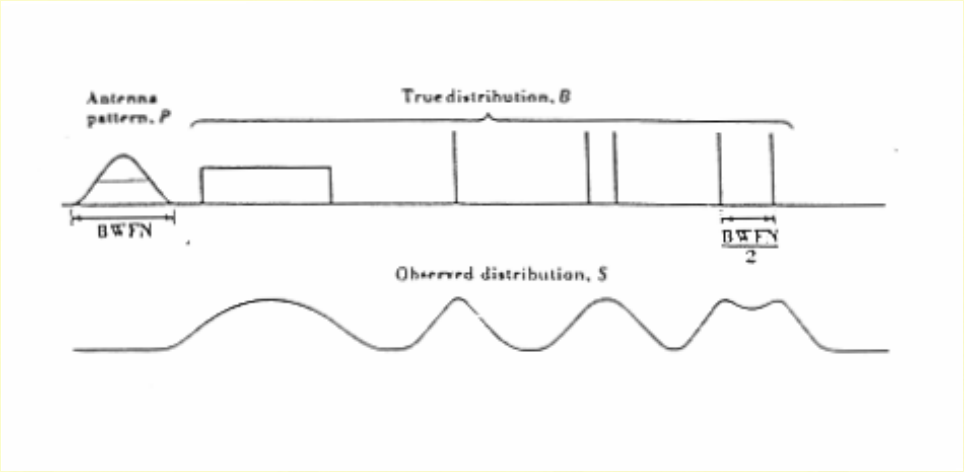
how well it can distinguish between two objects in space which are separated by a small angular distance - it is the telescope ability to resolve fine detail

Resolving power or angular resolution



Influence of angular resolution on far-infrared (FIR – 160 μm) observations of the same region of the sky

Observation is convolution of source by beam



Smoothed distribution S observed with an antenna pattern P

Given the sky brightness B(φ) , and the diffraction pattern of the system P(φ)=|E(φ)|^2, we can write the observed brightness at a position φ_0:

$$B_0(\varphi_0)=\int_{-\infty}^{+\infty} B(\varphi)\,P(\varphi_0,\varphi)\,d\varphi$$

If the diffraction pattern is space independent:

$$B_0(\varphi_0)=\int_{-\infty}^{+\infty} B(\varphi)\,P(\varphi_0-\varphi)\,d\varphi$$

Angular resolution: a comparison

Resolving power of an instrument characterized by:

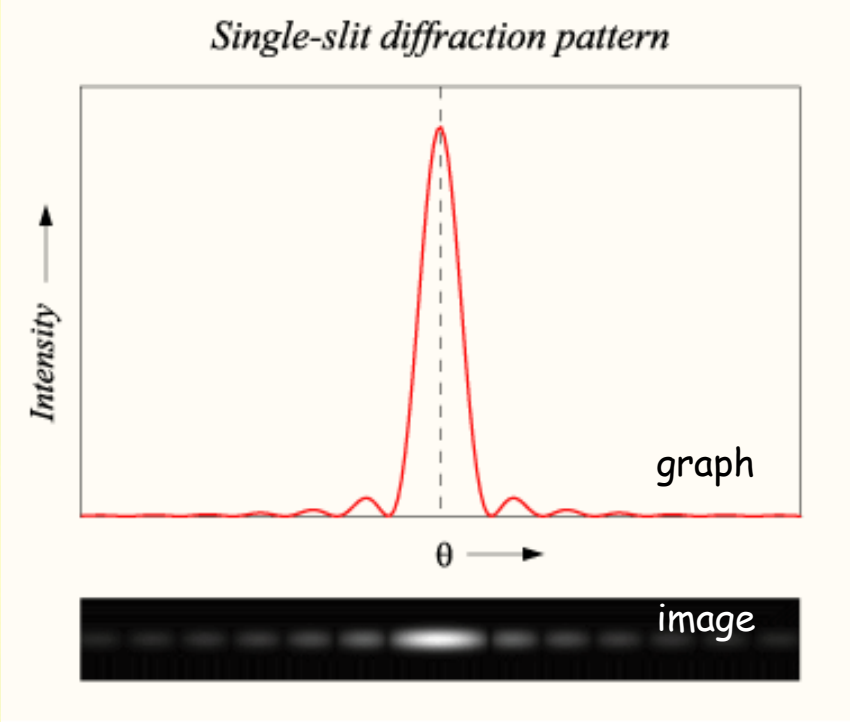
- full width at half maximum (FWHM)

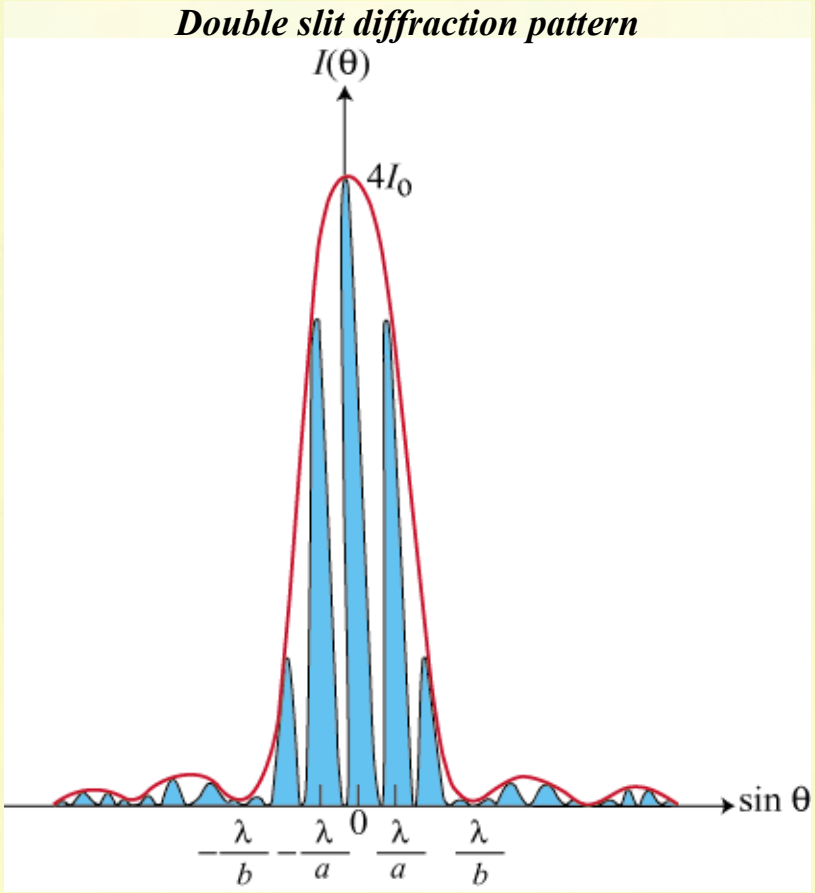
- distance between the first two zeros

The latter have been estimated in the case of the three kind of instruments considered above (*u* being the total size of each instrument):

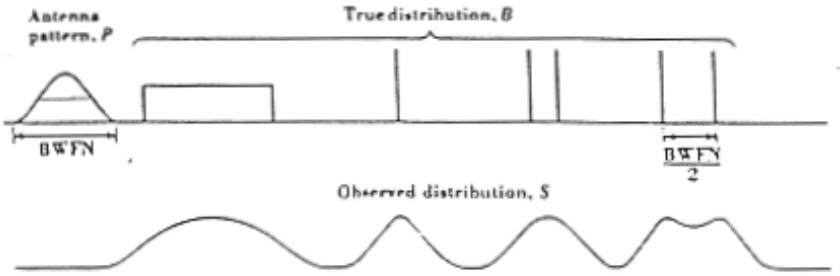
- Single aperture: $2/u$
- Two antennae interferometer: $1/u$
- N antennae interferometer: $2/Nd = 2/u$

→ for a given value of u, the **simple interferometer** (2 antennae) has a resolving power that is twice better than the other two instrument configurations


Single-slit diffraction pattern
Intensity
graph
θ
image


Double slit diffraction pattern
I(θ)
4I0
sin θ
λ/b
λ/a
0
λ/a
λ/b

Quality of an observed image: what else?



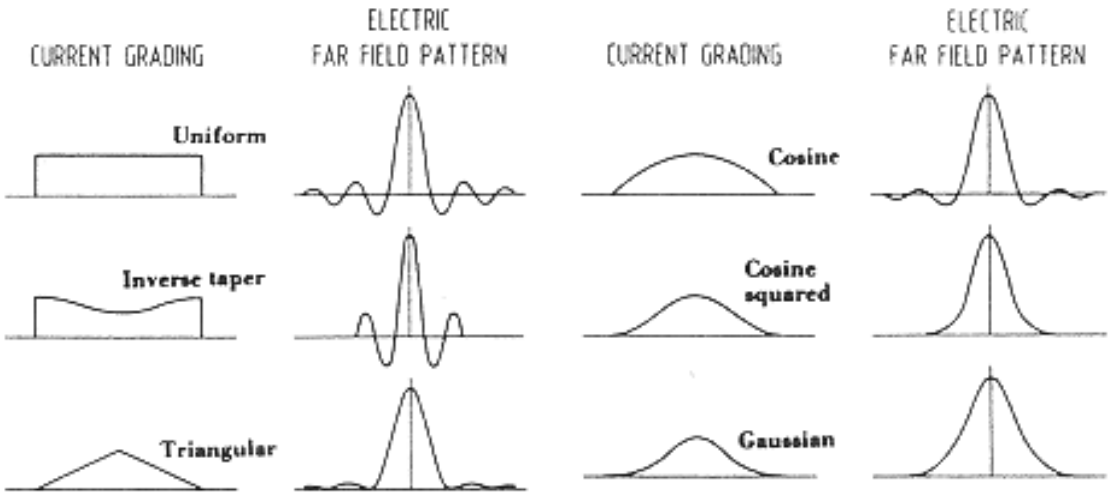
Smoothed distribution S observed with an antenna pattern P

The **quality of an image** depends also on the **diffraction image of the telescope**:

high grating lobes can significantly compromise the observations and all the chain of operations to treat them

This is particularly true in the case of observations of extended radio sources: if high grating lobes are present and the source extends beyonds the first of them, the radiation received by the grating lobes will be combined with that from the main lobe, and the resulting image will be a greatly distorted version of the true image

Distribution and weight of spatial frequencies



Pairs of current gradings and corresponding electric far field pattern.

