Chiara Ferrari



Observatoire de la CÔTE d'AZUR

Emission mechanisms

Introduction to Radioastronomy

Second Part

January 2011

- Fundamentals of radio observations: first part
- **Physical mechanisms responsible for emission at radio frequencies: second part**
- Galactic radio sources: third part
- Extra-galactic radio sources: third part

Lecture Outline

- **Emission mechanisms**
  - Thermal processes: Black Body & Bremsstrahlung
  - Non-thermal processes: Synchrotron & Inverse Compton
  - Line processes: just very few and basic concepts!!!

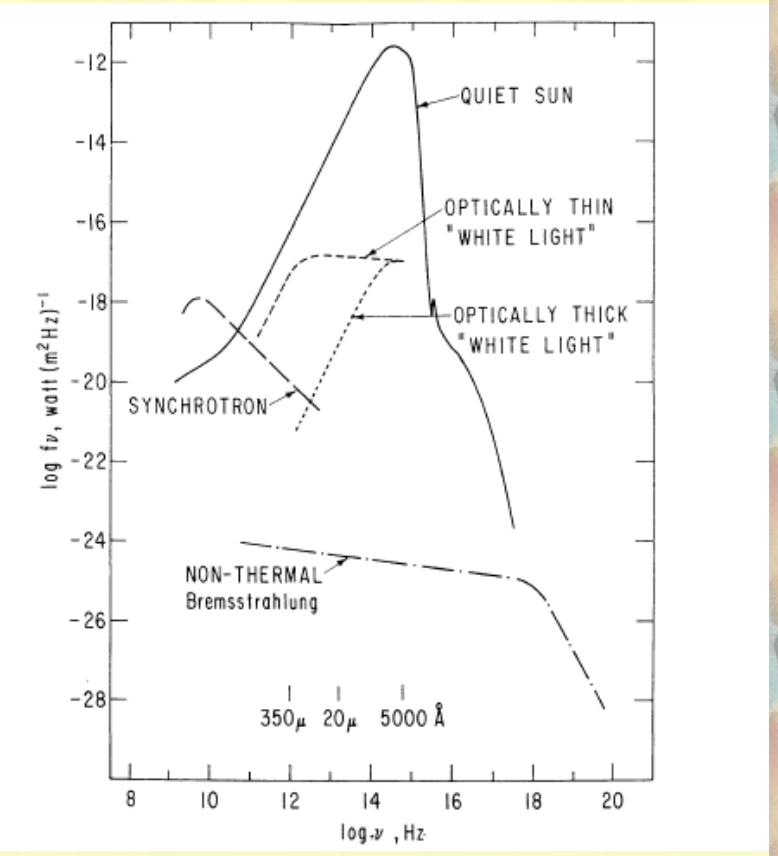
Emission mechanisms

- **Continuum processes:**
  - ✔ Black Body
  - ✔ Bremsstrahlung

  } **THERMAL**

  - ✔ Synchrotron
  - ✔ Inverse Compton

  } **NON-THERMAL**



- **Line processes:**
  - ✔ Electronics transitions (allowed and prohibited)

Lecture Outline

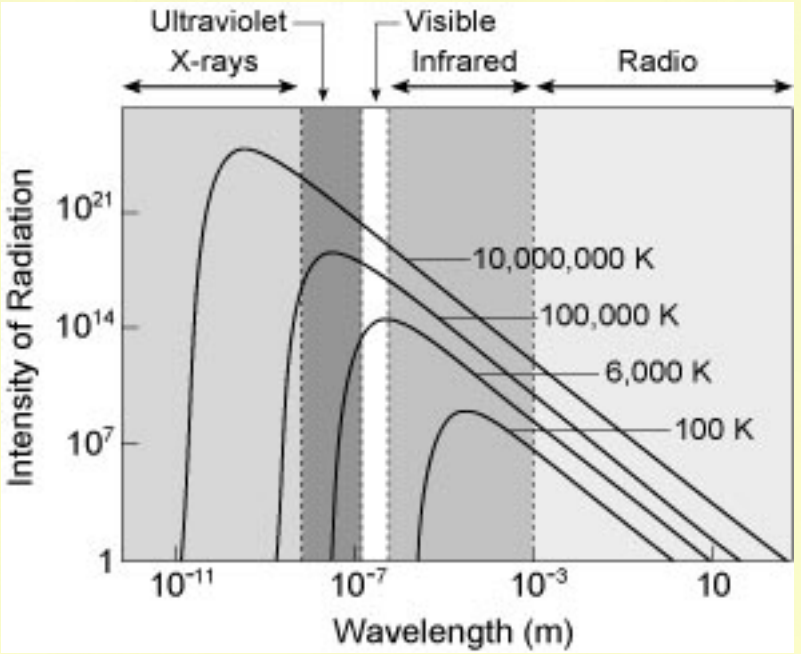
- Emission mechanisms
  - **Thermal processes: Black Body** & Bremsstrahlung
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Plank law:

spectral distribution of a black body in thermal equilibrium

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad \left(B_\nu(T)d\nu = -B_\lambda(T)d\lambda\right) \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/k\lambda T}-1}$$

- **Bν** : energy per unit time, per unit surface area, per unit solid angle, per unit frequency or wavelength (W m^{-2} sr^{-1} Hz^{-1} or W m^{-2} sr^{-1} μm^{-1})
- **T** : absolute temperature of the black body (K)
- **λ** : wavelength (μm)
- **ν** : frequency (Hz)
- **c** : velocity of light (2.998 *10^8 m s^{-1})
- **h** : Planck's constant (6.626 *10^{-34} J s)
- **k** : Boltzmann's constant (1.380 *10^{-23} J K^{-1})

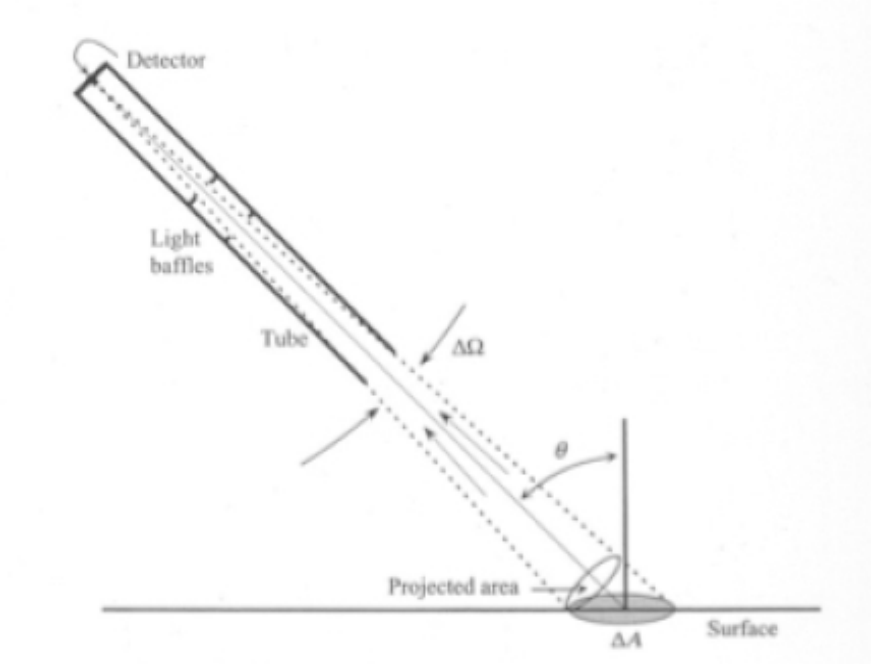


Black body irradiance E [W m^{-2}]: Stefan-Boltzmann law

Integrating the Plank law:

$$B_\nu(T)=\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad E(T)=\int_0^\infty\int B_\nu(T)\,d\nu\,d\Omega$$

$$x=\frac{h\nu}{kT} \quad + \quad \int_0^\infty \frac{x^3}{e^x-1}dx=\frac{\pi^4}{15} \quad + \quad d\Omega=\sin\theta\,d\theta\,d\phi \quad +$$



$$E_\theta(T)=B_\nu(T)\cos\theta\,d\Omega$$

$$E_\theta(T)=B_\nu(T)\cos\theta\sin\theta\,d\theta\,d\phi$$

Lambert's cosine law

Black body irradiance [W m^{-2}]: Stefan-Boltzmann law

$$E(T) = \sigma T^4 \qquad \sigma = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.67 \times 10^{-8}\, W\, m^{-2}\, K^{-4}$$

Stefan-Boltzmann radiation law

- ✔ found experimentally by J. Stefan (1879)
- ✔ derived theoretically by L. Boltzmann (1884)
- ✔ before Planck's radiation law was known!!!

Wien's displacement law

$$B_\nu(T)=\frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

Plank law

$$B_\lambda(T)=\frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/k\lambda T}-1}$$

$$\partial B_\nu/\partial\nu=0 \qquad \partial B_\lambda/\partial\lambda=0$$

$$x=\frac{h\nu}{kT}$$

$$3(1-e^{-x})-x=0 \qquad 5(1-e^{-x})-x=0$$

$$\frac{\nu_{max}}{GHz}=58.789\left(\frac{T}{K}\right)$$

Wien's displacement law

$$\left(\frac{\lambda_{max}}{cm}\right)\left(\frac{T}{K}\right)=0.28978$$

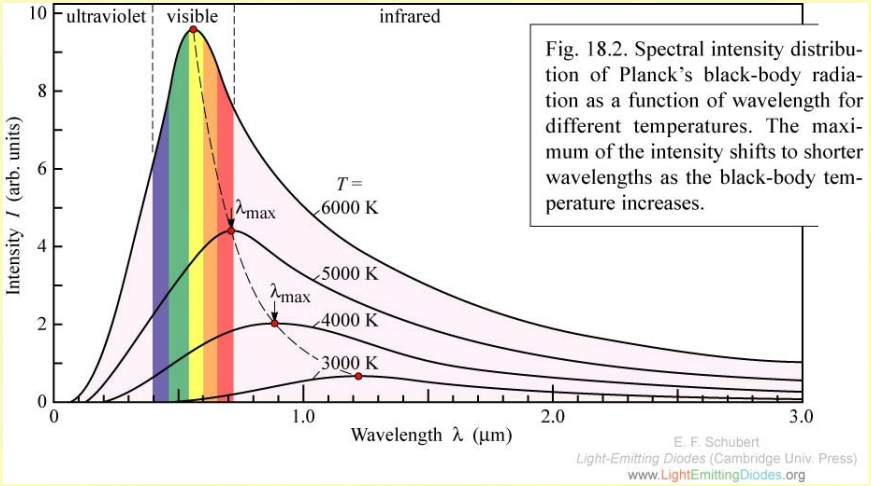
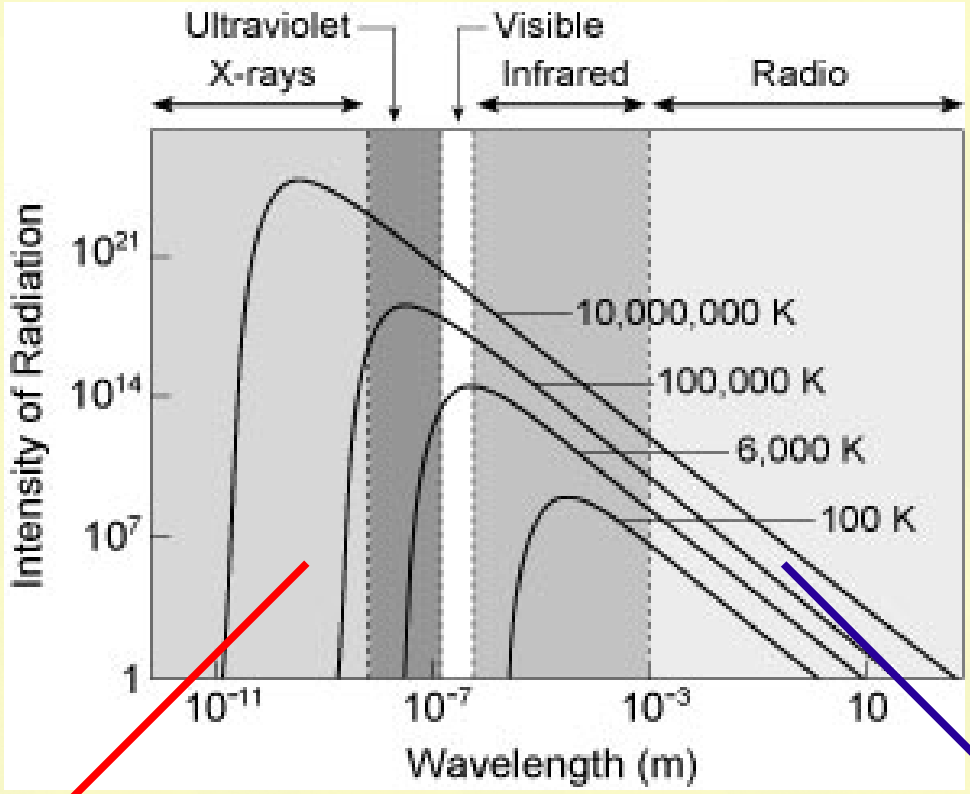


Fig. 18.2. Spectral intensity distribution of Planck's black-body radiation as a function of wavelength for different temperatures. The maximum of the intensity shifts to shorter wavelengths as the black-body temperature increases.

Rayleigh-Jeans and Wien's laws



Brightness temperature!

Wien's Law

$$h\nu \gg kT$$

$$B_W(\nu, T) = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$$

Rayleigh-Jeans' Law

$$h\nu \ll kT$$

$$B_{RJ}(\nu, T) = \frac{2\nu^2}{c^2} kT$$

Lecture Outline

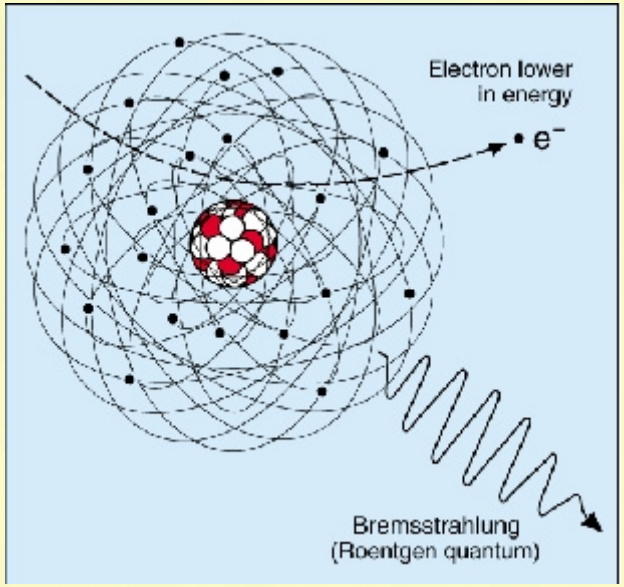
- Emission mechanisms
  - **Thermal processes:** Black Body & **Bremsstrahlung**
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Bremsstrahlung

Radiation emitted by a charged particle moving in an electric field.

The radiation is due to the acceleration of a charge in the Coulomb field of another charge. The particle emits energy in the form of e.m. radiation, at the expense of its kinetic energy, hence the name

"braking radiation"

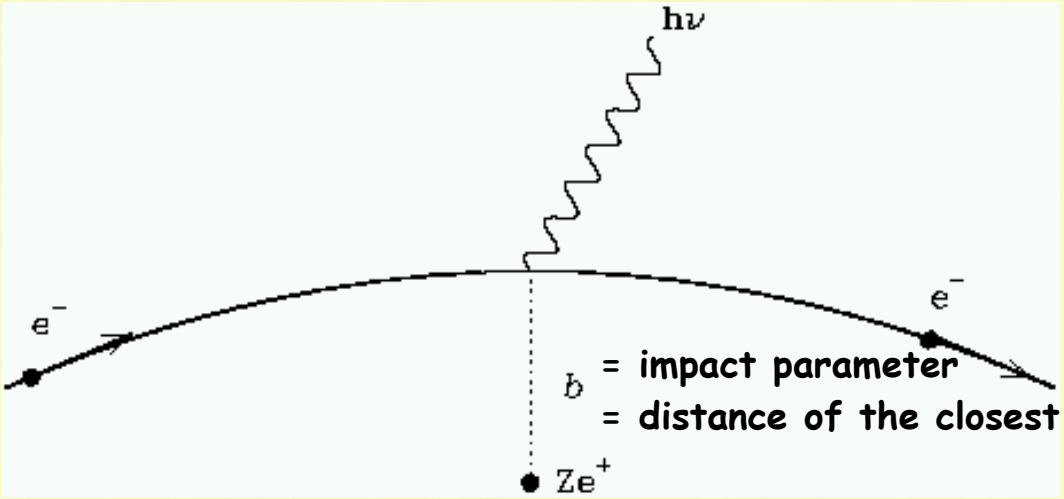


Bremsstrahlung

Free electrons are accelerated in the Coulomb field

of ionized nuclei and radiate energy

- Free-free emission (the e^- is free and remains free)
- Thermal emission: electrons and ions ⇔ thermodynamic equilibrium
- Acceleration not uniform ⇔ e^- emits γ with a range of λ (i.e. a spectrum)



Bremsstrahlung: radiated power (1)

Radiated power: Larmor's formula

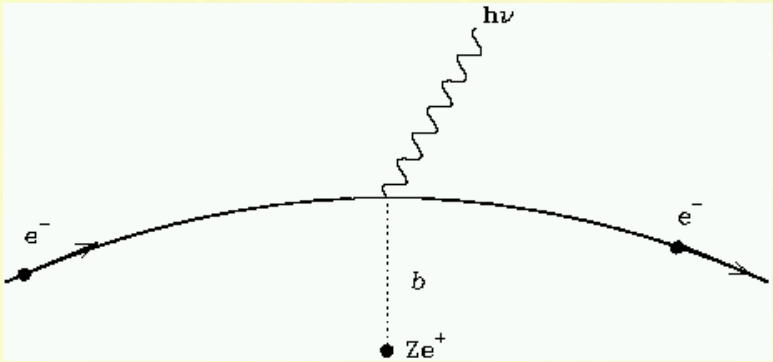
$$P = -\frac{dW}{dt} = \frac{2}{3}\frac{e^2}{c^3}a^2 \qquad [W]$$

This radiation is polarized with the electric field parallel to the acceleration direction

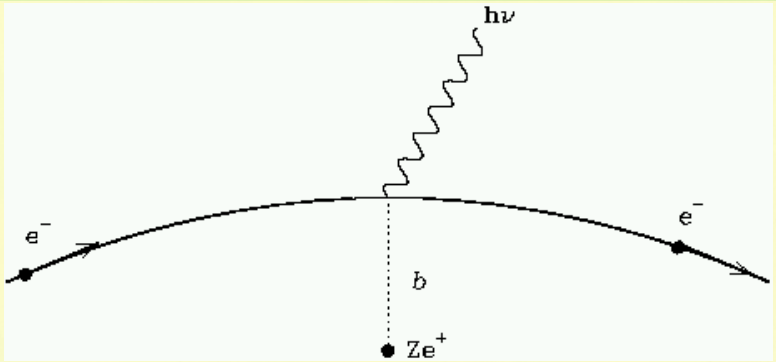
$$F = m_e a \simeq \frac{Z e^2}{x^2} \longrightarrow a \simeq \frac{Z e^2}{m_e x^2}$$

= actual distance between e^- and Ze^+

$$P = \frac{2}{3}\frac{e^6}{c^3}\frac{Z^2}{m_e^2 x^4}$$



Bremsstrahlung: radiated power (2)



$$P = \frac{2}{3}\frac{e^6}{c^3}\frac{Z^2}{m_e^{\,2}x^4}$$

- $P \propto m^{-2}$
- $P \propto x^{-4}$

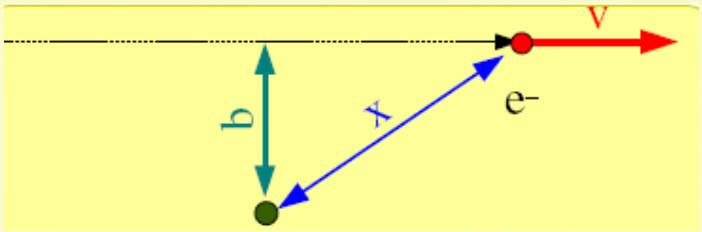
↓

P is relevant for **electrons only** and when particles are **very close** : $x \approx b$

↓

The interaction lasts **very shortly**: $\Delta t = 2b/v$

(v = velocity of the e^-)

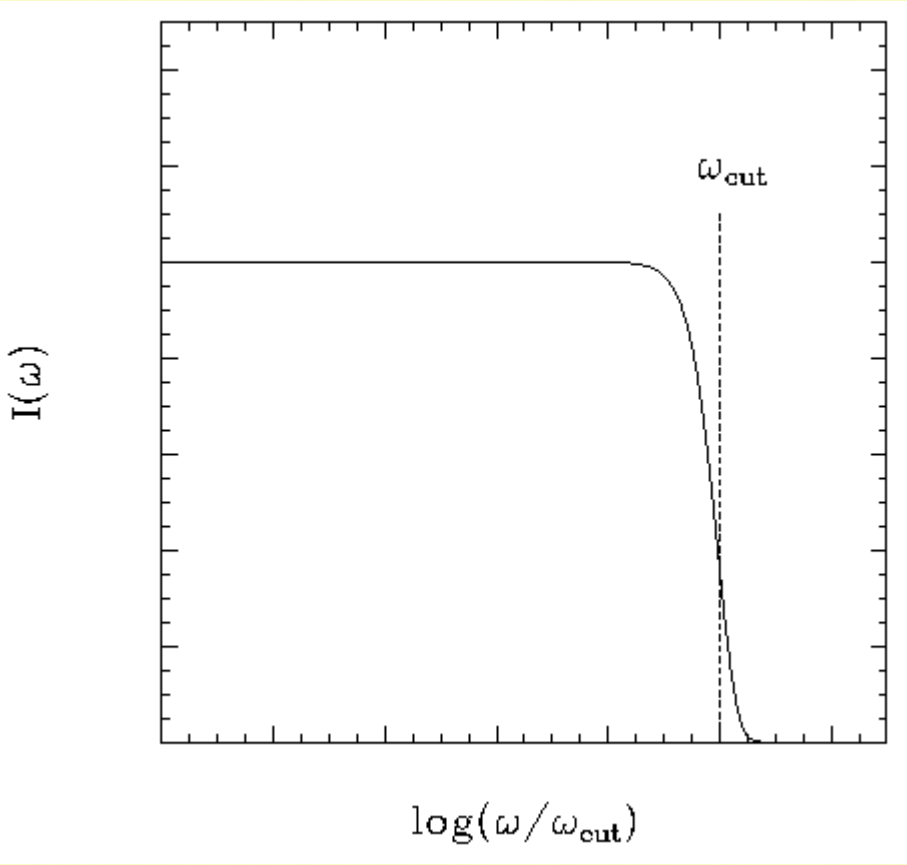


Bremsstrahlung: radiated power (3)

$$P = \frac{2}{3}\frac{e^6}{c^3}\frac{Z^2}{m_e^{\,2} x^4} \quad + \quad \Delta t = 2b/\mathrm{v} =$$

$$P\,\Delta t = \frac{4}{3}\frac{e^6}{c^3}\frac{Z^2}{m_e^{\,2} b^3 \mathrm{v}}$$

Total emitted energy in a single interaction collision



- Radiation comes as pulses Δt long
- The Fourier analysis of the pulse provides the spectral distribution of the radiated energy
- The spectral energy distribution is flat up to a frequency (exponential decay beyond it):

$$\nu_{max} \simeq \frac{1}{2\Delta t} = \frac{\mathrm{v}}{4b}$$

Bremsstrahlung: radiated power (4)

$$P\,\Delta t = \frac{4}{3}\frac{e^6}{c^3}\frac{Z^2}{m_e^{\,2} b^3 \mathrm{v}}$$

+

$$\nu_{max} \simeq \frac{1}{2\,\Delta t} = \frac{\mathrm{v}}{4\,b}$$

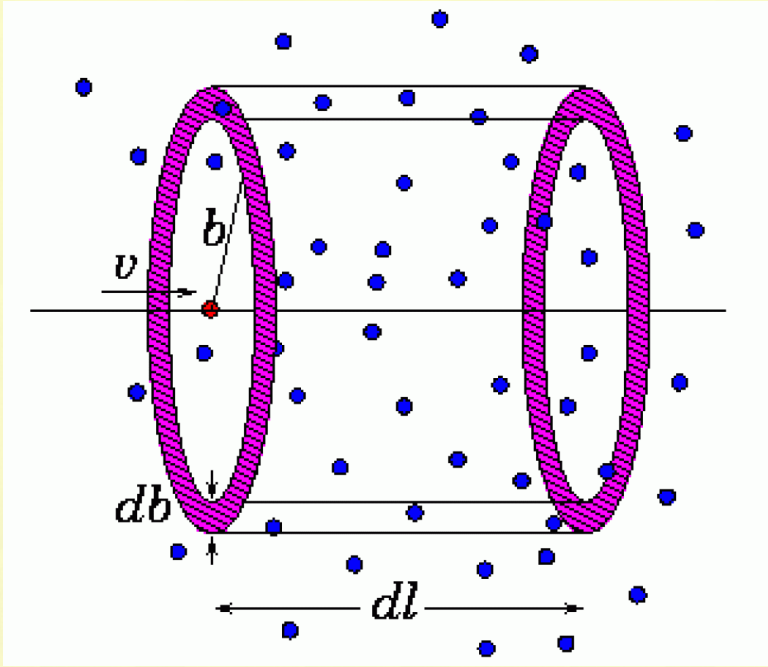
=

$$\frac{P\,\Delta t}{\Delta \nu} \approx \frac{P\,\Delta t}{\nu_{max}} = 2\,P\,\Delta t^2 \sim \frac{16}{3}\frac{e^6}{c^3}\frac{Z^2}{m_e^{\,2} b^2 \mathrm{v}^2}$$

Energy radiated by a single event per unit frequency
(assuming a flat spectrum)

Bremsstrahlung: the real case

Cloud with ion density n_Z
and electron density n_e,
moving with velocity v



Total emissivity for bremsstrahlung =
energy radiated by a single event ×
total number of collisions per second

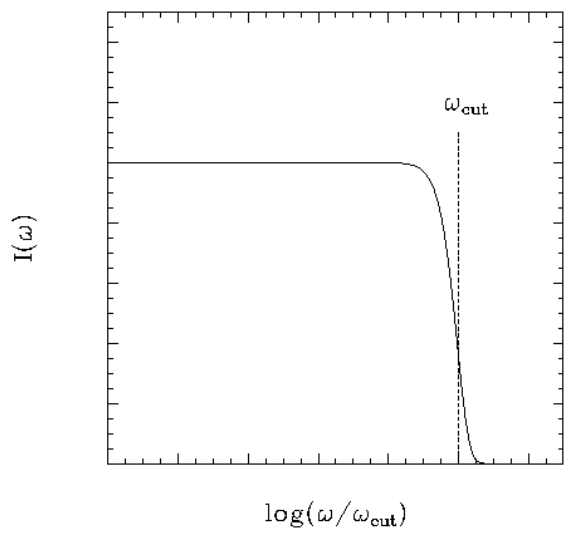
$$\frac{P\Delta t}{\Delta\nu} \approx 2P\Delta t^2 \sim \frac{16}{3}\frac{e^6}{c^3}\frac{Z^2}{m_e^{\,2} b^2 \mathrm{v}^2} \quad \times \quad \int_{b_{min}}^{b_{max}} 2\pi\, b \times n_e \times n_Z \times \mathrm{v}\, db$$

$$\epsilon_{\mathrm{v},\nu} = 2\pi\, n_e n_Z \mathrm{v} \int_{b_{min}}^{b_{max}} \left(2P\Delta t^2\right) b\, db \simeq \frac{32\pi}{3}\frac{\mathrm{e}^6}{m_e^{\,2} c^3}\frac{1}{\mathrm{v}} n_e n_Z Z^2 \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Gaunt Factor: $g_{ff}(\nu, T) = \frac{\pi}{3}\ln\left(\frac{b_{max}}{b_{min}}\right)$

Bremsstrahlung: b_{min} and b_{max}

Evaluation of b_{max}:



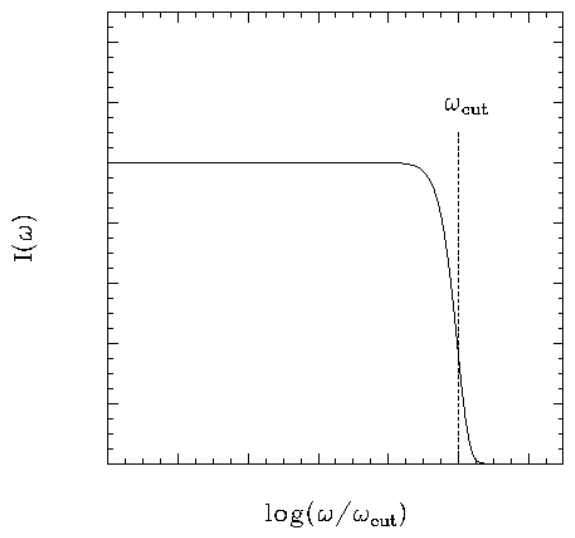
at a given frequency ν, only those interactions where electrons have impact parameters corresponding to $\nu_{max} > \nu$ contribute

$$\nu_{max} \simeq \frac{1}{2\Delta t} = \frac{v}{4b}$$

$$\nu_{max} = \frac{v}{4b} > \nu \rightarrow b_{max} = v/4\nu$$

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Evaluation of b_{min}:

- Classical physics requires that the variation of the electron velocity orbiting around the nucleus is lower than its original velocity:

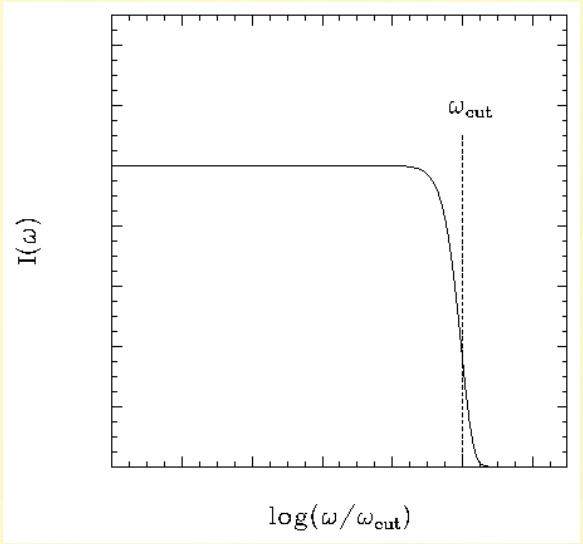
$$\Delta v \sim a\,\Delta t = \frac{Ze^2}{m_e b^2}\frac{2b}{v} \leq v \rightarrow b_{min,\,class.} \geq \frac{2Ze^2}{m_e v^2}$$

- Quantum physics requires that the Heisenberg principle is satisfied

$$\Delta p = \approx m_e v \geq \frac{h}{2\pi\Delta x} \sim \frac{h}{2\pi b_{min}} \rightarrow b_{min,\,quant.} \geq \frac{h}{2\pi m_e v}$$

Bremsstrahlung: b_{min} and b_{max}

Evaluation of b_{max}:



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Which b_{min}?