- Softest gradings → worst resolving power & lower grating lobes (if any!)
- Taper (grading with a lower amplitude at the borders than at the center) → lower grating lobes & loss in resolution
- Inverse taper → higher grating lobes & increase in the angular resolution

→ the resolving power of an instrument does not depend only on its size, but also on the distribution of the spatial frequencies covered by the telescope and on their relative weight

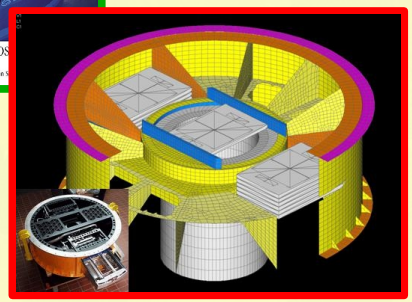
Lecture Outline

- Why radioastronomy?
- Electromagnetic waves
- Some basic definitions: brightness & flux density
- Fraunhofer diffraction and its applications
- **The radio telescope: hints to its basic properties**
- The need of resolution: interferometry
- Data reduction: imaging & deconvolution

Telescopes

Telescopes
- Sensor
- Receiver





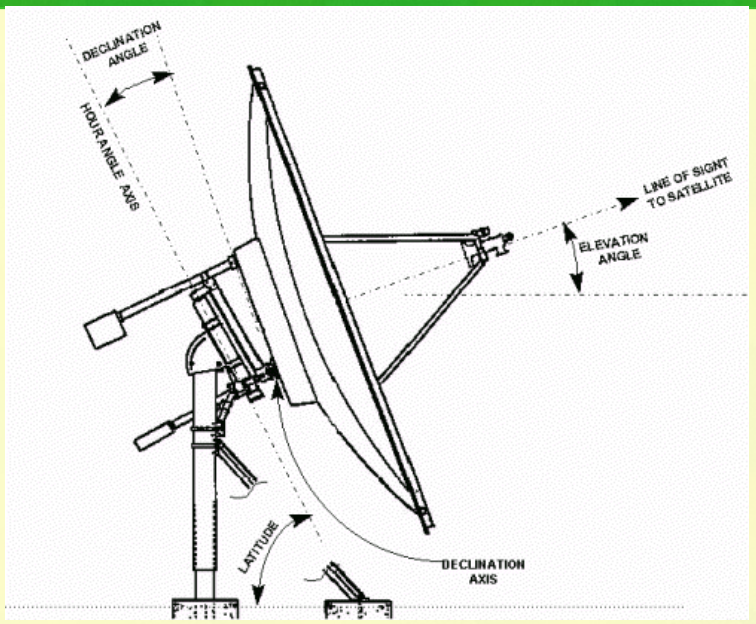
Optical telescope
- Mirror
- Phot.plate or CCD



Radio telescope
- Antenna
- Feed



Antenna's mounts

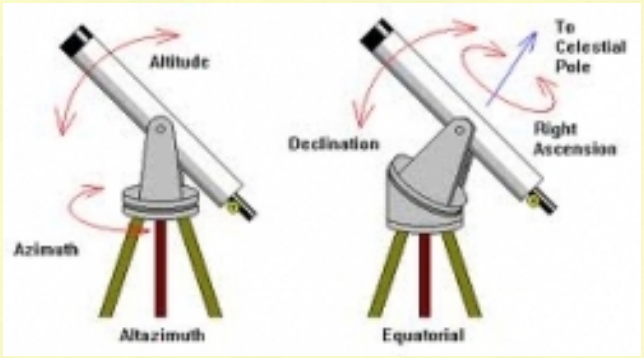


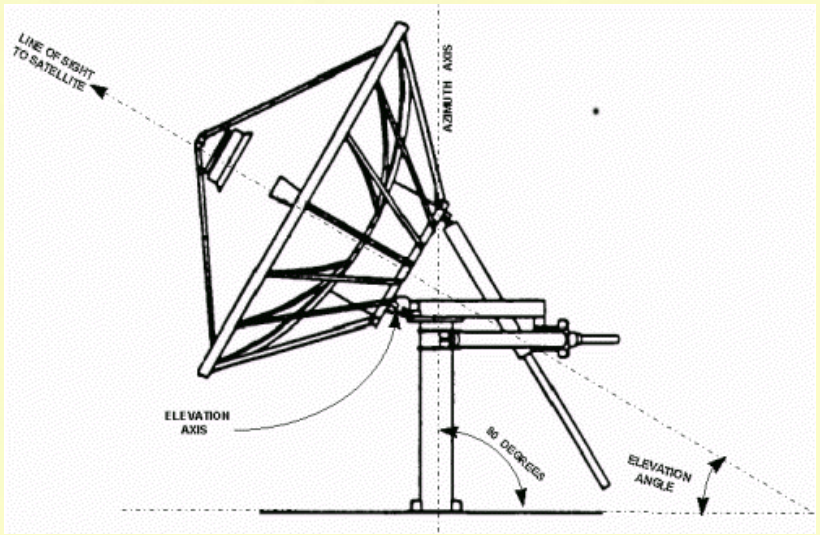
Equatorial or polar mount:

- axis parallel to the Earth's rotation (polar)
- axis orthogonal to the Earth's rotation (declination)

Antenna:

device that transform incoming e.m. waves into electric tension





Altitude azimuth mount:

- horizontal axis (elevation axis)
- vertical axis (azimuth axis)

Main properties of an antenna

A radio telescope is characterized by its **diffraction pattern**, or **antenna diagram**, or **power pattern** or **beam**

The presence of a beam means that an antenna receives different amounts of power in different directions

$$P_n(\theta,\varphi)=\frac{P(\theta,\varphi)}{P_{max}(\theta,\varphi)}$$ Normalized power pattern

$$\Omega_A=\iint_{4\pi} P_n(\theta,\varphi)\,d\Omega$$ Beam solid angle

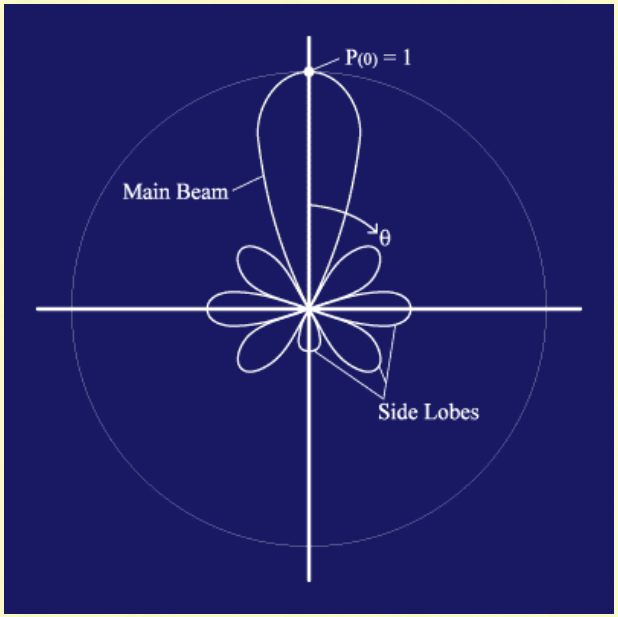
$$\Omega_{MB}=\iint_{main-lobe} P_n(\theta,\varphi)\,d\Omega$$ Main beam solid angle

$$\eta_B=\frac{\Omega_{MB}}{\Omega_A}$$ (Main) beam efficiency

$$D=\frac{4\pi}{\Omega_A}$$ Directivity

$$A_e=\frac{w(\theta,\varphi)}{S}$$ Effective aperture

$$\eta_A=\frac{A_e}{A_g}$$ Aperture efficiency



Noise temperature T_N

The signal from radio sources is very low.
Receiver noise must be kept as low as possible and also constant, so that it can be treated as a constant background and subtracted

The receiver noise is mainly due to thermal motions within the electronic circuitry and can always be expressed in terms of the **thermal noise power from a resistor at some temperature**.

$$P_N = kT_N$$ Nyquist formula

= thermal noise power per unit frequency from a resistor at temperature T_N

Antenna temperature T_A

Antenna receiving e.m. radiation from a source characterized by a (monochromatic) brightness distribution $B_\nu(\theta,\varphi)$:

$$w = m A_e \iint B_\nu(\theta,\varphi) P_n(\theta,\varphi) d\Omega$$

Total power per unit bandwidth

Incoming radiation is ~ not polarized & antennae can receive only one component of polarization $\rightarrow m=1/2$

- $P = kT$ Nyquist formula

$$w = k T_A$$

This definition of antenna temperature relates the output of the antenna to the power of a matched resistor. When these two power levels are equal, then the antenna temperature is given by the temperature of the resistor.

Antenna and brightness temperatures

Antenna receiving e.m. radiation from a source characterized by a (monochromatic) brightness distribution $B_\nu(\theta,\varphi)$:

$$w = m\,A_e \iint B_\nu(\theta,\varphi)\,P_n(\theta,\varphi)\,d\Omega$$

+ $w = k\,T_A$

+ $A_e\,\Omega_A = \lambda^2$

= $$T_A(\theta,\varphi) = \frac{w}{k} = m\frac{A_e}{k}\iint B_\nu(\theta,\varphi)\,P_n(\theta,\varphi)\,d\Omega = m\frac{\lambda^2}{k\,\Omega_A}\iint B_\nu(\theta,\varphi)\,P_n(\theta,\varphi)\,d\Omega$$

+ $$T_b = \frac{c^2}{2k}\frac{1}{\nu^2}B_\nu = \frac{\lambda^2}{2k}B_\nu \quad \& \quad \Omega_A = \iint_{4\pi} P_n(\theta,\varphi)\,d\Omega$$

= $$T_A(\theta,\varphi) = \frac{2m\iint T_b(\theta,\varphi)\,P_n(\theta,\varphi)\,d\Omega}{\iint P_n(\theta,\varphi)\,d\Omega}$$

Antenna and brightness temperatures

$$T_A(\theta,\varphi)=\frac{2m\iint T_b(\theta,\varphi)\,P_n(\theta,\varphi)\,d\Omega}{\iint P_n(\theta,\varphi)\,d\Omega}$$

Since based on their definitions, T_A is the quantity that we measure, while T_b is the one desired, this equations must be inverted . It is an integral equation of the first kind, which in theory can be solved if $T_A(\theta,\varphi)$ and $P_n(\theta,\varphi)$ are known. In practice this inversion is possible only approximately. As we will see in more details in the following, usually T_A and P_n known only for a limited range of θ and φ, and the measured data are affected by errors. Therefore an approximated deconvolution of previous equation is only possible

Antenna gain

$$\eta_A = \frac{A_e}{A_g}$$ Aperture efficiency

- $w = m\, A_e \iint B_\nu(\theta,\varphi) P_n(\theta,\varphi)\, d\Omega$
- $w = k\, T_A$

$$S_\nu = \iint_{source} B_\nu(\theta,\varphi) P_n(\theta,\varphi)\, d\Omega$$

$$S = \frac{k\, T_A}{m\, A_e} \propto \frac{k\, T_A}{\eta_A A_G} = \frac{T_A}{G}$$

$$G = \frac{\eta_A A_G}{k}$$ = antenna gain (°K / Jy)

As the effective aperture, the antenna gain measures the capability of an antenna to collect the flux. For the same value of the source flux, the higher is the antenna gain, the higher is the antenna temperature, making the signal detection easier

System temperature

Receivers are affected by different sources of system noise:

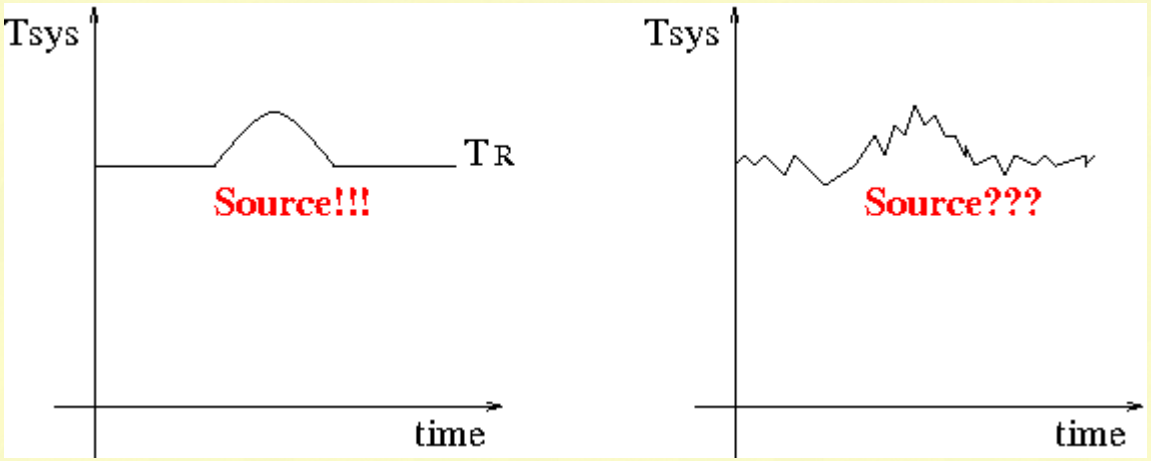
- Noise of the receiver itself
- Atmosphere
- Spillover noise of the antenna
- Noise produced in guides and cables

$$T_{sys} = \sum T_i$$

= system temperature

$$T_{tot} = T_A + T_{sys} \quad \text{where:} \quad T_A \ll T_{sys} \rightarrow \quad T_{tot} \approx T_{sys}$$

How is it possible to make radio astronomical observations???