- Quantum physics requires that the Heisenberg principle is satisfied

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Bremsstrahlung: which b_{min} ?

$$\frac{b_{min,\,quant.}}{b_{min,\,class.}} \sim \frac{h/(2\pi m_e \mathrm{v})}{2Ze^2/(m_e \mathrm{v}^2)} = \frac{hc}{4\pi e^2}\frac{\mathrm{v}}{c}\frac{1}{Z} \sim \frac{137}{Z}\frac{\mathrm{v}}{c}$$

≥ 1 when v ~ 0.01 c

$$\frac{1}{2}m\mathrm{v}^2 = \frac{3}{2}kT \rightarrow \mathrm{v} = \sqrt{3kT/m_e} \rightarrow \boxed{\frac{b_{min,\,quant.}}{b_{min,\,class.}} \sim \frac{137}{Zc}\sqrt{\frac{3k}{m_e}}T^{1/2} \approx 2.3\times10^{-3}\,T^{1/2}}$$

→ We use $b_{min,class}$ or $b_{min,quant}$ depending on the temperature!

Examples:

- HII regions: T~10^4 K → $b_{min,class}$ ($\frac{b_{min,\,quant.}}{b_{min,\,class.}} < 1$)
- Gas in galaxy clusters: T~10^8 K → $b_{min,quant}$ ($\frac{b_{min,\,quant.}}{b_{min,\,class.}} > 1$)

Thermal (Maxwellian) velocity distribution:

$$f(\mathrm{v})d\,\mathrm{v} = 4\pi\left(\frac{m_e}{2\pi k T}\right)^{3/2} e^{-m_e \mathrm{v}^2/2kT}\,\mathrm{v}^2 d\,\mathrm{v}$$



$n_e(\mathrm{v}) = n_e f(\mathrm{v})d\,\mathrm{v}$ = **number density of electrons whose velocity is between v and v+dv**

Thermal bremsstrahlung

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$\downarrow$

$n_e(\mathrm{v}) = n_e f(\mathrm{v})d\,\mathrm{v}$ = **number density of electrons whose velocity is between v and v+dv**

$$\epsilon_{\nu,T} = \int_{\mathrm{v}_{\min}}^{\infty} \epsilon_{\mathrm{v},\nu} f(\mathrm{v})d\,\mathrm{v} = 6.6\times10^{-38} T^{-1/2} e^{-h\nu/kT} n_e n_Z Z^2 \overline{g_{ff}(\nu,T)}\ erg\,cm^{-3}\,s^{-1}\,Hz^{-1}$$

~ particle energy ~ 1-15

where: $\frac{1}{2} m_e \mathrm{v}_{\min} \geq h\nu \rightarrow \mathrm{v}_{\min} \geq \sqrt{\frac{2h\nu}{m_e}}$

since the kinetic energy of the electrons cannot

be lower than the energy of the emitted photons

Thermal bremsstrahlung

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$\downarrow$

$n_e(\mathrm{v}) = n_e f(\mathrm{v})d\,\mathrm{v}$ = **number density of electrons whose velocity is between v and v+dv**

- Spectrum nearly flat up to $\nu = kT/h$
- Then the emissivity decreases exponentially as a function of frequency
- For typical astrophysical objects, exponential nearly constant for a large frequency range
- Emission in a wide part of the e.m. spectrum
- Typically at radio wavelengths for intermediate temperatures
- Up to X-ray frequencies for higher temperatures (e.g. intra-cluster medium)

$$\epsilon_{\nu,T} = \int_{\mathrm{v}_{\min}}^{\infty} \epsilon_{\mathrm{v},\nu} f(\mathrm{v})d\,\mathrm{v} = 6.6\times10^{-38} T^{-1/2} e^{-h\nu/kT} n_e n_Z Z^2 \overline{g_{ff}(\nu,T)}\ erg\ cm^{-3}\ s^{-1}\ Hz^{-1}$$

$T^{-1/2}$: ~ particle energy

$\overline{g_{ff}(\nu,T)}$: ~ 1-15

where: $\frac{1}{2} m_e \mathrm{v}_{\min} \geq h\nu \rightarrow \mathrm{v}_{\min} \geq \sqrt{\frac{2h\nu}{m_e}}$

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Thermal bremsstrahlung

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Integrating over the whole spectrum:

$$\epsilon_T = \int \epsilon_{\nu,T}\, d\nu \simeq 2.4\times10^{-27} T^{1/2} n_e n_Z Z^2 \overline{g_{ff}(\nu,T)}\ erg\, cm^{-3}\, s^{-1}$$

Thermal bremsstrahlung

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Cooling time:

$$t_{br} \sim \frac{3/2(n_e+n_Z)kT}{\epsilon_T} \approx \frac{3 n_e k T}{\epsilon_T} = \frac{1.8\times10^{11}}{n_e g_{ff}(\nu,T)} T^{1/2}\,\mathrm{sec} = \frac{6\times10^{13}}{n_e g_{ff}(\nu,T)} T^{1/2}\,\mathrm{yr}$$

= Total thermal energy / Radiative losses

Thermal bremsstrahlung

$$\epsilon_{\nu,T} = \int_{\nu_{min}}^{\infty} \epsilon_{\nu,\nu} f(\nu) d\nu = 6.6\times10^{-38} T^{-1/2} e^{-h\nu/kT} n_e n_Z Z^2 \overline{g_{ff}(\nu,T)}\ erg\,cm^{-3}\,s^{-1}\,Hz^{-1}$$

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Exemples:

- HII regions: n_e~10^2-10^3 cm^{-3} & T~10^3-10^4 K → t_{br}~100-1000 yr
- Gas in galaxy clusters: n_e~10^{-3} cm^{-3} & T~10^7-10^8 K → t_{br}~10^{10} yr

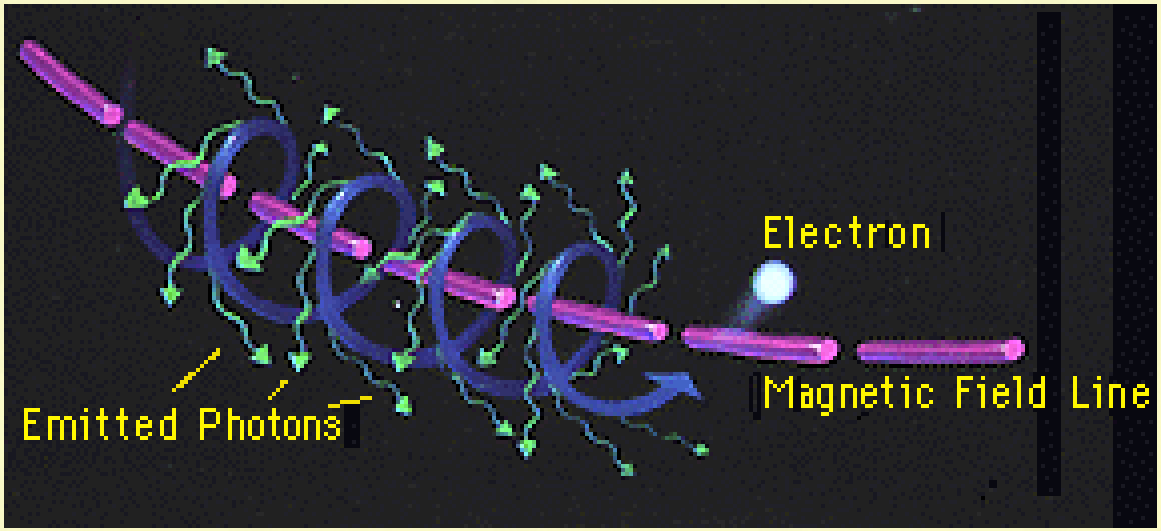
Main cooling process at temperatures above 10^7 K

Lecture Outline

- Emission mechanisms
  - Thermal processes: Black Body & Bremsstrahlung
  - **Non-thermal processes: Synchrotron** & Inverse Compton
  - Line processes: just very few and basic concepts!!!

Synchrotron radiation

Particles accelerated by a magnetic field will radiate. For non-relativistic velocities the radiation is called **cyclotron radiation**. Its frequency of emission is the frequency of gyration in the magnetic field. For extremely relativistic particles the frequency spectrum is much more complex and can extend to many times the gyration frequency. This radiation is known as **synchrotron radiation**.



Motion of a charged particle in a magnetic field

Uniform magnetic field

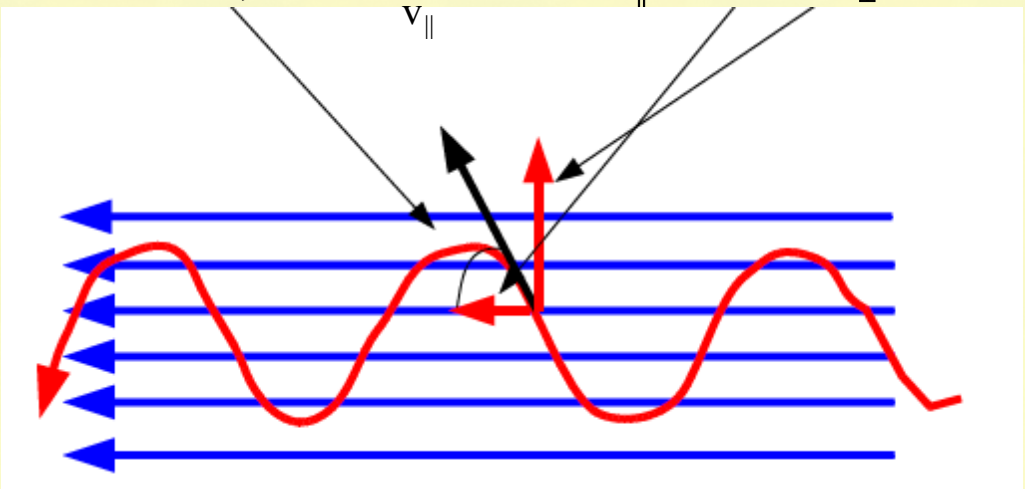
From **Lorentz force** we know that the equation of motion of a charge *q* in a uniform magnetic field ***B*** is:

$$\frac{d\vec{p}}{dt} = \frac{q}{c}\vec{v}\times\vec{H}$$

$$\vec{p} = m\vec{v} \qquad \vec{B} = \frac{\vec{H}}{c}$$

θ = pitch angle = angle between the directions of $\vec{v}$ and $\vec{H}$

$$\theta;\ \tan\theta = \frac{v_\perp}{v_\parallel} \qquad v_\parallel = v\cos\theta \qquad v_\perp = v\sin\theta$$



Motion of a charged particle in a uniform B

$$\frac{d(m\mathrm{v}_{\parallel})}{dt}=0 \longrightarrow \mathrm{v}_{\parallel} \text{ is constant}$$

- uniform motion along **B** direction (♣)
- since v is constant, also $\mathrm{v}_{\perp}$ is constant (♦)

the pitch angle remains constant

$$\frac{d(m\mathrm{v}_{\perp})}{dt}=\frac{q}{c}\vec{v}_{\perp}\times\vec{H} \quad + (\blacklozenge) \longrightarrow$$

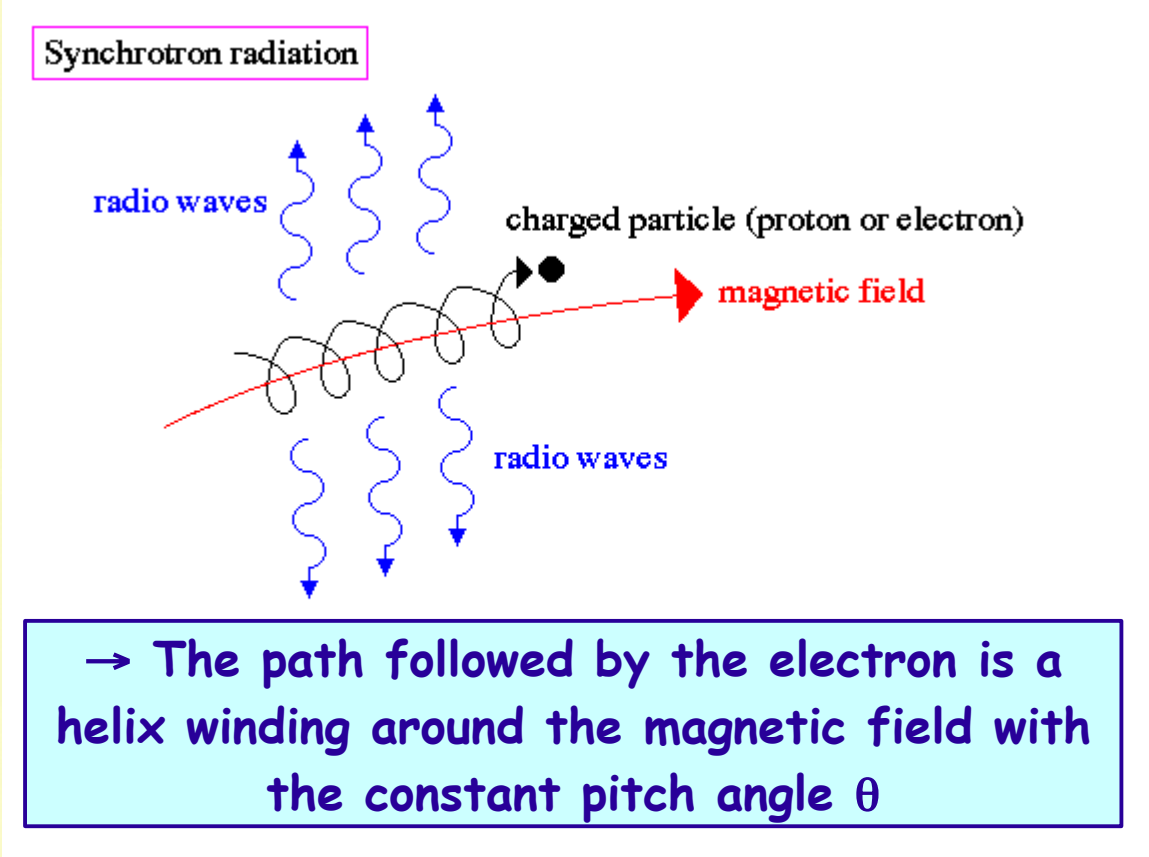
circular motion with a constant orbital velocity $\mathrm{v}_{\perp}$ and:

- curvature radius: $r_L = \frac{mc}{qH}v_{\perp}$
- revolution period: $T_L = \frac{2\pi r_L}{v_{\perp}} = \frac{2\pi mc}{qH}$
- frequency of the gyration: $\omega_L = \frac{2\pi}{T_L} = \frac{qH}{mc}$

Motion of a charged particle in a uniform B

$$\frac{d(m v_{\parallel})}{dt} = 0 \longrightarrow v_{\parallel} \text{ is constant} \longrightarrow \underline{\text{uniform motion}} \text{ along } \mathbf{B} \text{ direction}$$

$$\frac{d(m v_{\perp})}{dt} = \frac{q}{c}\vec{v}_{\perp} \times \vec{H} \longrightarrow \underline{\text{circular motion}} \text{ with a constant orbital velocity } v_{\perp}$$



→ The path followed by the electron is a helix winding around the magnetic field with the constant pitch angle θ

Relativistic case

The relativistic case is dealt with by considering the appropriate expression of the particle mass:

$$m = \frac{m_0}{\sqrt{1-\beta^2}} = m_0\gamma \qquad \gamma = 1/\sqrt{1-v^2/c^2} = 1/\sqrt{1-\beta^2}$$

$$r_{rel} = \frac{\gamma m_0 c}{qH} v_\perp = \gamma r_L$$

$$T_{rel} = \frac{2\pi\, r_L}{v_\perp} = \frac{2\pi m_0 c\gamma}{qH} = \gamma T_L$$

$$\omega_{rel} = \frac{2\pi}{T_rel} = \frac{qH}{\gamma mc} = \frac{\omega_L}{\gamma}$$

v ~ c

$$r_{rel} = \frac{\gamma\, m_0 c^2}{q\,H} = \frac{\epsilon}{q\,H}$$

total energy of the particle

Synchrotron radiation of a relativistic particle

Let's consider a charge q in a magnetic field **B**.
From the Larmor's formula you can demonstrate that:

$$P = -\frac{dW}{dt} = \frac{2}{3}\frac{q^2}{m^2 c^3}\gamma^2\left(\frac{d\vec{p}}{dt}\right)^2$$

$$+\ \frac{d(m\mathrm{v}_{\parallel})}{dt} = 0 \quad + \quad \frac{d(m\mathrm{v}_{\perp})}{dt} = \frac{q}{c}\vec{v}_{\perp}\times\vec{H} \quad + \quad v \approx c \rightarrow \beta \approx 1$$

=

$$P = -\frac{dW}{dt} \approx \frac{2}{3}\frac{q^2}{m^2 c^3}\gamma^2\frac{q^2}{c^2}c^2(\sin^2\theta)H^2 = \frac{2}{3}\frac{q^4}{m^2 c^3}\gamma^2 H_{\perp}{}^2$$

$$+\ \epsilon = \gamma m c^2$$

=

$$P = -\frac{dW}{dt} \approx \frac{2}{3}\frac{q^4}{m^2 c^3}\left(\frac{\epsilon}{mc^2}\right)^2 H_{\perp}{}^2 \propto \epsilon^2 H_{\perp}^2$$

$$P = -\frac{dW}{dt} \approx \frac{2}{3}\frac{q^4}{m^2 c^3}\left(\frac{\epsilon}{mc^2}\right)^2 H_\perp{}^2 \propto \epsilon^2 H_\perp^2$$

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65\times 10^{-25}\, cm^{-2}$$

$$P = -\frac{dW}{dt} = 2c\sigma_T\left(\frac{m_e}{m_0}\right)^2\left(\frac{q}{e}\right)^4 \gamma^2 \frac{H_\perp^2}{8\pi} \xrightarrow{\text{for e-}} P = 2c\sigma_T\gamma^2\frac{H_\perp^2}{8\pi}$$

Synchrotron radiation

In the reference frame of the observer

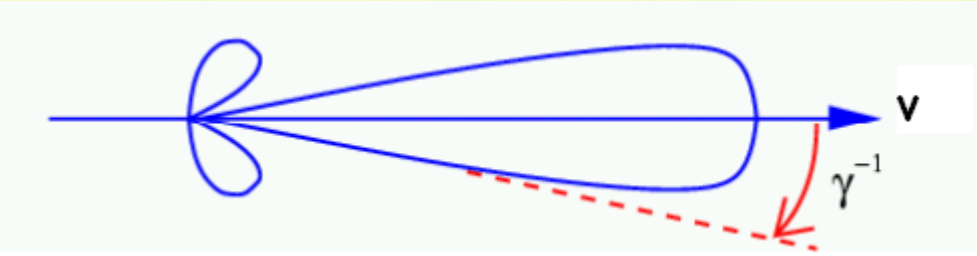
In the reference frame of the particle

$$\frac{dW}{d\Omega} = \frac{1}{\gamma^3 (1-\beta\cos\theta)^3} \frac{dW'}{d\Omega'}$$

$$\frac{dW}{dt\, d\Omega} = \frac{1}{\gamma^4 (1-\beta\cos\theta)^4} \frac{dW'}{dt'\, d\Omega'}$$

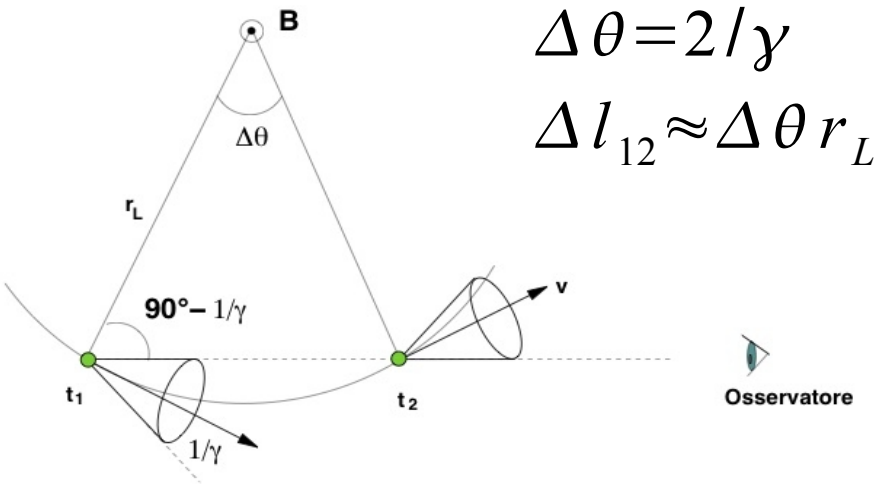
$$dt = \gamma (1-\beta\cos\theta)\, dt'$$

$$\gamma \gg 1 \wedge \beta \sim 1 \rightarrow \frac{1}{\gamma^4 (1-\beta\cos\theta)^4} \approx \left(\frac{2\gamma}{1+\gamma^2\theta^2} \right)^4 \quad \theta \approx 0$$



For relativistic electrons, half of the radiation is concentrated in a very narrow cone of semi-aperture ≈ 1/γ

Synchrotron radiation: pulse duration



$$\Delta\theta = 2/\gamma$$

$$\Delta l_{12} \approx \Delta\theta\, r_L$$

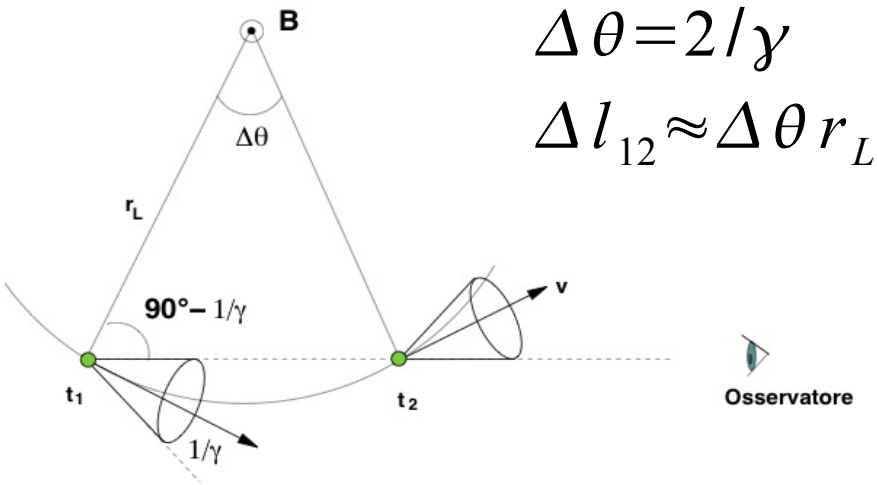
Time necessary to the electron to move from 1 to 2

$$\Delta t_e = \frac{\Delta\theta\, r_L}{v} = \frac{\Delta\theta}{\omega_L}$$

Time necessary to the radiation to move from 1 to 2

$$\Delta t_f = \frac{\Delta\theta\, r_L}{c} = \frac{\Delta\theta}{\omega_L}\beta$$

Synchrotron radiation: pulse duration



$$\Delta\theta = 2/\gamma$$

$$\Delta l_{12} \approx \Delta\theta\, r_L$$

Time necessary to the electron to move from 1 to 2

$$\Delta t_e = \frac{\Delta\theta\, r_L}{v} = \frac{\Delta\theta}{\omega_L}$$

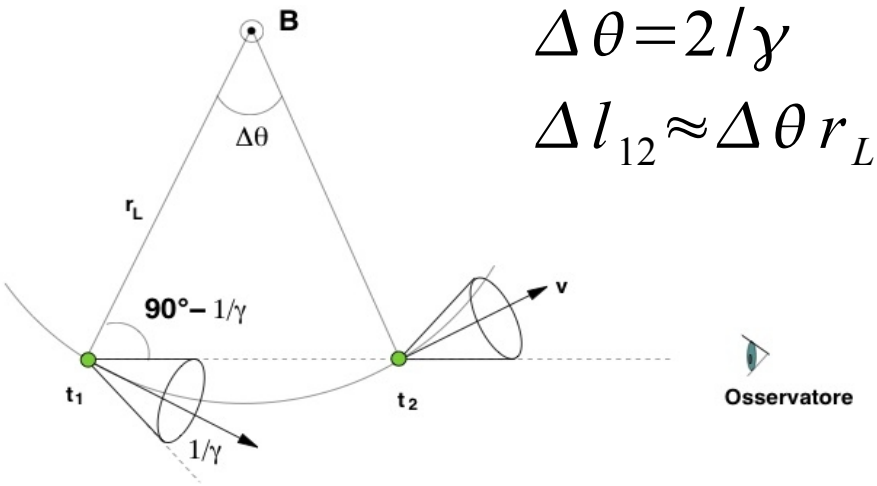
Time necessary to the radiation to move from 1 to 2

$$\Delta t_f = \frac{\Delta\theta\, r_L}{c} = \frac{\Delta\theta}{\omega_L}\beta$$

Pulse duration:

$$\tau = \Delta t_e - \Delta t_f = \frac{\Delta\theta}{\omega_L}(1-\beta) = \frac{\Delta\theta}{\omega_L}\left(\frac{1-\beta^2}{1+\beta}\right)$$

Synchrotron radiation: pulse duration



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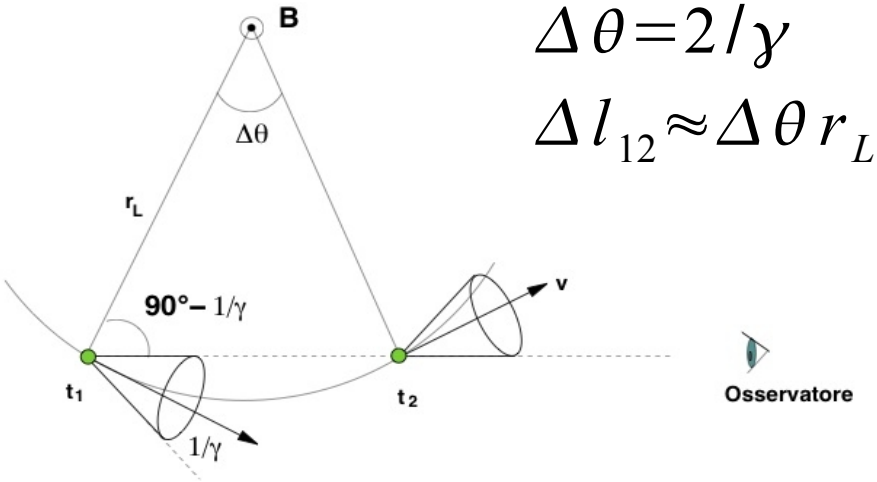
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$$\beta \simeq 1 \wedge \Delta\theta = 2/\gamma \rightarrow \tau = \frac{1}{\gamma^3 \omega_L} \sim \frac{T_L}{\gamma^3}$$

Synchrotron radiation: pulse duration



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Pulse duration:

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$$\beta \simeq 1 \wedge \Delta\theta = 2/\gamma \rightarrow \tau = \frac{1}{\gamma^3 \omega_L} \sim \frac{T_L}{\gamma^3}$$

$$\omega_L = qH/m_0\gamma c \rightarrow \tau \propto m_e \frac{c}{\gamma^2 eH}$$

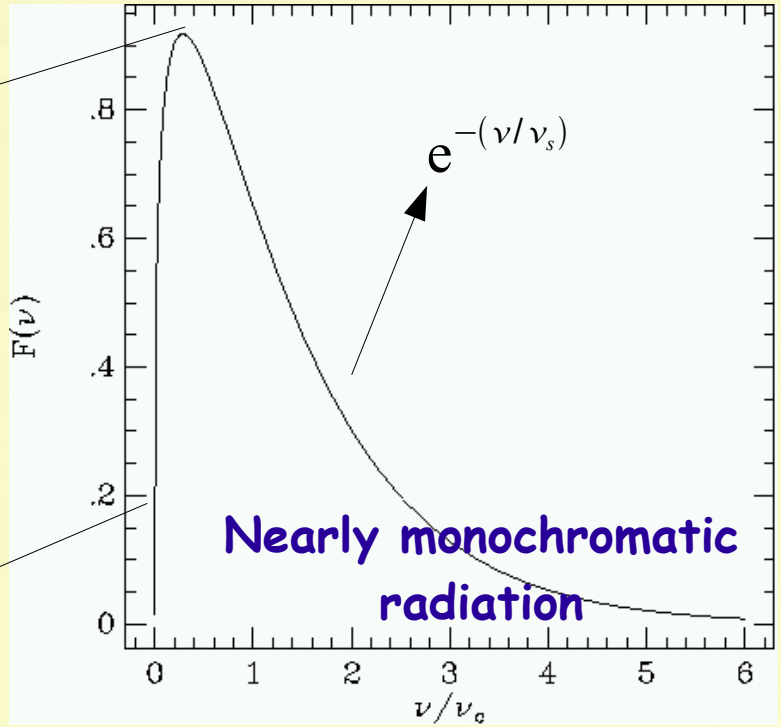
$$\tau = \frac{5\times10^{-8}}{\gamma^2 (H/Gauss)}\ sec$$

Synchrotron radiation of a single charge: summary

- Charge + magnetic field (constant or not)
- The path followed by the charge is a helix, whose axis is parallel to the magnetic field line of forces
- In the case of relativistic particles, the emission is within a cone with semi aperture $1/\gamma$, centered on the instantaneous velocity direction. The observer receive the radiation when his line of size is inside this cone
- It can be shown that the spectral distribution of the radiation is that shown below:

$$\nu_m = \frac{1}{3}\nu_s = \frac{1}{4\pi}\gamma^2\frac{eH_\perp}{m_e c} = \frac{1}{4\pi}\frac{eH_\perp}{m_e^3 c^5}\epsilon^2$$

Critical frequency



Synchrotron radiation from a set of relativistic e⁻

In astrophysical conditions we have always to deal not with one electron, but with a set of electrons characterized by very different energies → γ values.

Since the characteristic frequency of radiation of a single electron is proportional to γ^2, it is clear that it is sufficient to slightly variate γ to cover the whole radio band.

If we assume that each particle radiates independently, the total emission from a sample of relativistic electrons is given by the sum of the single contributions.

$N(\epsilon)d\epsilon$ = number of particles per unit volume whose energy is between ε and ε+dε

$$(\epsilon_\nu) = J_s(\nu) = P(\nu)N(\epsilon)\frac{d\epsilon}{d\nu} = -\frac{dW(\nu,\epsilon)}{dt}\frac{N(\epsilon)d\epsilon}{d\nu}$$ = specific emissivity

$$P = 2c\sigma_T\gamma^2\frac{H_\perp^2}{8\pi} \quad = \quad J_S(\nu) = 2c\sigma_T\gamma^2\frac{H^2}{8\pi}\sin^2\theta\frac{N(\epsilon)d\epsilon}{d\nu}$$

Synchrotron radiation: power law energy distribution

$$J_S(\nu) = 2 c \sigma_T \gamma^2 \frac{H^2}{8\pi} \sin^2\theta \frac{N(\epsilon) d\epsilon}{d\nu}$$

Mean angle between the line of sight and the magnetic field

We assume: $N(\epsilon) d\epsilon = N_0 \epsilon^{-\delta}$

- Energy spectrum of cosmic rays
- In agreement with observations of galactic and extra-galactic radio sources
- Some acceleration mechanisms give rise to a power law energy distribution

$$\rightarrow \quad J_S(\nu) = 2 c \sigma_T \gamma^2 \frac{H^2}{8\pi} \sin^2\theta \frac{N_0 \epsilon^{-\delta} d\epsilon}{d\nu}$$

There are various approximate ways to derive the total emissivity:

1) all the energy is radiated at the characteristic frequency ν_m

2) the energy is radiated at a constant rate over a small frequency range

Synchrotron radiation: power law energy distribution

$$J_s(\nu) = P(\nu) N(\epsilon) \frac{d\epsilon}{d\nu} = -\frac{dW(\nu,\epsilon)}{dt} \frac{N_0 \epsilon^{-\delta} d\epsilon}{d\nu}$$ = specific emissivity

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1) all the energy is radiated at the characteristic frequency ν_m

$$P = -\frac{dW}{dt} = \frac{2}{3} \frac{q^4}{m^2 c^3} \left(\frac{\epsilon}{m c^2} \right)^2 H_\perp{}^2 \qquad (1)$$

Synchrotron radiation: power law energy distribution

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$$\nu \approx \nu_m = \frac{1}{3}\nu_s = \frac{1}{4\pi} \gamma^2 \frac{e H_\perp}{m_e c} = \frac{1}{4\pi} \frac{e H_\perp}{m_e^3 c^5} \epsilon^2 \quad (2)$$

Synchrotron radiation: power law energy distribution

$$J_s(\nu) = P(\nu) N(\epsilon) \frac{d\epsilon}{d\nu} = -\frac{dW(\nu,\epsilon)}{dt} \frac{N_0 \epsilon^{-\delta} d\epsilon}{d\nu}$$ = specific emissivity

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$$\epsilon^2 = \underbrace{\left(\frac{4\pi m_e^3 c^5}{e}\right)}_{a} \frac{\nu}{H_\perp} \quad (3)$$

Synchrotron radiation: power law energy distribution

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$$\rightarrow \quad \epsilon = a^{1/2} \nu^{1/2} H_\perp^{\,-1/2} \quad (4)$$

Synchrotron radiation: power law energy distribution

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Synchrotron radiation: power law energy distribution

$$J_s(\nu) = P(\nu) N(\epsilon) \frac{d\epsilon}{d\nu} = -\frac{dW(\nu,\epsilon)}{dt} \frac{N_0 \epsilon^{-\delta} d\epsilon}{d\nu}$$ = specific emissivity

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$$\epsilon^2 = \underbrace{\left(\frac{4\pi m_e^3 c^5}{e}\right)}_{a} \frac{\nu}{H_\perp} \quad (3)$$

$$\rightarrow \quad \epsilon = a^{1/2} \nu^{1/2} H_\perp{}^{-1/2} \quad (4) \qquad \rightarrow \quad \frac{d\epsilon}{d\nu} = \frac{1}{2} a^{1/2} \nu^{-1/2} H_\perp{}^{-1/2} \quad (5)$$

(1) + (3) $\rightarrow$

$$P = -\frac{dW}{dt} = \frac{2}{3} \frac{e^4}{m_e^2 c^3} \left(\frac{1}{m_e c^2}\right)^2 H_\perp{}^2 \left(\frac{4\pi m_e^3 c^5}{e}\right) \frac{\nu}{H_\perp} = \frac{8\pi}{3} \frac{e^3}{m_e c^2} \nu H_\perp \quad (6)$$

Synchrotron radiation: power law energy distribution

$$J_s(\nu)=P(\nu)N(\epsilon)\frac{d\epsilon}{d\nu}=-\frac{dW(\nu,\epsilon)}{dt}\frac{N_0\epsilon^{-\delta}d\epsilon}{d\nu}$$ = specific emissivity

1) all the energy is radiated at the characteristic frequency ν_m

$$P=-\frac{dW}{dt}=\frac{2}{3}\frac{q^4}{m^2c^3}\left(\frac{\epsilon}{mc^2}\right)^2 H_\perp{}^2 \quad (1)$$

$$\nu\approx\nu_m=\frac{1}{3}\nu_s=\frac{1}{4\pi}\gamma^2\frac{eH_\perp}{m_ec}=\frac{1}{4\pi}\frac{eH_\perp}{m_e^3c^5}\epsilon^2 \quad (2)$$

$$\epsilon^2=\underbrace{\left(\frac{4\pi m_e^3c^5}{e}\right)}_{a}\frac{\nu}{H_\perp} \quad (3)$$

$$\rightarrow \quad \epsilon=a^{1/2}\nu^{1/2}H_\perp{}^{-1/2} \quad (4) \qquad \rightarrow \quad \frac{d\epsilon}{d\nu}=\frac{1}{2}a^{1/2}\nu^{-1/2}H_\perp{}^{-1/2} \quad (5)$$

(1) + (3) $\rightarrow$

$$P=-\frac{dW}{dt}=\frac{2}{3}\frac{e^4}{m_e^2c^3}\left(\frac{1}{m_ec^2}\right)^2 H_\perp{}^2\left(\frac{4\pi m_e^3c^5}{e}\right)\frac{\nu}{H_\perp}=\frac{8\pi}{3}\frac{e^3}{m_ec^2}\nu H_\perp \quad (6)$$

$$J_s(\nu)=\underbrace{\frac{8\pi}{3}\frac{e^3}{m_ec^2}\nu H_\perp}_{(6)}N_0\epsilon^{-\delta}\underbrace{\frac{1}{2}a^{1/2}H_\perp{}^{-1/2}\nu^{-1/2}}_{(5)}=\underbrace{\frac{4\pi}{3}\frac{e^3}{m_ec^2}}_{b}\nu H_\perp N_0\underbrace{a^{-\delta/2}\nu^{-\delta/2}H_\perp{}^{\delta/2}}_{(4)}a^{1/2}H_\perp{}^{-1/2}\nu^{-1/2}$$

Synchrotron radiation: power law energy distribution

1) all the energy is radiated at the characteristic frequency ν_m

$$J_s(\nu)=\frac{8\pi}{3}\frac{e^3}{m_e c^2}\nu H_\perp N_0 \epsilon^{-\delta}\frac{1}{2}a^{1/2}H_\perp{}^{-1/2}\nu^{-1/2}=b\nu H_\perp N_0 a^{-\delta/2}\nu^{-\delta/2}H_\perp^{\delta/2}a^{1/2}H_\perp{}^{-1/2}\nu^{-1/2}$$

$$a=\frac{4\pi m_e^3 c^5}{e}\wedge b=\frac{4\pi e^3}{3 m_e c^2}$$

$$J_s(\nu)=f(\delta)N_0\nu^{-\frac{\delta-1}{2}}H_\perp^{\frac{\delta+1}{2}}\propto N_0\nu^{-\alpha}H_\perp^{\alpha+1}$$

where: $\alpha=\frac{\delta-1}{2}$ = spectral index of the synchrotron radiation

Synchrotron radiation: power law energy distribution

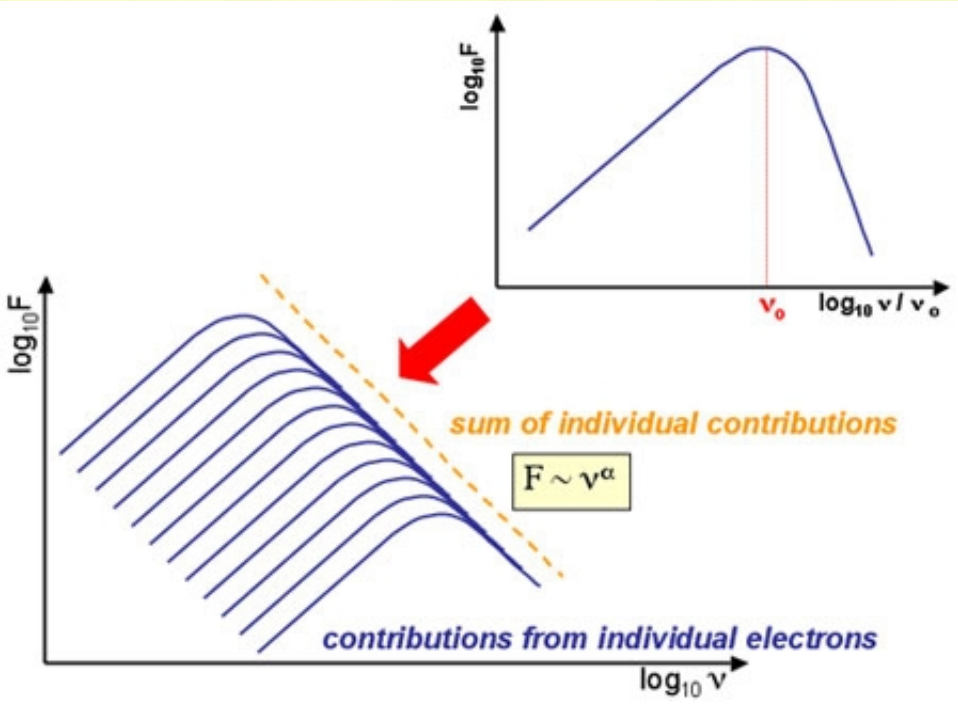
2) the energy is radiated at a constant rate over a small frequency range

$$J_s(\nu) = f'(\delta) N_0 \nu^{-\frac{\delta-1}{2}} H_\perp^{\frac{\delta+1}{2}} \propto N_0 \nu^{-\alpha} H_\perp^{\alpha+1}$$

where: $f'(\delta) \approx f(\delta)$

Synchrotron radiation: power law energy distribution

$$J_s(\nu) = b\, a^{-\frac{\delta-1}{2}} N_0 \nu^{-\frac{\delta-1}{2}} H_\perp^{\frac{\delta+1}{2}} \propto N_0 \nu^{-\alpha} H_\perp^{\alpha+1} \qquad \alpha = \frac{\delta-1}{2}$$



The total spectrum is interpreted as the superposition of many contributions from the various electrons each emitting at its own characteristic frequency