The flux to be observed has to be *n* times (*n* = 3 or 5) higher than the noise associated to $T_{tot} \approx T_{sys}$

!!!

Noise of an instrument

If we indicate with Δν the observed bandwidth, from Fourier Transform theory we know that the time resolution Δt for independent samples is given by ΔνΔt =1. This means that two samples taken at time interval less than 1/Δν are not independent. An observation lasting a total time τ contains thus N=τ/Δt=τΔν independent samples . Gaussian statistics shows that the total error for N samples is $N^{-1/2}$ of that of a single sample

αT_{sys} with $\alpha \simeq 1$ error on the single sample

→ $\Delta T_{r.m.s.} = \frac{\alpha T_{sys}}{\sqrt{\Delta \nu \tau}}$ mean quadratic error associated to the total observation

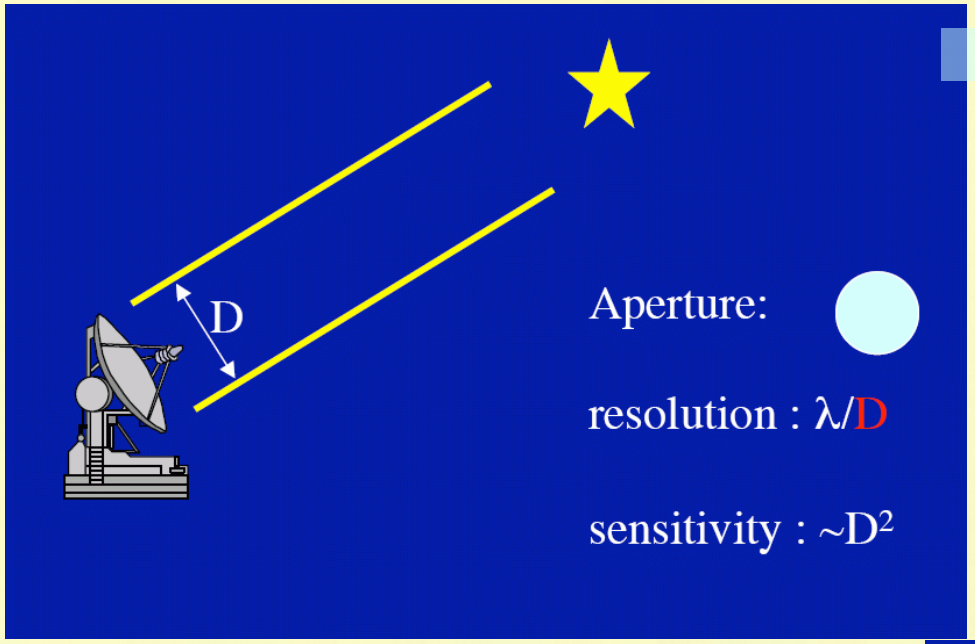
✚ $S = \frac{k T_A}{m A_e} \propto \frac{k T_A}{\eta_A A_G} = \frac{T_A}{G}$

$\Delta S_{r.m.s.} = \frac{\Delta T_{r.m.s.}}{G} = \frac{\alpha (T_{As} + T_R)}{G\sqrt{\Delta \nu \tau}} \approx \frac{\alpha T_R}{G\sqrt{\Delta \nu \tau}}$ → $S_{source} > 5 \times \Delta S_{r.m.s.} \approx \frac{5 \alpha T_R}{G\sqrt{\Delta \nu \tau}}$

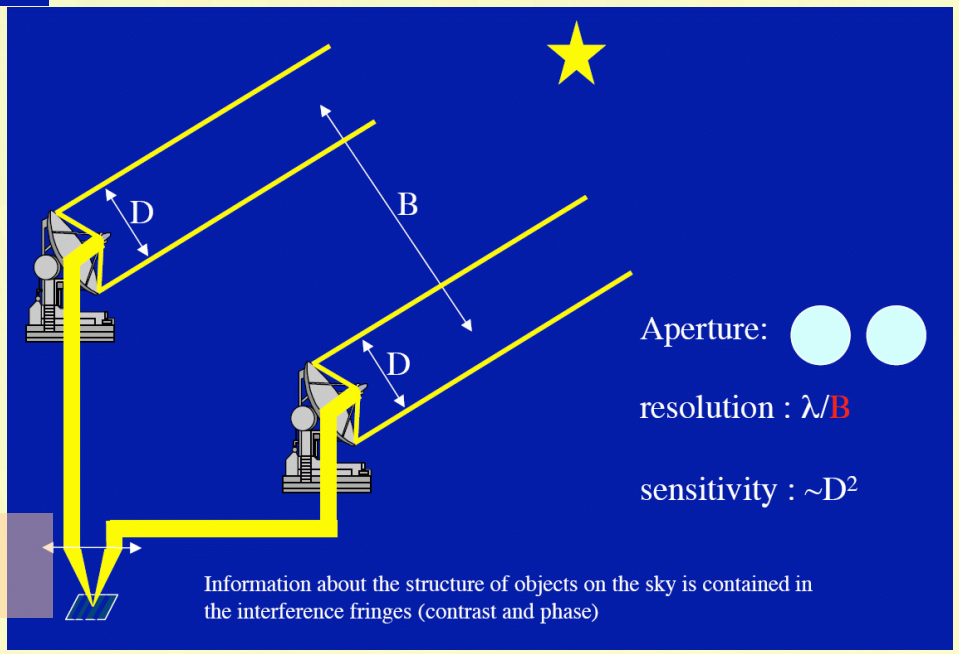
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Sensitivity & resolution

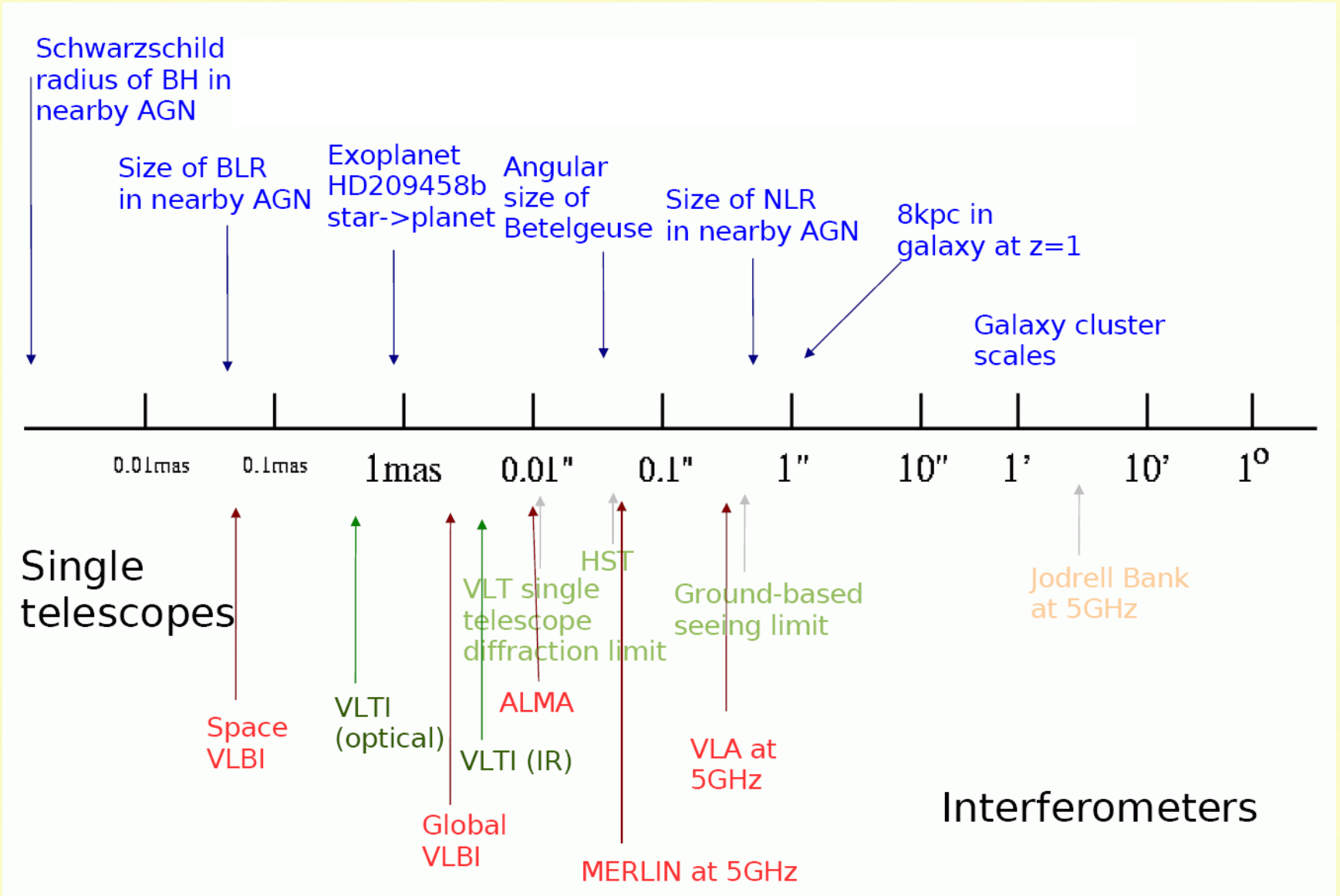


Sensitivity & resolution of a single antenna



Sensitivity & resolution of a simple interferometer (2 antennae)

The need for resolution



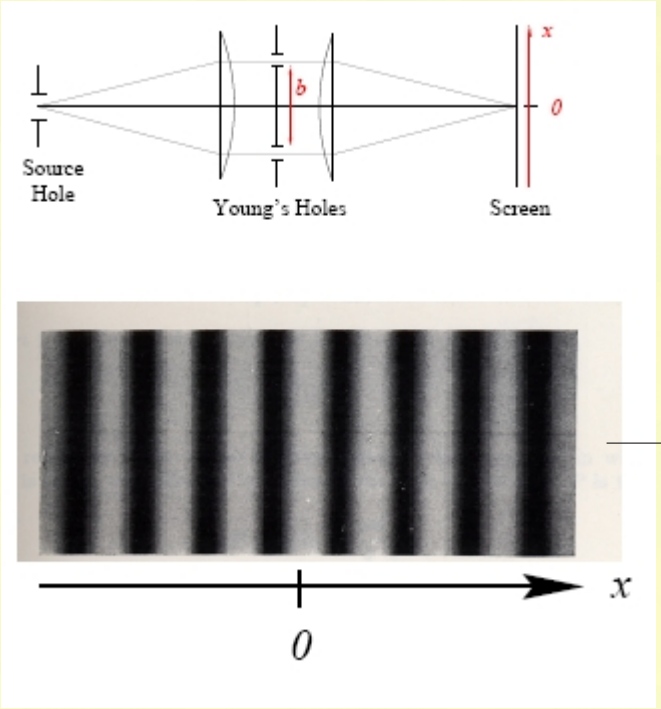
Visibility

Output of the double interferometer formed by correlating the voltages received by the two antennae → **two independent numbers** (cosine and sine outputs) = real and imaginary components of a quantity called **complex visibility** or just **visibility V**

$$V = R_C - i\,R_S = A\,e^{-i\Phi}$$

$$A = \sqrt{R_C^2 + R_S^2}$$

$$\Phi = \arctan\left(\frac{R_S}{R_C}\right)$$



fringe visibility

The power received from a source offset by an angle Θ(θ,φ) from the pointing direction not its "true" brightness, but it is reduced by the beam:

$$I'(\Theta) = A(\Theta) I(\Theta)$$

modified sky brightness distribution

beam

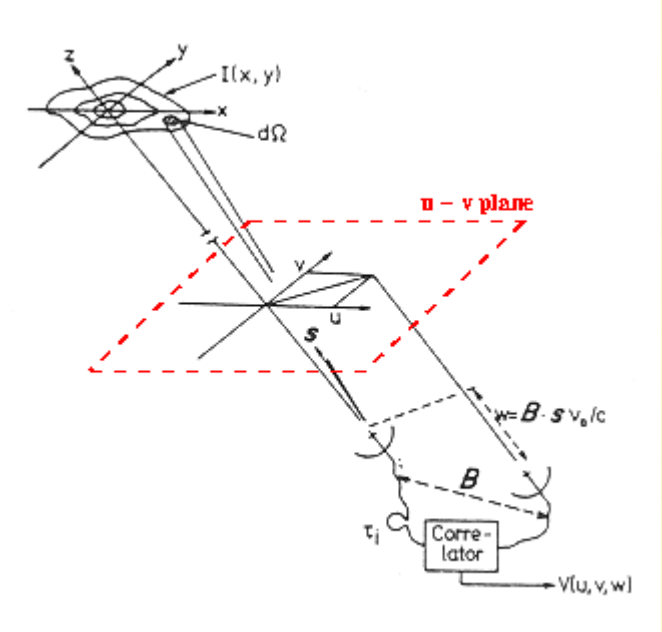
"true" brightness distribution

van Citter-Zernike theorem:

the visibility V is the F.T. of the modified sky brightness distribution I'(Θ)

$$V = F.T.[I'(\Theta)] = F.T.[A(\Theta) I(\Theta)]$$

uv-plane



uv-plane:
plane perpendicular to a reference direction that is the telescope pointing center