Synchrotron self-absorption

Synchrotron self-absorption (SSA):

a photon gives back its energy to an electron

NOT in thermal equilibrium

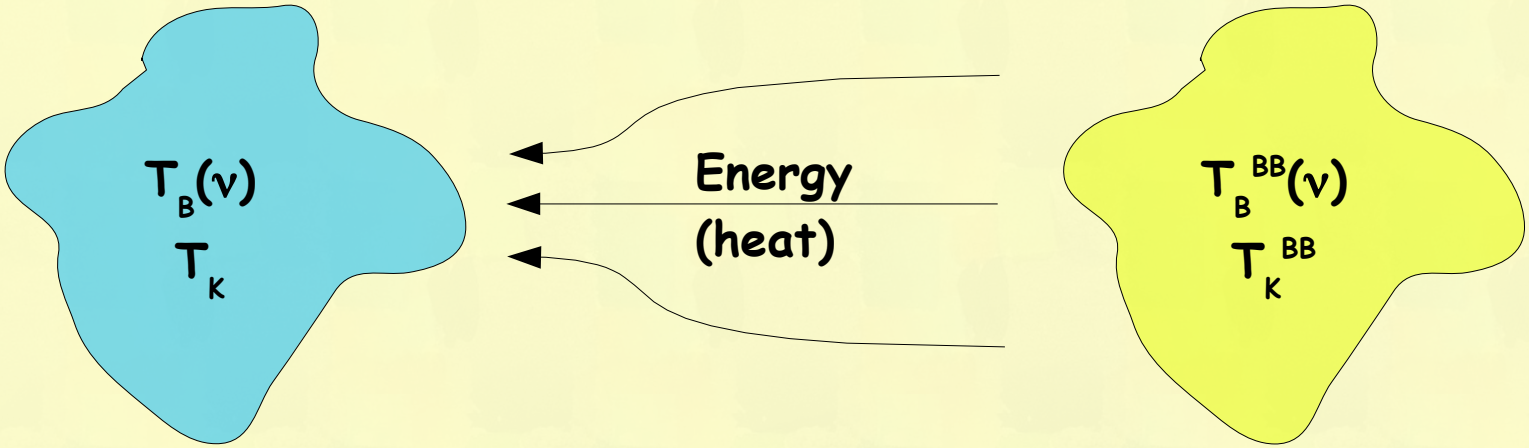
- The concept of brightness temperature T_B still holds
- Electron temperature = kinetic temperature of electrons: $3 k T_k = \gamma m_e c^2 = \epsilon$

relativistic particles

SSA applies when $T_B \sim T_k$: T_B can never exceed T_k

Brightness and kinetic temperatures

The brightness $T_B(\nu)$ temperature never exceeds the kinetic temperature T_K



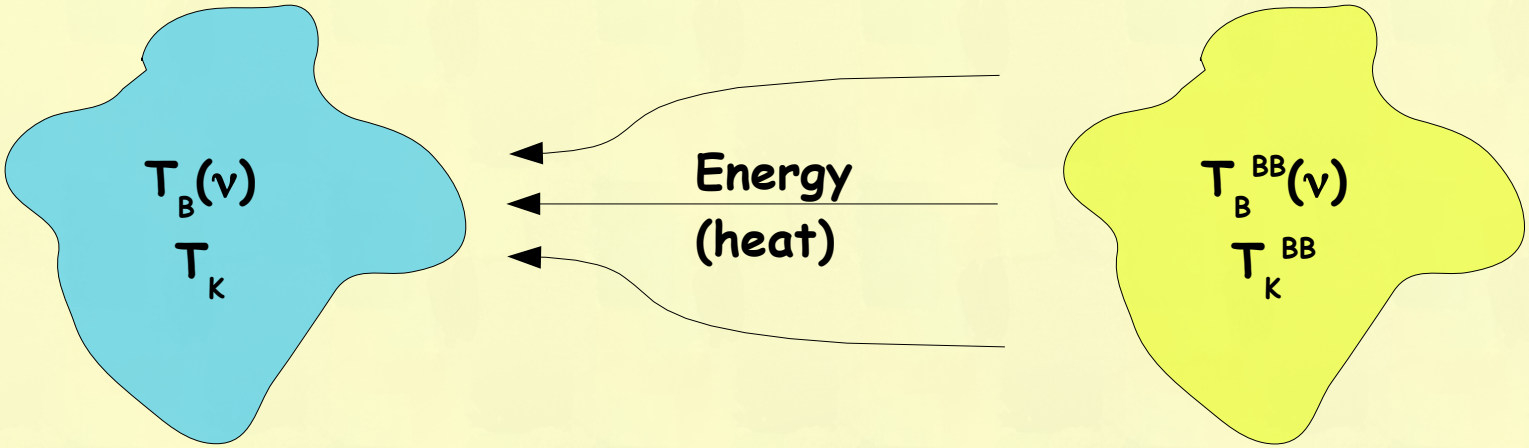
$T_B(\nu) > T_K$ $\qquad$ $T_B^{BB}(\nu) = T_K^{BB}$

$T_B^{BB}(\nu) = T_K^{BB}$ are chosen to be between $T_B(\nu)$ and T_K

Then, let's couple the two bodies:

Brightness and kinetic temperatures

The brightness $T_B(\nu)$ temperature never exceeds the kinetic temperature T_K



$T_B(\nu) > T_K$ $\qquad$ $T_B^{BB}(\nu) = T_K^{BB}$

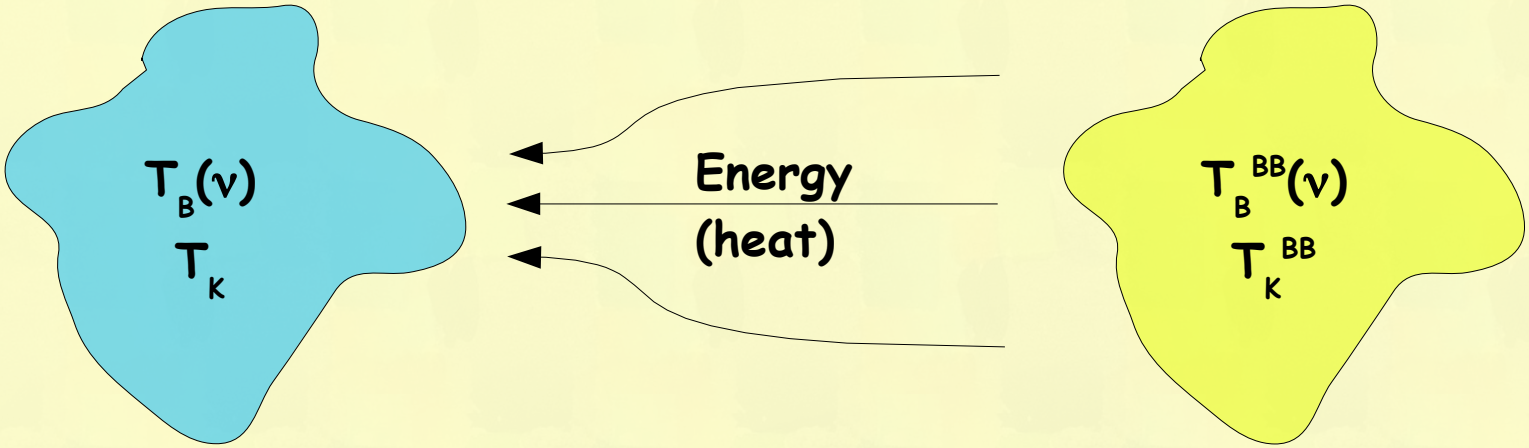
$T_B^{BB}(\nu) = T_K^{BB}$ are chosen to be between $T_B(\nu)$ and T_K

Then, let's couple the two bodies:

$$T_K^{BB}(\nu) > T_K$$

Brightness and kinetic temperatures

The brightness $T_B(\nu)$ temperature never exceeds the kinetic temperature T_K



$T_B(\nu) > T_K$ $\qquad$ $T_B^{BB}(\nu) = T_K^{BB}$

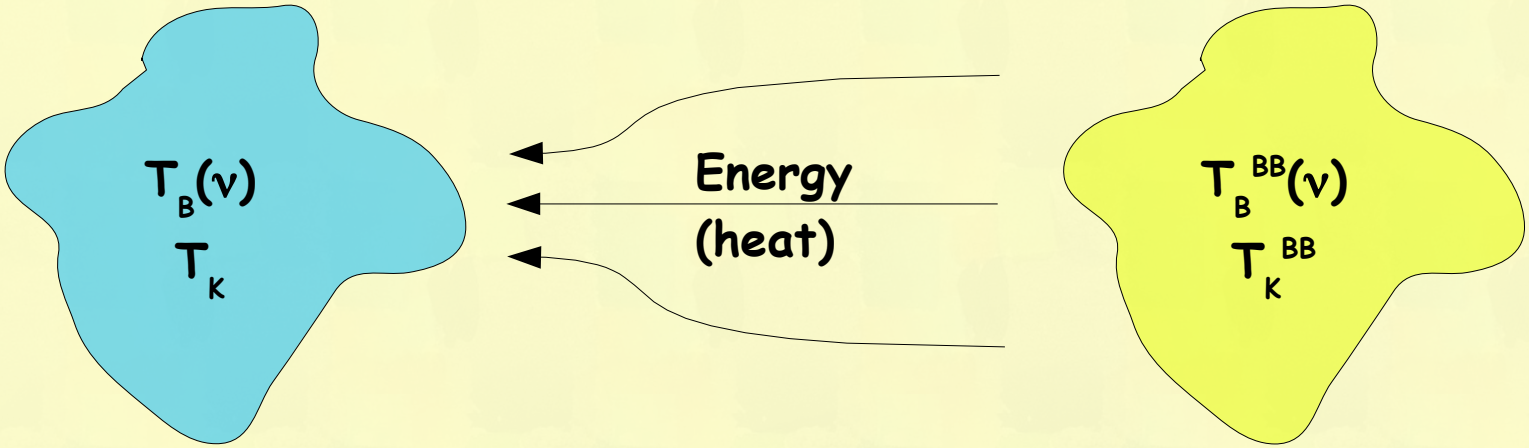
$T_B^{BB}(\nu) = T_K^{BB}$ are chosen to be between $T_B(\nu)$ and T_K

Then, let's couple the two bodies:

$$T_B^{BB}(\nu) = T_K^{BB}(\nu) > T_K$$

Brightness and kinetic temperatures

The brightness $T_B(\nu)$ temperature never exceeds the kinetic temperature T_K



$T_B(\nu) > T_K$ $\qquad$ $T_B^{BB}(\nu) = T_K^{BB}$

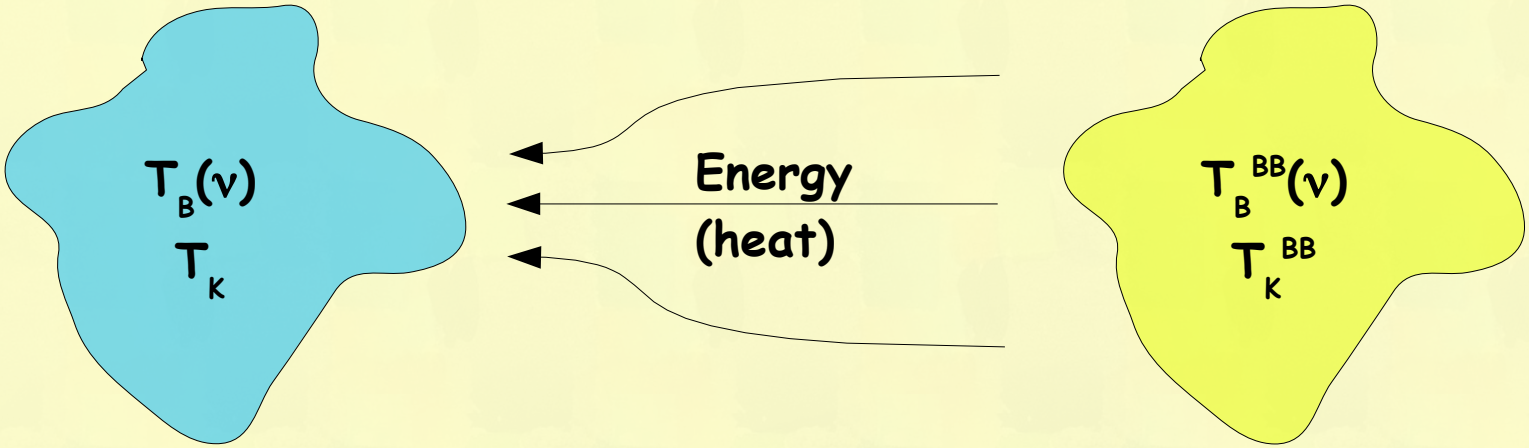
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Then, let's couple the two bodies:

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Brightness and kinetic temperatures

The brightness $T_B(\nu)$ temperature never exceeds the kinetic temperature T_K



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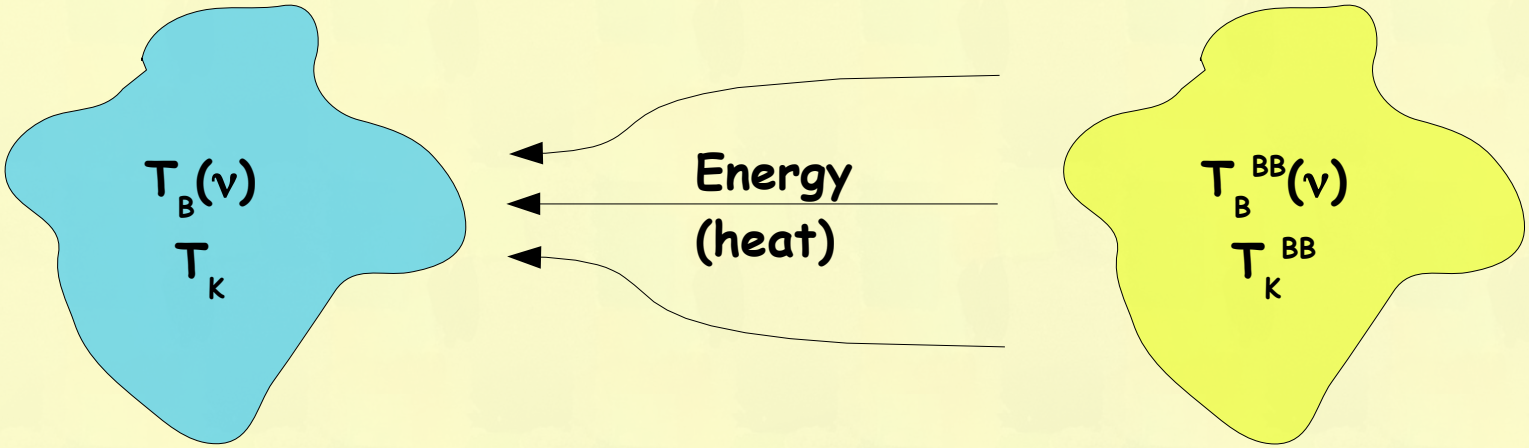
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Synchrotron self-absorption

Synchrotron self-absorption (SSA):

a photon gives back its energy to an electron

NOT in thermal equilibrium, the Kirchoff's law does not apply

SSA applies when $T_B \sim T_k$: T_B can never exceed T_k

$$T_B = \frac{1}{2k} B(\nu) \frac{c^2}{\nu^2} \propto \nu^{-\alpha-2}$$

$$B(\nu) \propto \nu^{-\alpha}$$

$$T_k \approx \frac{1}{3} \frac{\epsilon}{k} \propto H^{-1/2} \nu^{1/2}$$

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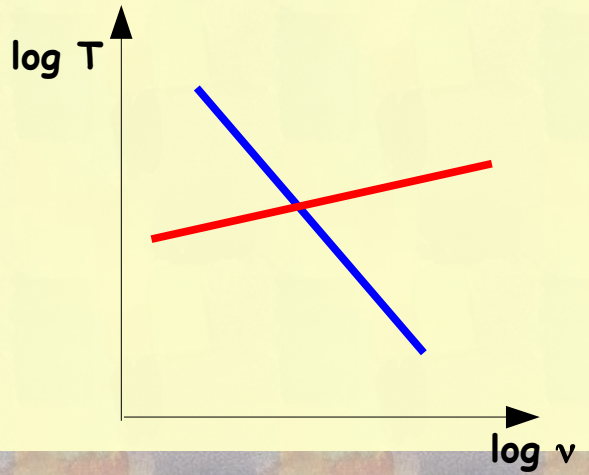
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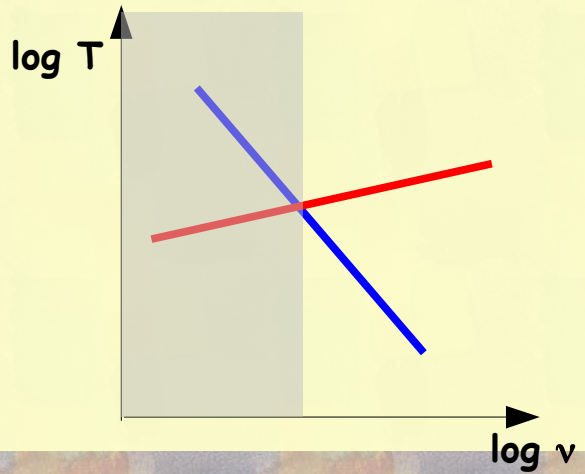
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At a certain frequency the brightness temperature would be higher than the kinetic temperature: IMPOSSIBLE!!!