Projected baseline:
distance between the two antennae (in units of wavelengths) in the uv-plane

Baselines in the uv-plane: coordinates (u,v) - u towards East & v towards North
Corresponding coordinates in the sky: directional cosines (x,y) relative to the u,v axes

$$V = F.T.[I'(\Theta)] = F.T.[A(\Theta) I(\Theta)]$$

$$V(u,v) = \int_{-\infty}^{+\infty}\int A(x,y) I(x,y) e^{-2\pi i(ux+vy)} dxdy$$

$$I'(x,y) = A(x,y) I(x,y) = \int_{-\infty}^{+\infty}\int V(u,v) e^{2\pi i(ux+vy)} dudv$$

Sampling function & uv-coverage

$$V(u,v) = \iint_{-\infty}^{+\infty} A(x,y)\, I(x,y)\, e^{-2\pi i(ux+vy)}\, dxdy$$

$$I'(x,y) = A(x,y)\, I(x,y) = \iint_{-\infty}^{+\infty} V(u,v)\, e^{2\pi i(ux+vy)}\, dudv$$

They are true only for a continuous sampling of the uv-plane, such that V is measured for **all** values of (u,v) !!!

two antennae → single spatial frequency

→ **V** from a pair of antennae = **one Fourier component** of the modified sky I'(x,y)

↓ **Sampling function**

S(u,v) = 1 at (u,v) points where visibilities are measured
S(u,v) = 0 elsewhere

uv-coverage = pattern of all measured (u,v) points

Dirty beam & dirty map

Limited uv-coverage

+

$$V = F.T.[I'(\Theta)] = F.T.[A(\Theta) I(\Theta)]$$

=

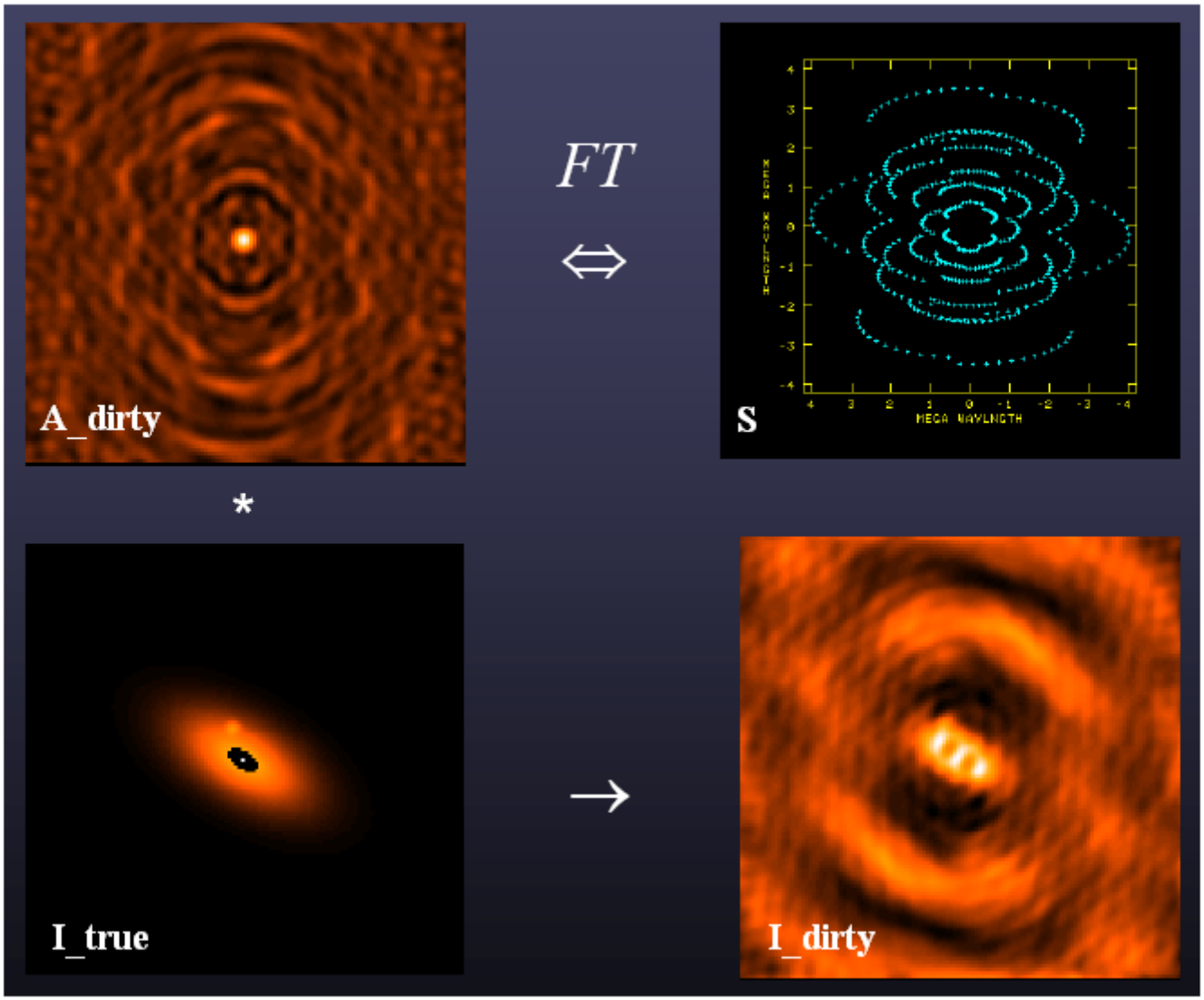
$$V \times S = F.T.[I' * A_{dirty}] = F.T.[I_{dirty}]$$

Dirty beam: $S = F.T.(A_{dirty})$

Dirty map

What we measure: the dirty map

$$V \times S = F.T.[I' * A_{dirty}] = F.T.[I_{dirty}]$$



Dirty beam & dirty map

Limited uv-coverage

+

$$V = F.T.[I'(\Theta)] = F.T.[A(\Theta) I(\Theta)]$$

=

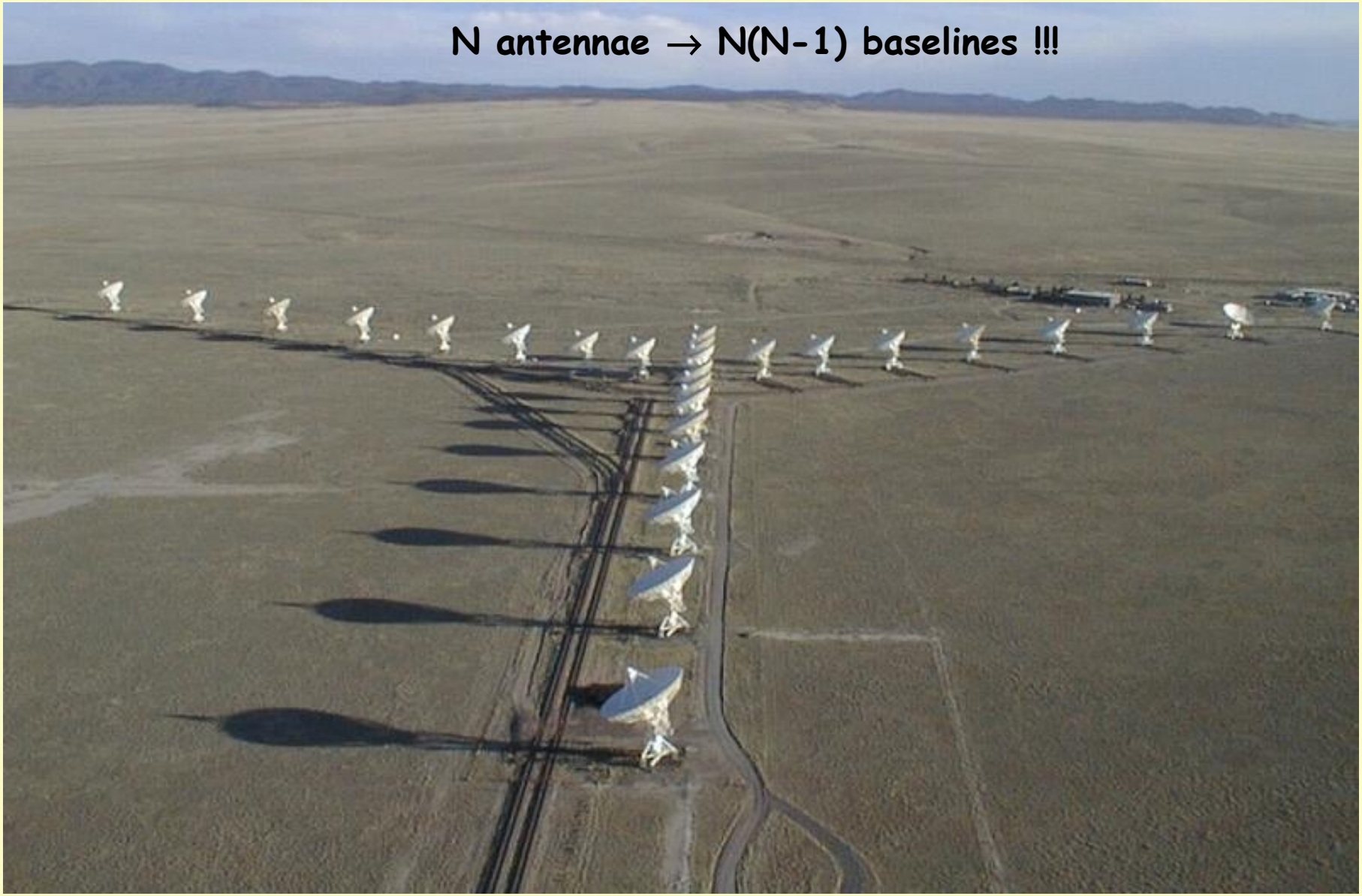
$$V \times S = F.T.[I' * A_{dirty}] = F.T.[I_{dirty}]$$

Dirty beam: S=F.T.(A_{dirty})

Dirty map

→ Need of a good coverage of the uv-plane

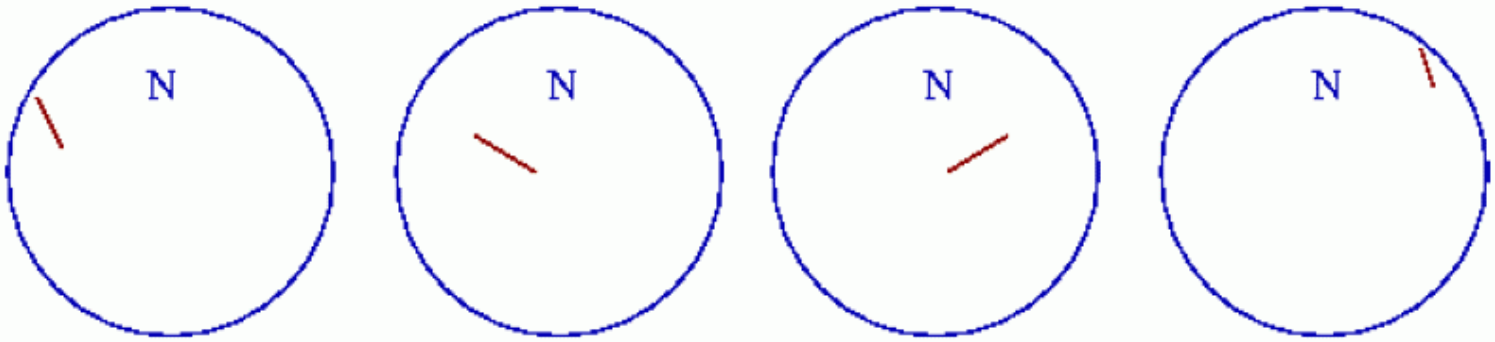
Aperture synthesis array



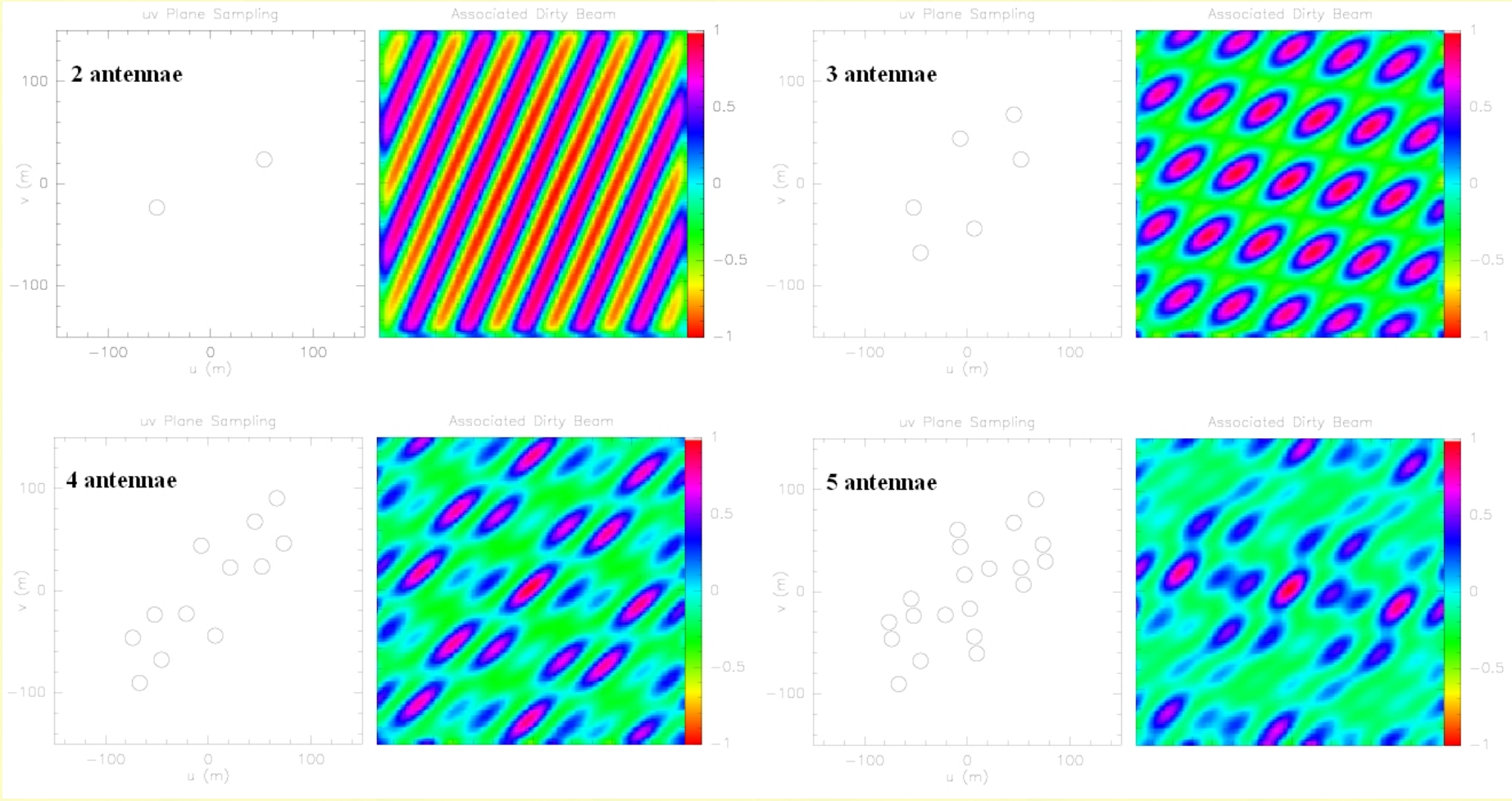
Earth-rotation synthesis or super synthesis