Synchrotron self-absorption

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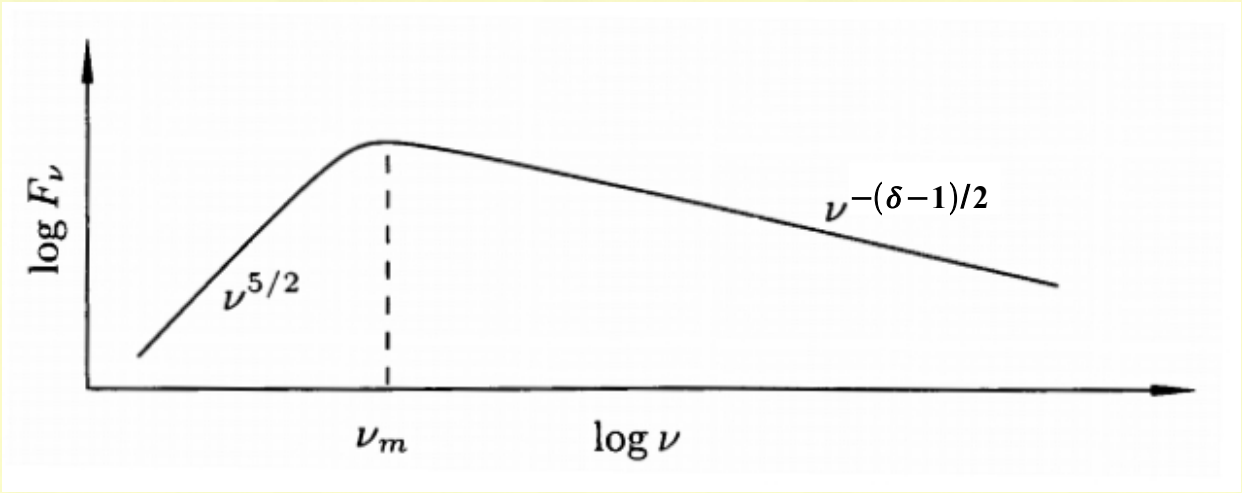
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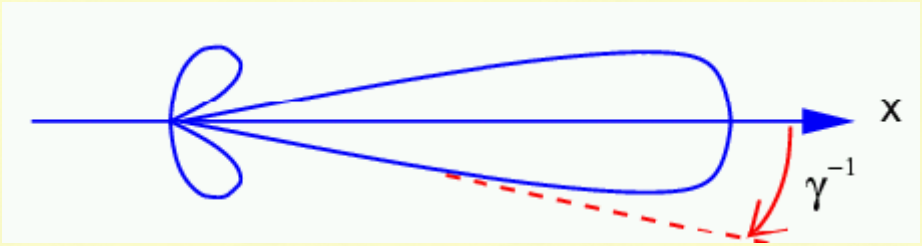
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Polarization

$\vec{E} \| \vec{a}$



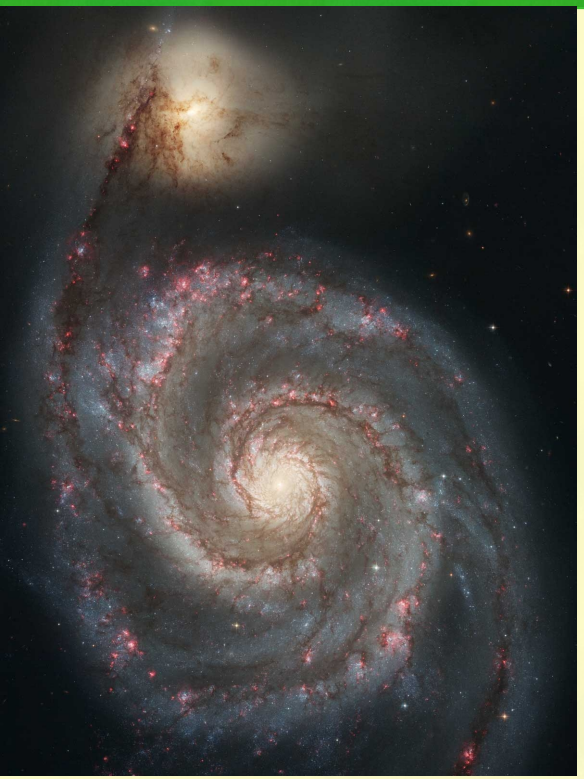
Elliptical polarization, a_{max} perpendicular to **B**

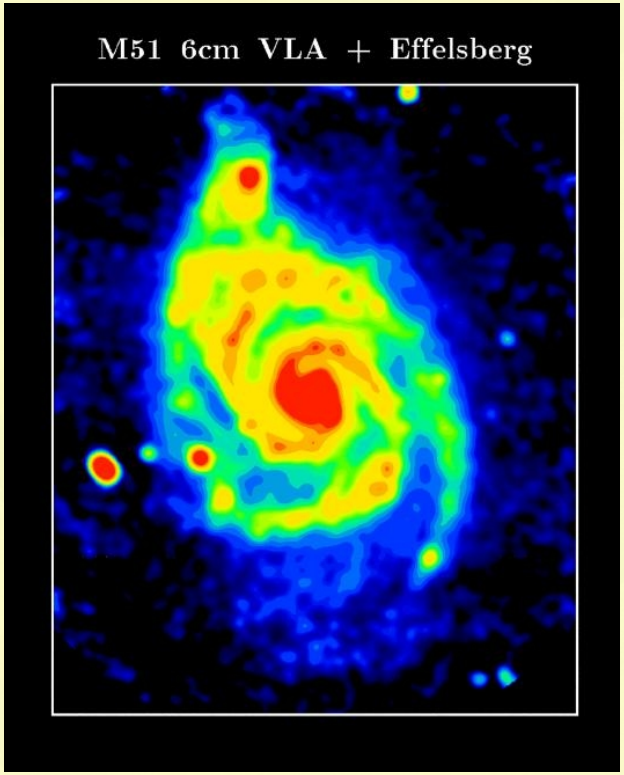
Linear polarization, perpendicular to **B**

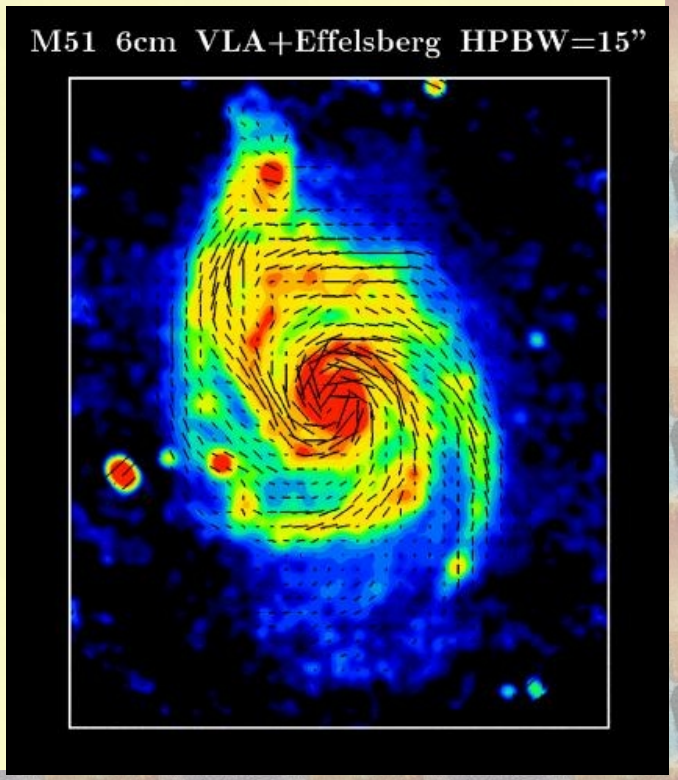
Set of relativistic electrons:

- We receive radiation only from those with velocities within a cone of semi-aperture $1/\gamma$ centered on the line of sight axis
- If the magnetic field is ordered the radiation is still polarized
- Since electron velocities are not ordered, the ellipses of polarization have different orientations, and they cancel each other. We will have therefore only linear polarization, perpendicular to the ordered magnetic field
- The polarized signal is thus intrinsically lower than the total one (maximum ~70%)

Polarization







Evolution of the synchrotron spectrum

Time evolution of a synchrotron emitting region with volume V, filled by a magnetized relativistic plasma

$$\frac{\partial \mathcal{N}(\varepsilon,t)}{\partial t} + \frac{\partial}{\partial \varepsilon}\left(\frac{d\varepsilon}{dt}\mathcal{N}(\varepsilon,t)\right) + \frac{\mathcal{N}(\varepsilon,t)}{T_{conf}} = Q(\varepsilon,t)$$

Particle flow | Energy losses | Leakage | Injection

$$\mathcal{N}(\varepsilon,0) = \mathcal{N}_o\,\varepsilon^{-\delta}$$

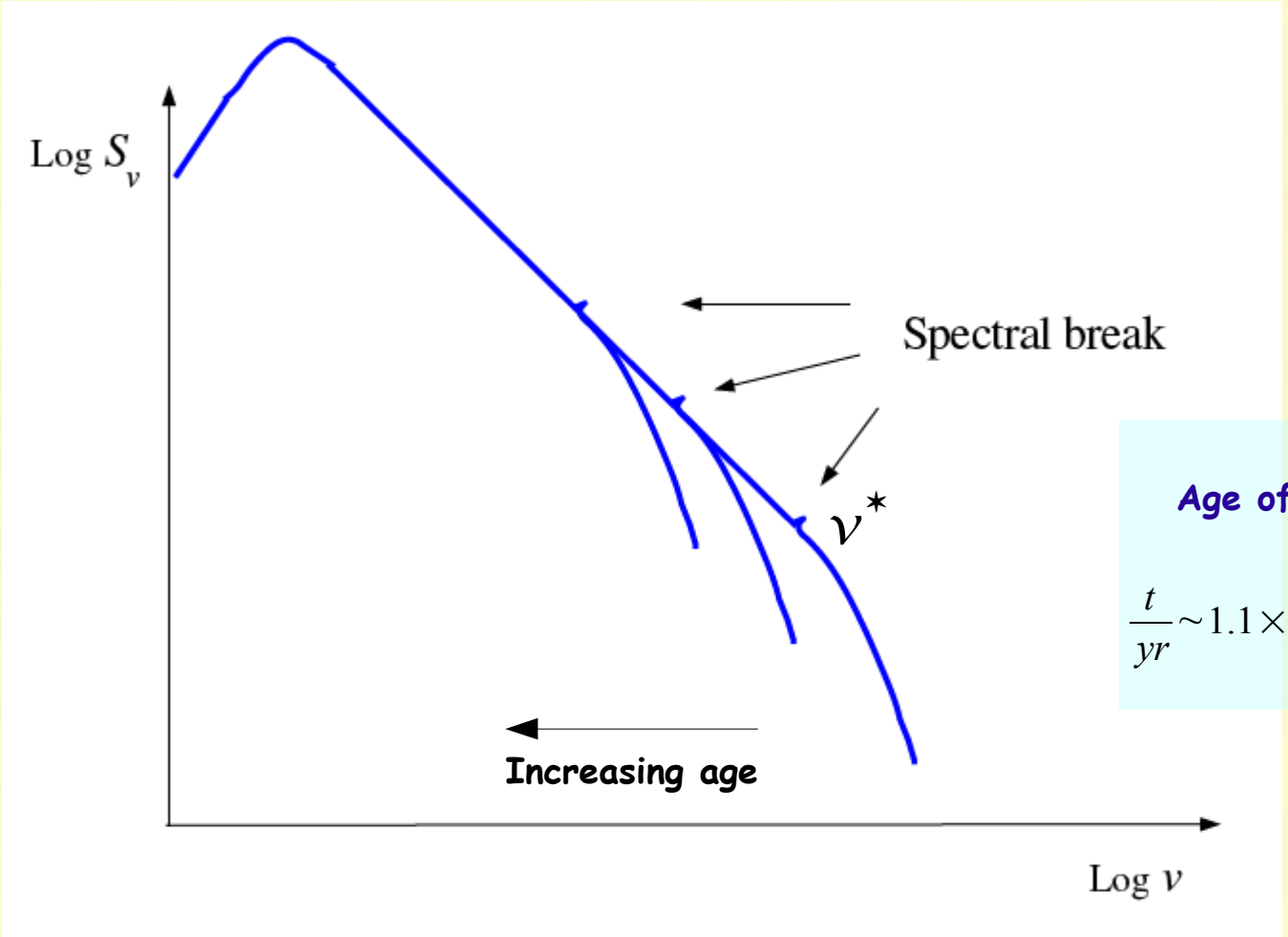
$$Q(\varepsilon,t) = \mathcal{A}\,\varepsilon^{-\delta}$$

Main processes responsible for energy losses:

- Adiabatic expansion of the radio source
- Radiative losses due to the synchrotron emission
- Radiative losses due to Inverse Compton emission (see below)

Evolution of the synchrotron spectrum

High energy (i.e. high frequency) particles have shorter radiative life time than low energy ones and the emission spectrum is modified accordingly due to the depletion of high energy particles from the energy distribution