1 2 3 4

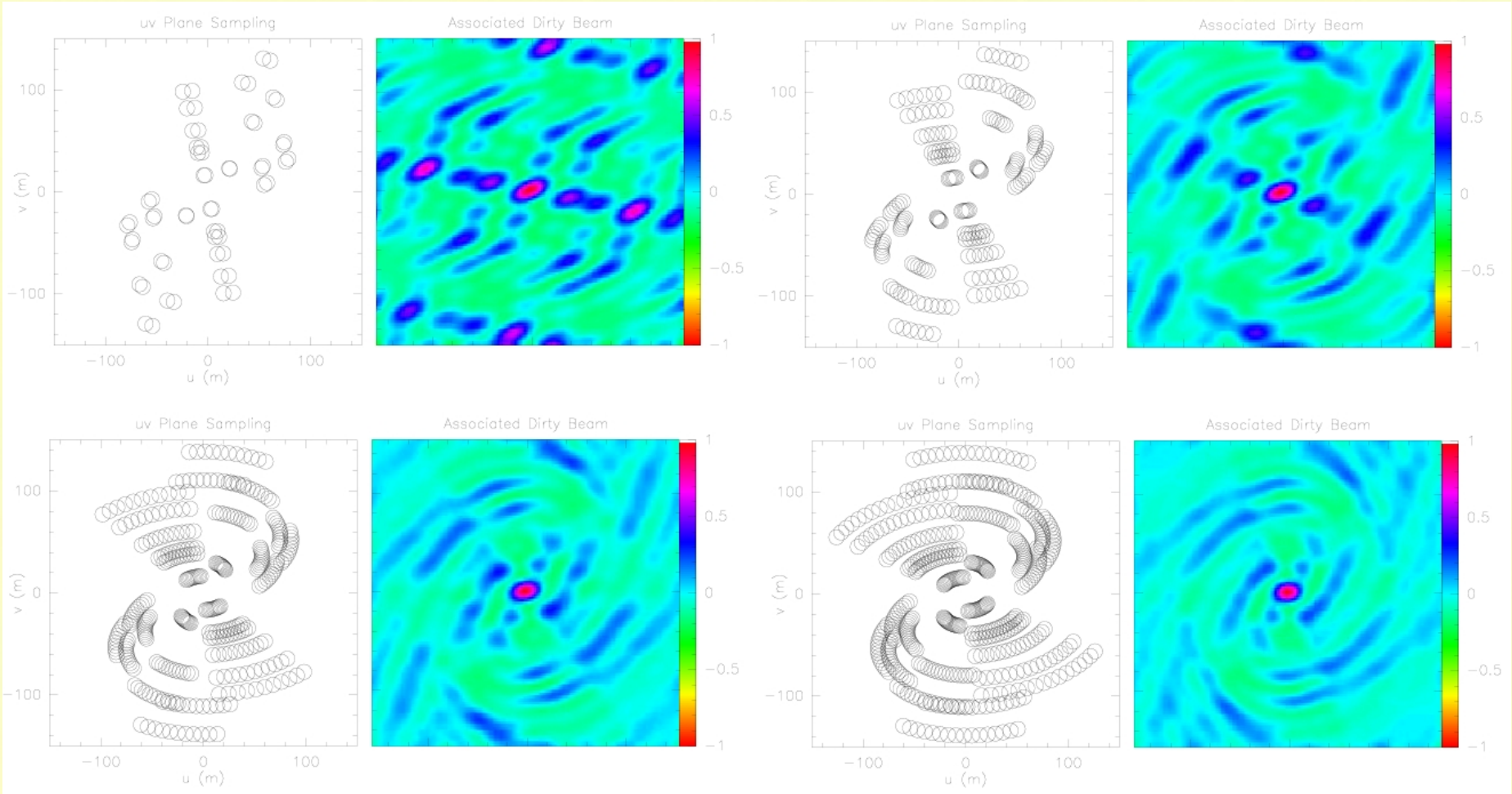


Dirty beam & number of antennae

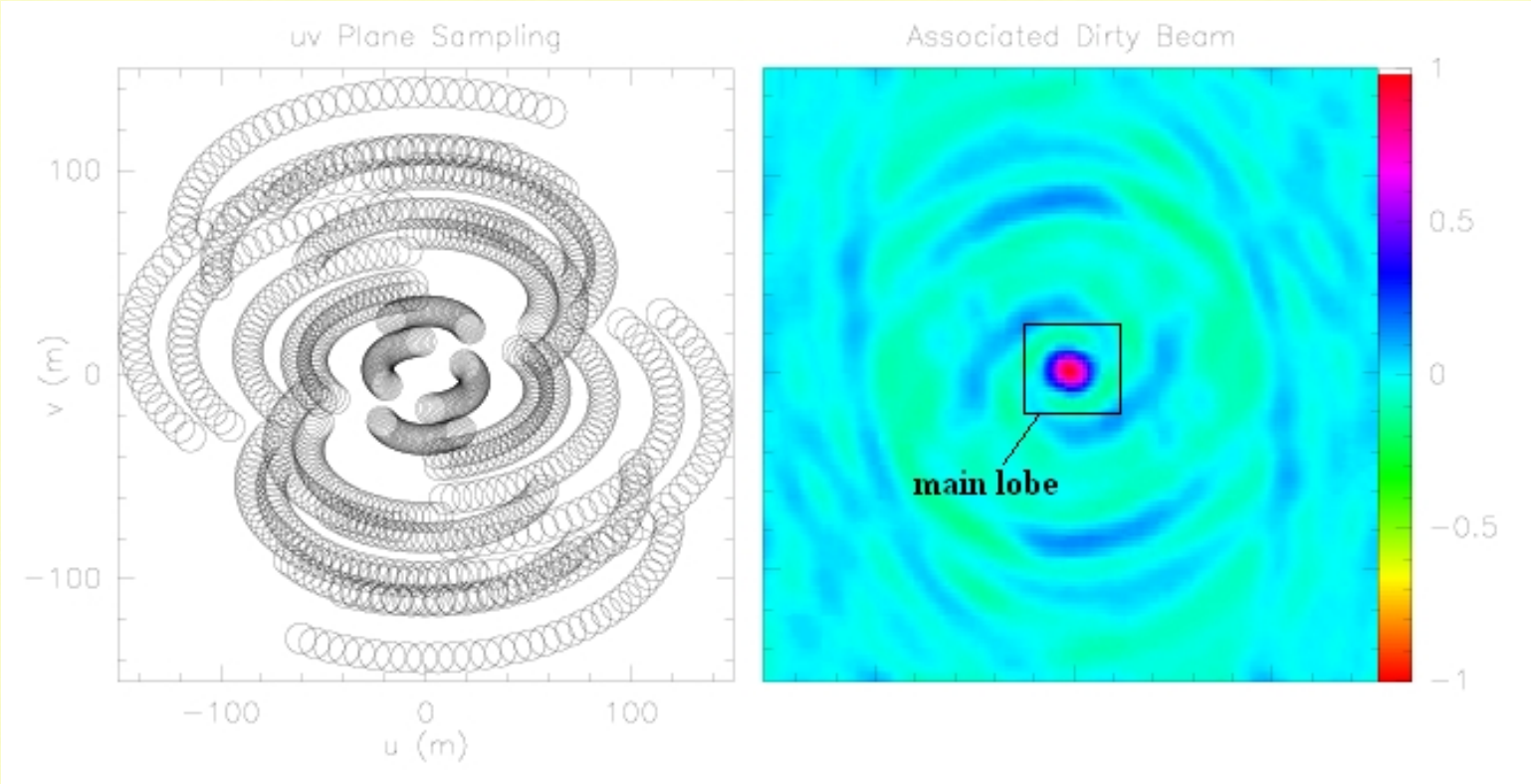


Dirty beam & super-synthesis

Integration time



Synthesized beam



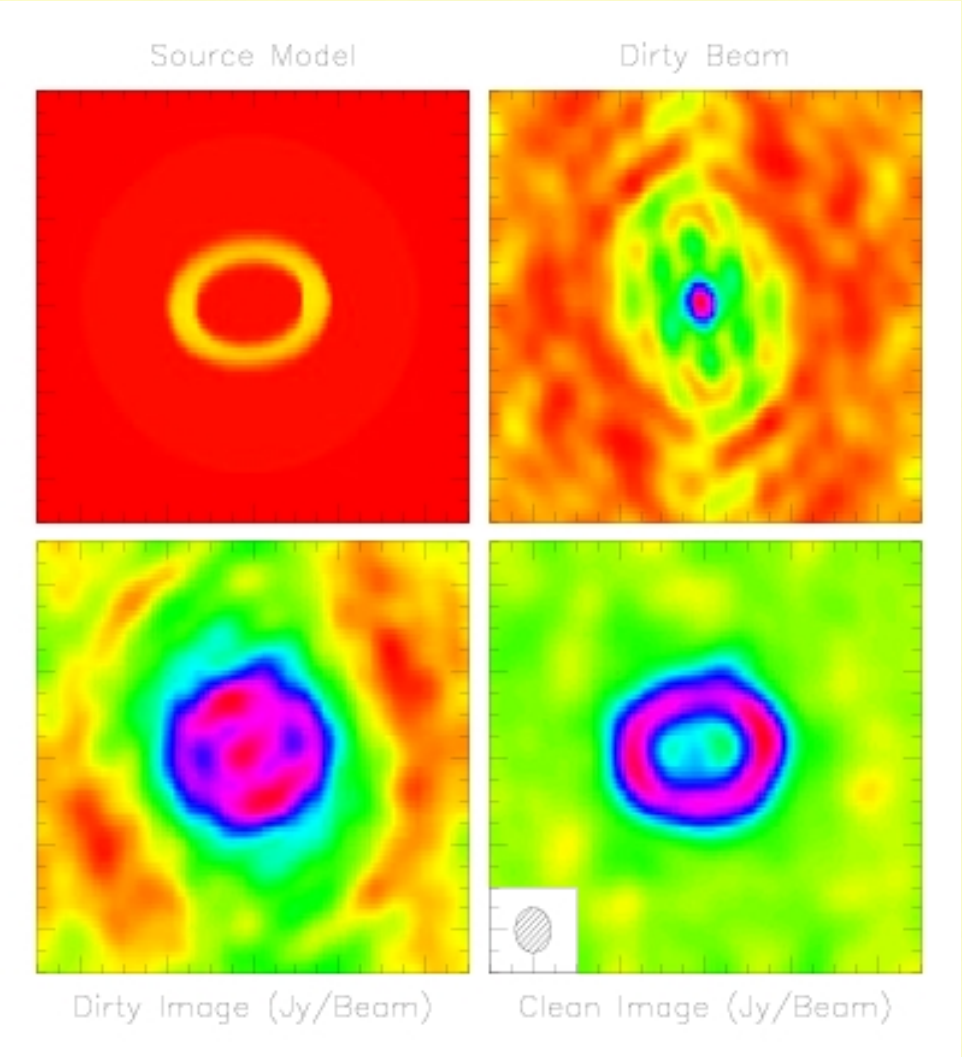
Dirty beam of an interferometer made up by N antennae and for a long observation time. The main lobe of the dirty beam can be fitted with an elliptical Gaussian

= synthesized beam

Short summary of basic lexicon

- **Baseline:** line segment between two antennae (normally measured in a direction perpendicular to the telescope pointing direction)
- **Configuration:** layout of the antennae
- **Beam:** instrument diffraction pattern
- **Primary beam:** beam of the single antenna (single dish beam)
- **Field of view of the interferometer:** primary beam size
- **Synthesized beam:** beam of the interferometers
- **Image resolution of the interferometer:** synthesized beam size

What else do we need? Imaging & deconvolution!



→ we have to **remove the effect of the dirty beam** from the map before using it for astrophysical studies

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- The radio telescope: hints to its basic properties
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- **Data reduction: imaging & deconvolution**

Imaging & deconvolution (just the basic concepts)

$$V \times S = F.T.[I' \star A_{dirty}] = F.T.[I_{dirty}]$$

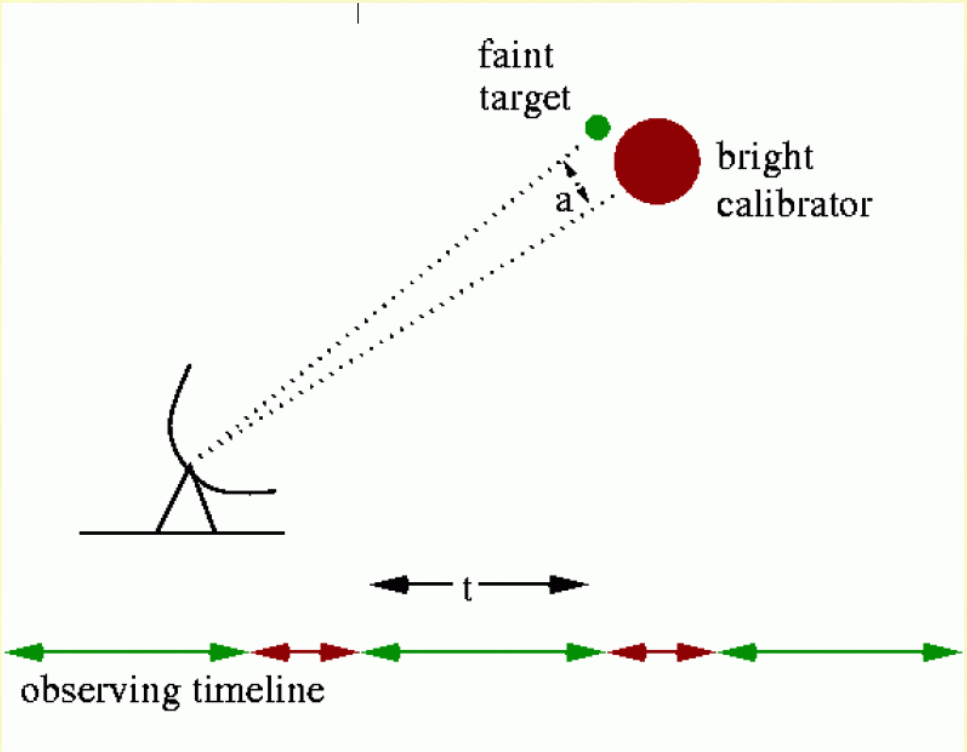
Dirty beam: $S = F.T.(A_{dirty})$

Dirty map

Stage
Calibrated Visibilities
⇓ Fourier Transform
Dirty beam & image
⇓ Deconvolution
Clean beam & image
⇓ Visualization
⇓ Image analysis
Physical information on your source

Calibrated visibilities

We need to calibrate visibilities since the output of an instrument is different from the astronomical visibility due to both instrumental effects and propagation effects in the Earth's atmosphere and ionosphere → **loss of coherence** → the measured visibility V_m is approximately given by the product of the true visibility V_t and a factor (system visibility, V_s)



- **Phase calibration:** we can derive V_s by observing calibration sources for which V_t is known
- **Amplitude calibration:** we can determine the absolute source flux density by observing a calibrator of known flux

Imaging = F.F.T.

$$V \times S = F.T.[I' * A_{dirty}] = F.T.[I_{dirty}]$$

Dirty beam: $S = F.T.(A_{dirty})$

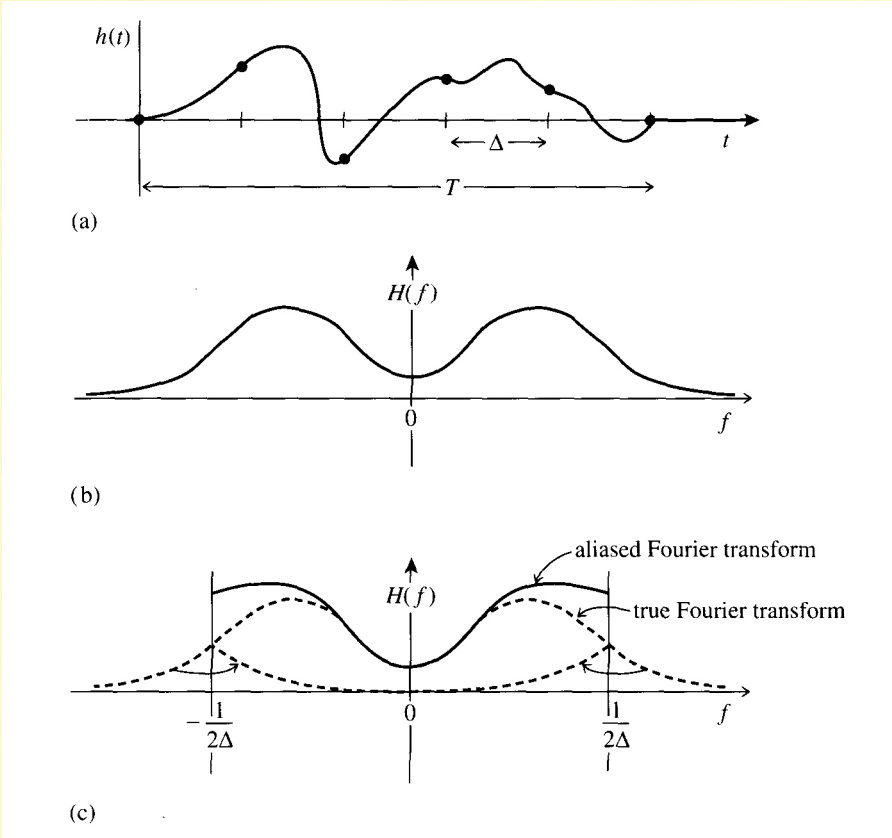
Dirty map

- **F.T.:** directly compute *sin* and *cos* functions in previous equation

  BUT! Prohibitive computation time

- **F.F.T.:** definite speed advantage

  BUT! It needs to re-grid visibilities on a regular grid to perform a 2D FFT→ distortions & aliasing!!!



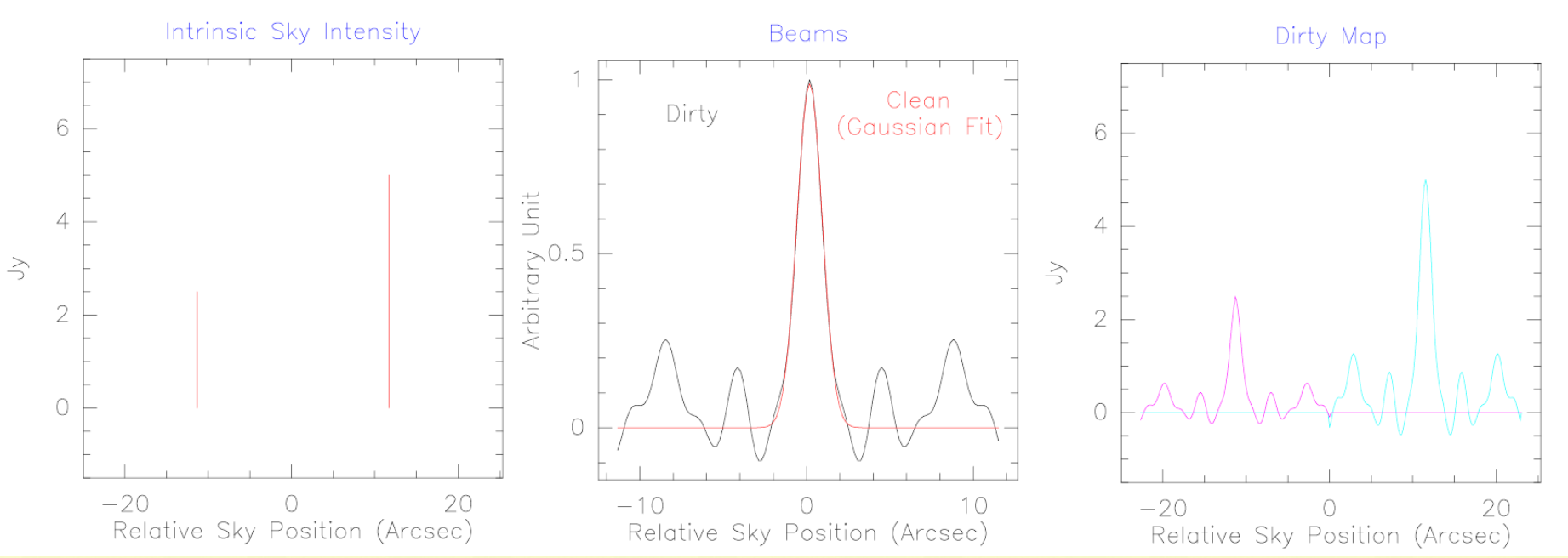
Deconvolution = clean

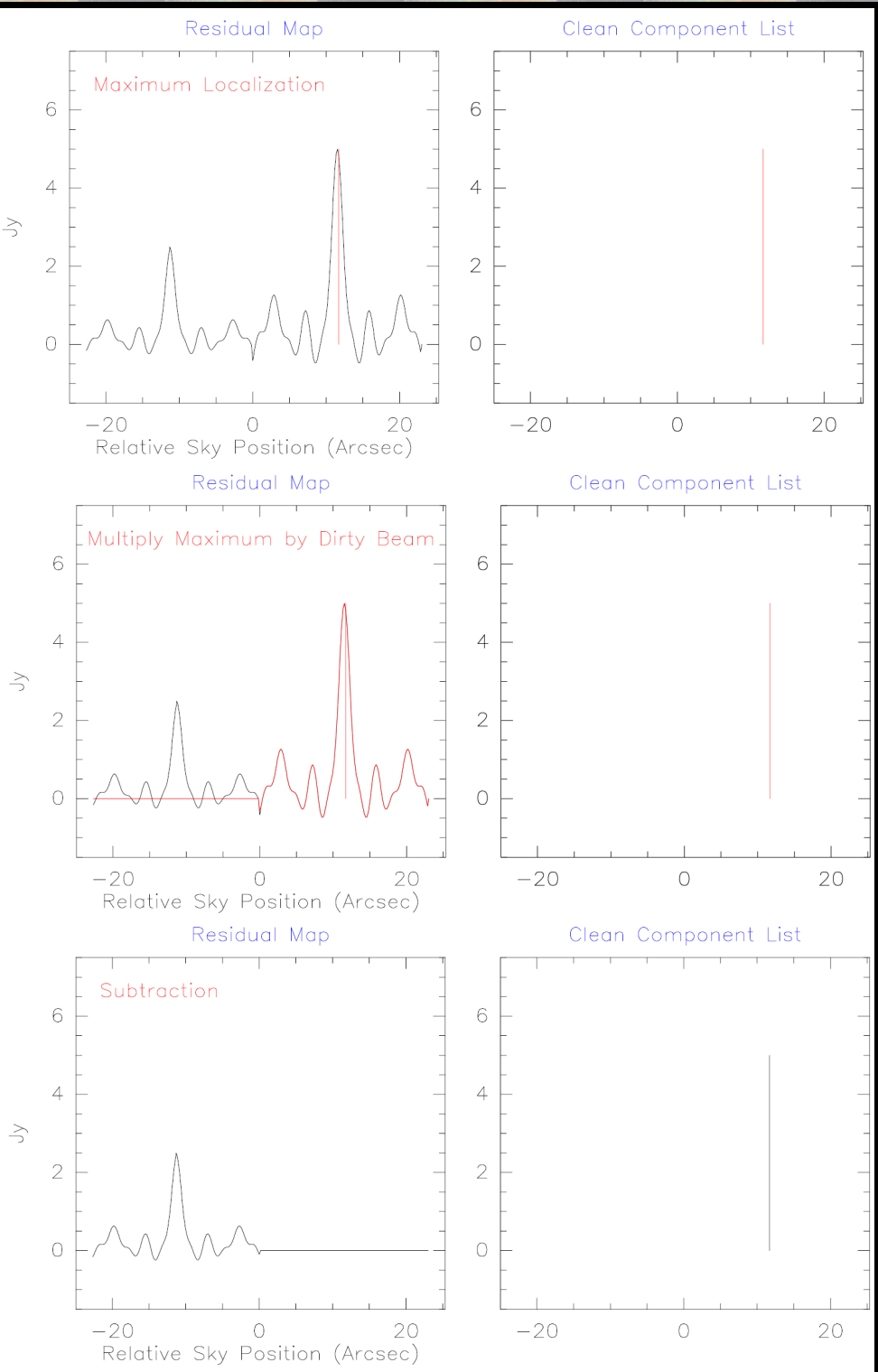
$$V \times S = F.T.[I \circledast A_{dirty}] = F.T.[I_{dirty}]$$

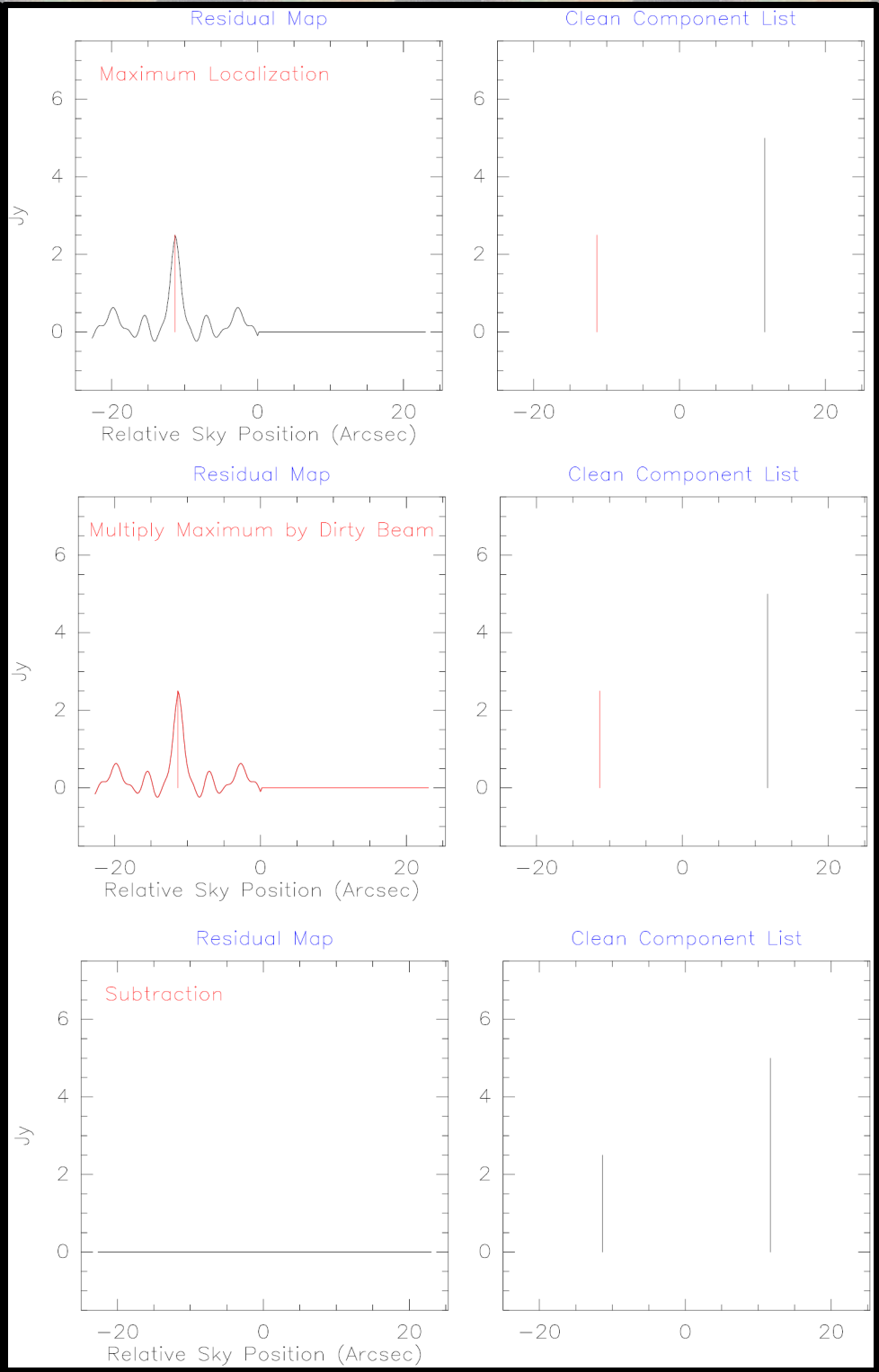
Dirty beam: $S = F.T.(A_{dirty})$ **Dirty map**

- **Deconvolution** has to remove as much as possible the noise due to the dirty beam and introduced by the FFT
- **Basic idea** (J. Högbom 1974):
  - ✔ Take the largest maximum of the dirty map & subtract the corresponding dirty beam
  - ✔ Take the second largest maximum of the dirty map & subtract the corresponding dirty beam
  - ✔ ...
  - ✔ Just noise is left behind
  - ✔ We have a list of point sources = CLEAN components
  - ✔ We replace each point source by a Gaussian (= clean beam)
  - ✔ We have the sky brightness !!!

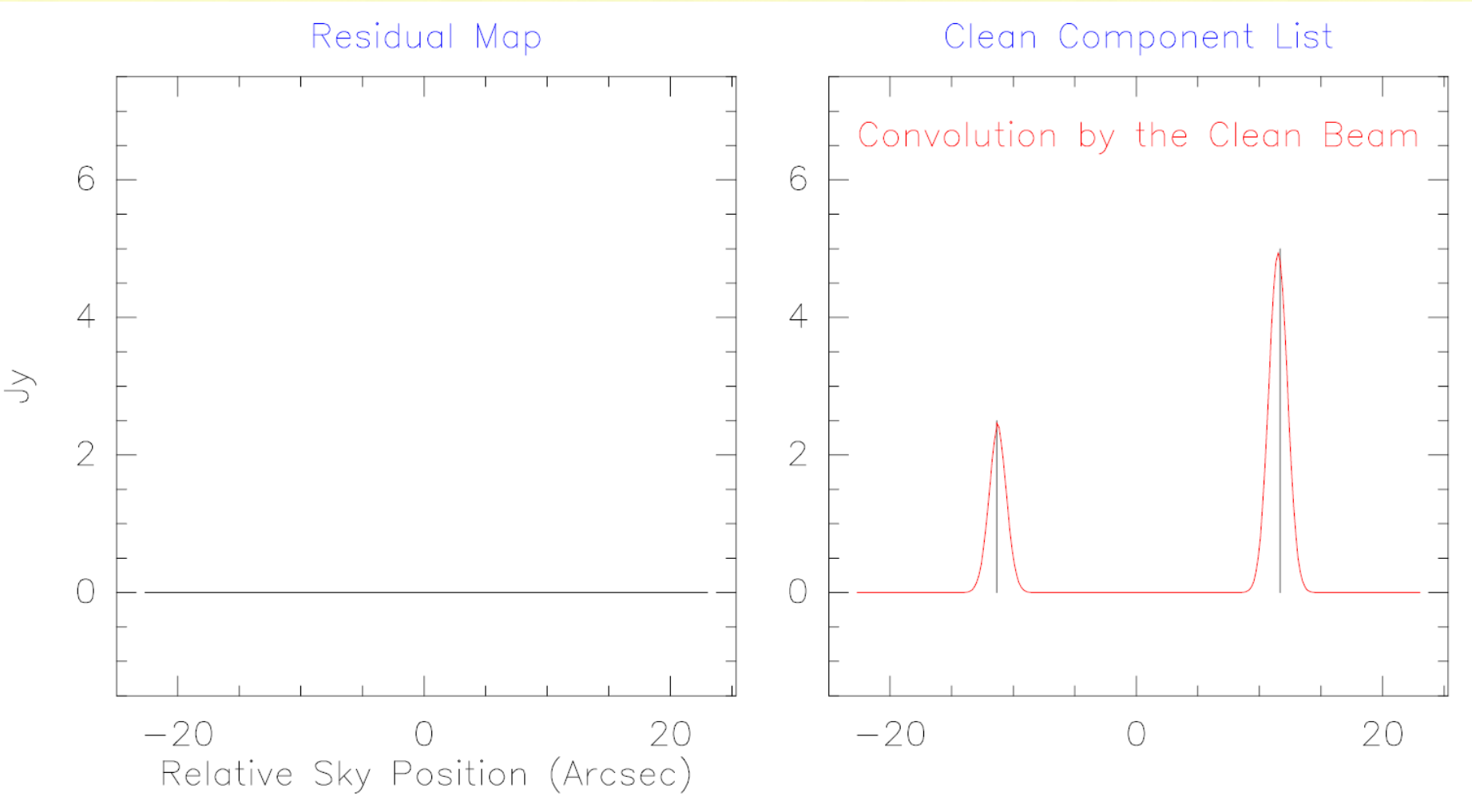
Deconvolution = clean




Residual Map
Clean Component List
Maximum Localization
Multiply Maximum by Dirty Beam
Subtraction
Jy
Relative Sky Position (Arcsec)

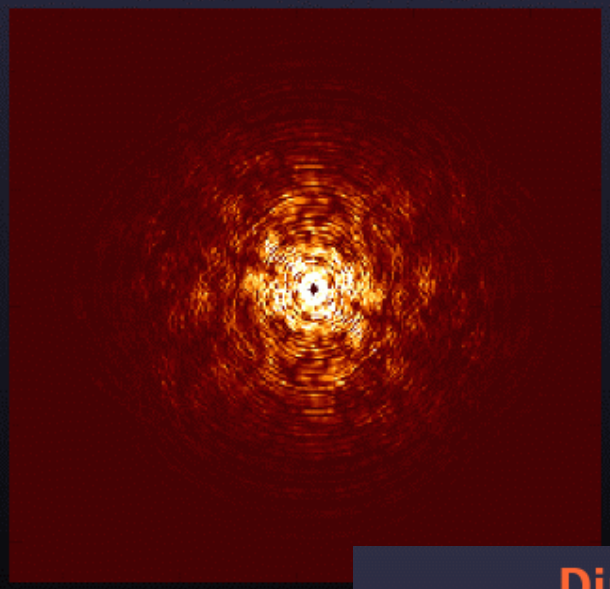

Residual Map
Clean Component List
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Multiply Maximum by Dirty Beam
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Relative Sky Position (Arcsec)

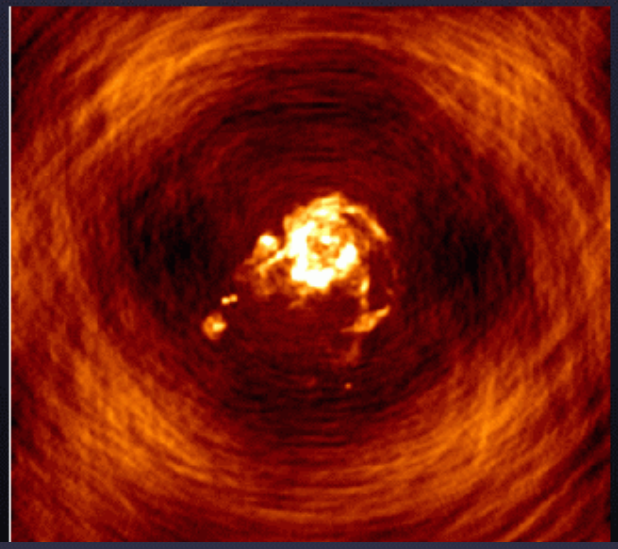
Deconvolution = clean



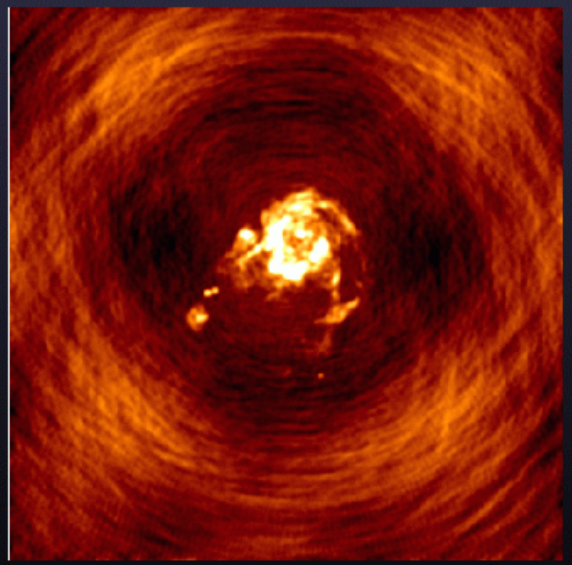
Imaging & deconvolution

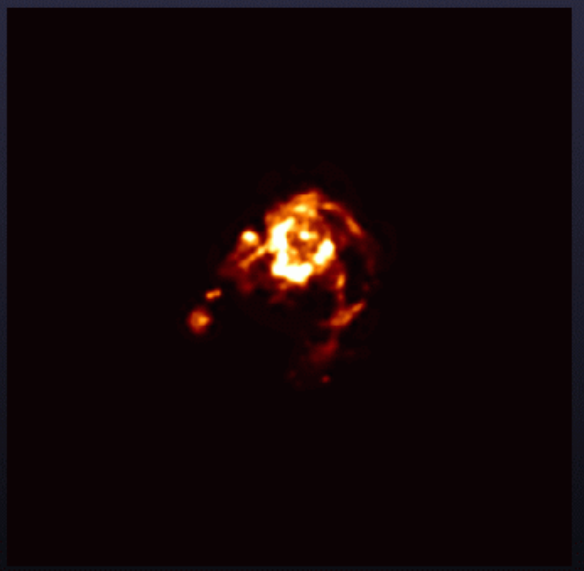
Measured Visibilities <--> Dirty Image



<--> 

Dirty image <--> Model of the sky



<--> 

The 76m Lovell radio telescope at Jodrell Bank



The 100m radio telescope in Effelsberg



The 90m radio telescope at Green Benk (USA)



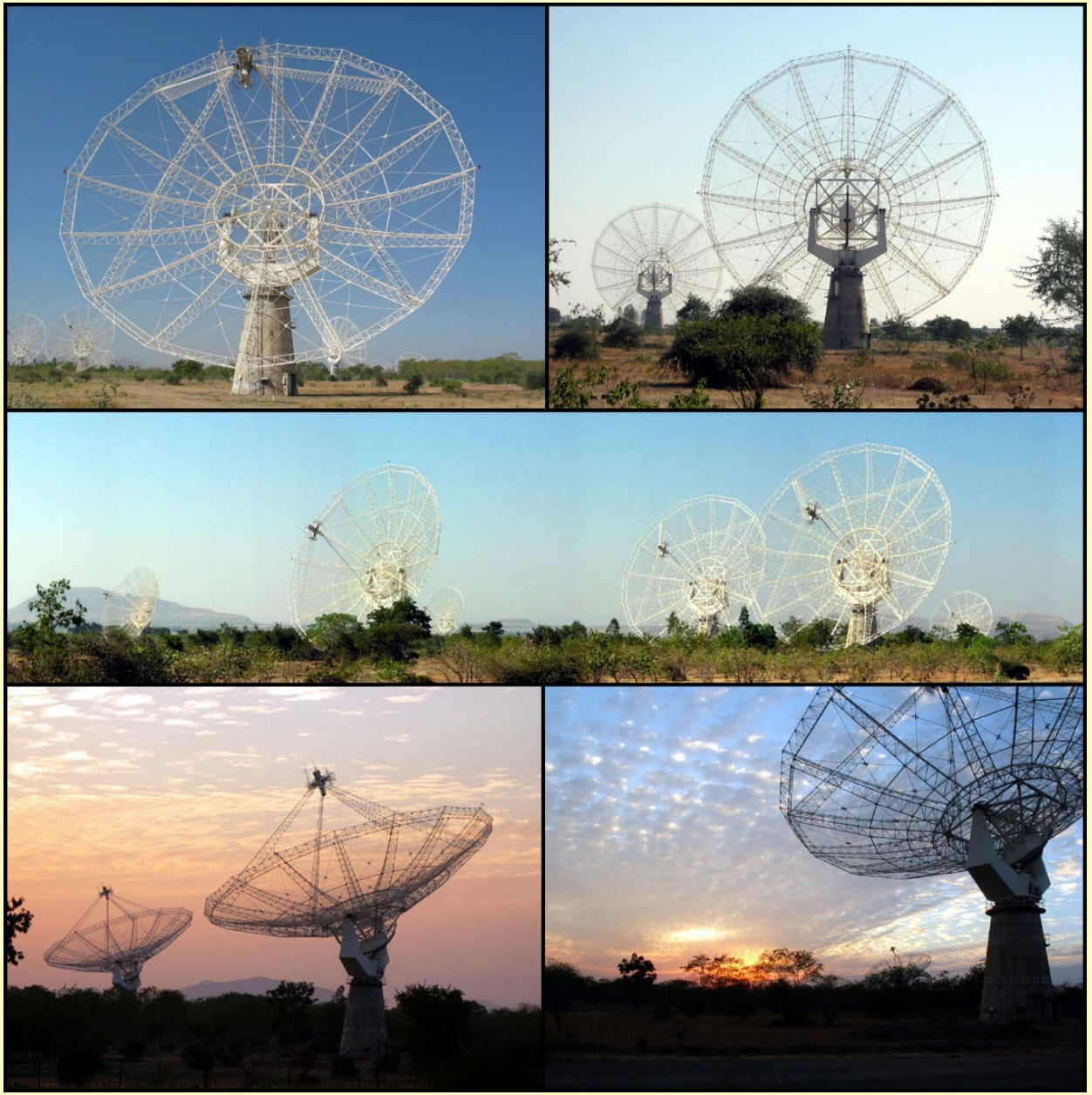
The 305m Arecibo radio telescope



The Very Large Array (VLA) at Socorro



The Giant Metrewave Radio Telescope (GMRT) at Pune



The Australia Telescope Compact Array (ATCA) at Narrabri



Very Long Baseline Interferometry (VLBI)