Age of the radio source

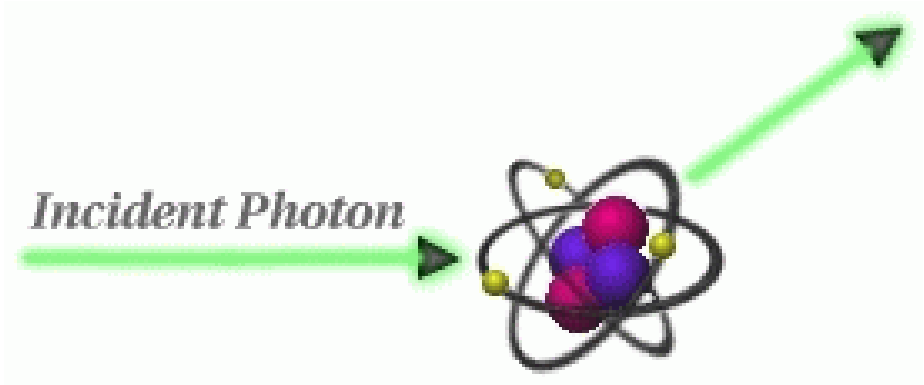
$$\frac{t}{yr} \sim 1.1 \times 10^9 \left(\frac{H}{\mu G}\right)^{-3/2} \left(\frac{\nu^*}{GHz}\right)^{-1/2}$$

Lecture Outline

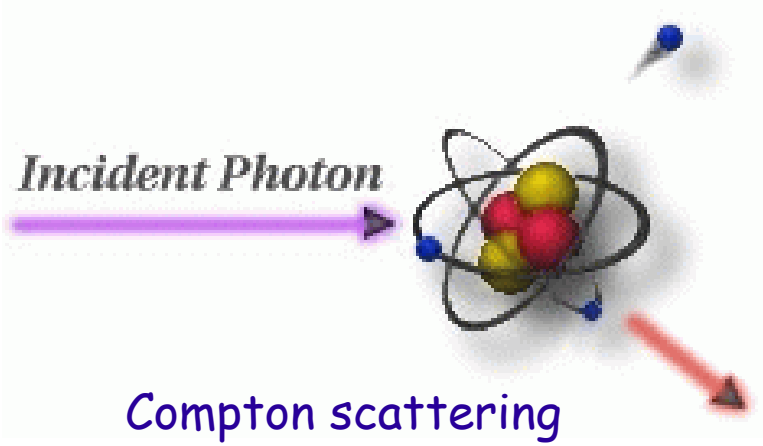
- Emission mechanisms
  - Thermal processes: Black Body & Bremsstrahlung
  - **Non-thermal processes:** Synchrotron & **Inverse Compton**
  - Line processes: just very few and basic concepts!!!

Thomson & Compton scattering
Inverse Compton effect

The interaction between particles and light is able to scatter photons from lower to higher energies (or vice versa) in interactions with electrons of higher (or lower) energies. The kind of interaction between photons and electrons depends from their relative energy: if the electrons is (nearly) not moving, we will speak about **Thomson** or **Compton scattering** as a function of the photon energy. If the electron is relativistic, we will be in the **Inverse Compton** case.



Thompson scattering



Compton scattering

Matter-photon interactions

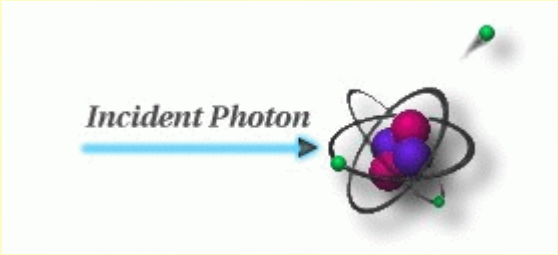
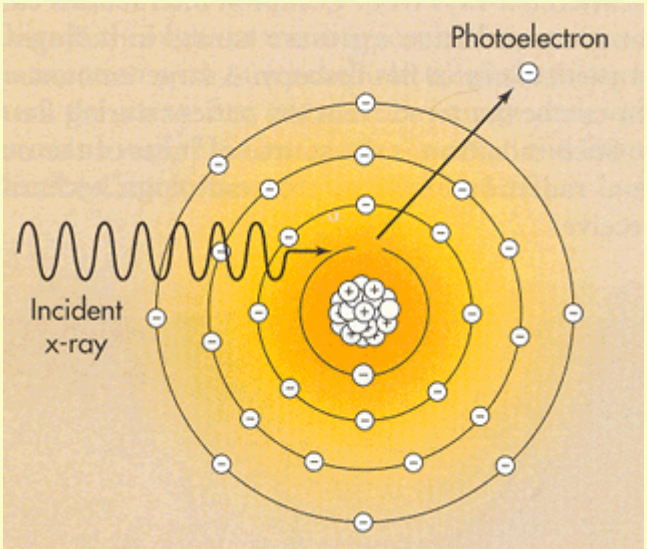
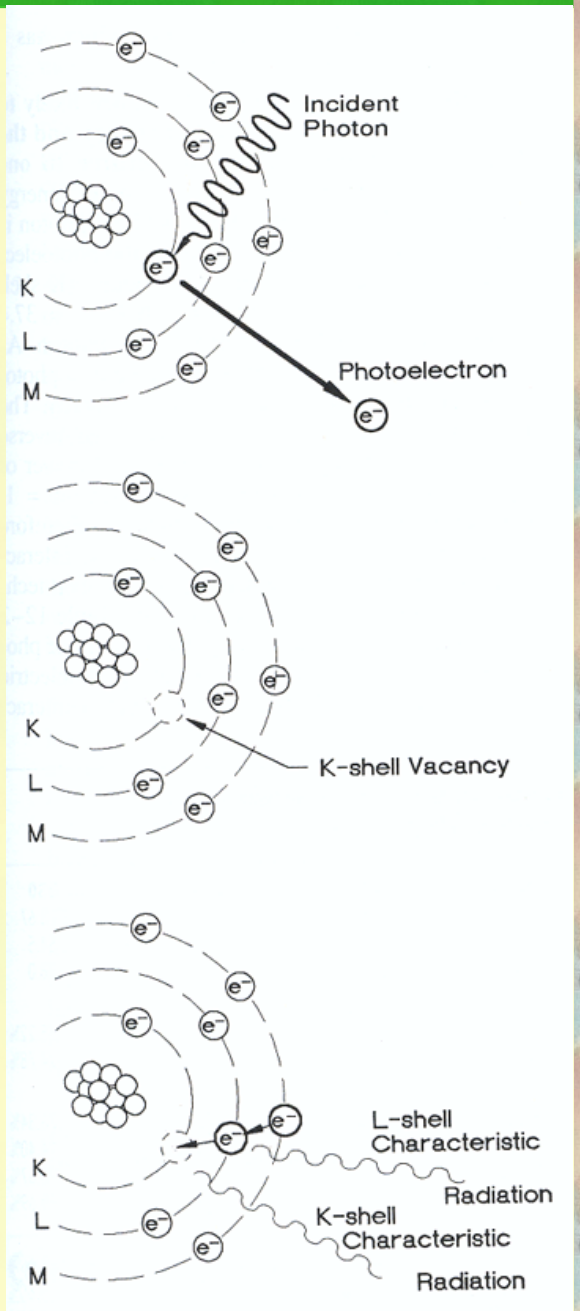
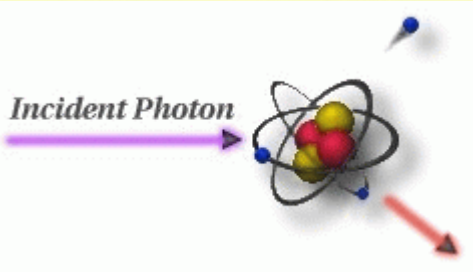


Photo-electric effect

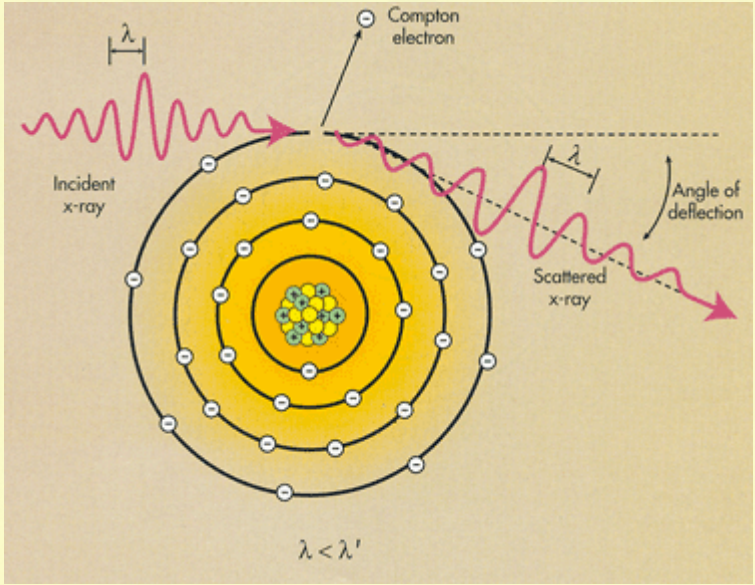


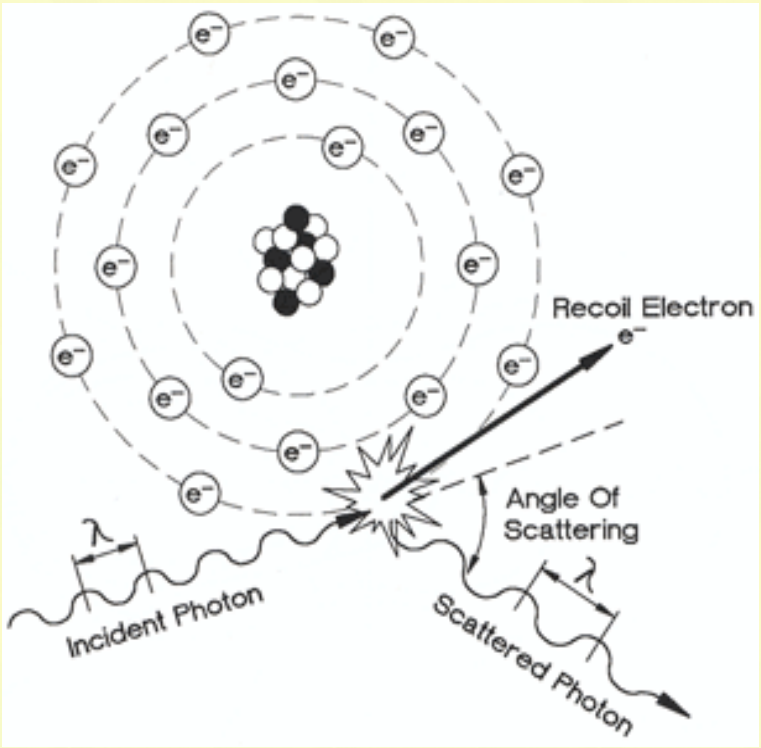


Matter-photon interactions



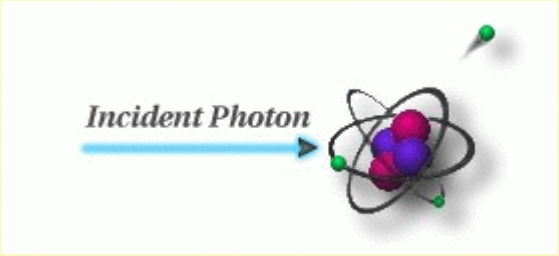
Compton scattering



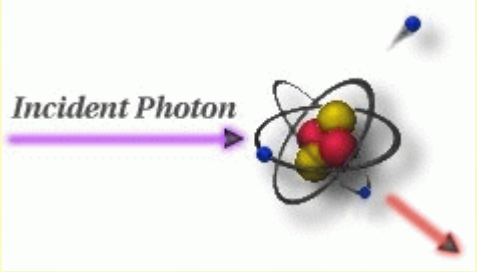


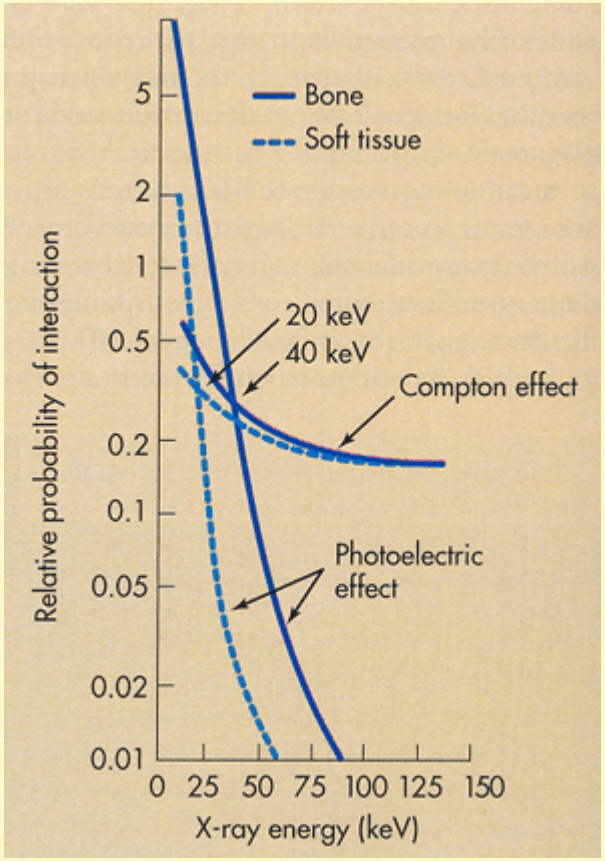
Matter-photon interactions: photo-electric effect or Compton scattering?

Photo-electric effect

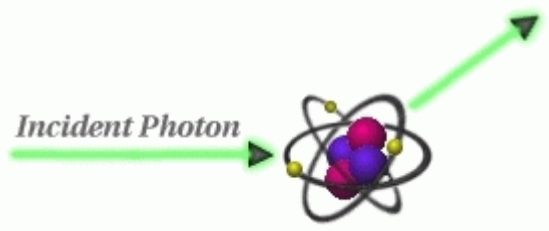


Compton scattering

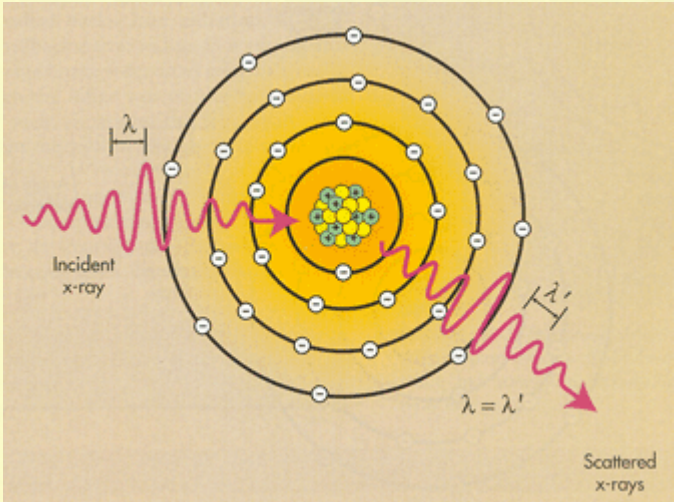


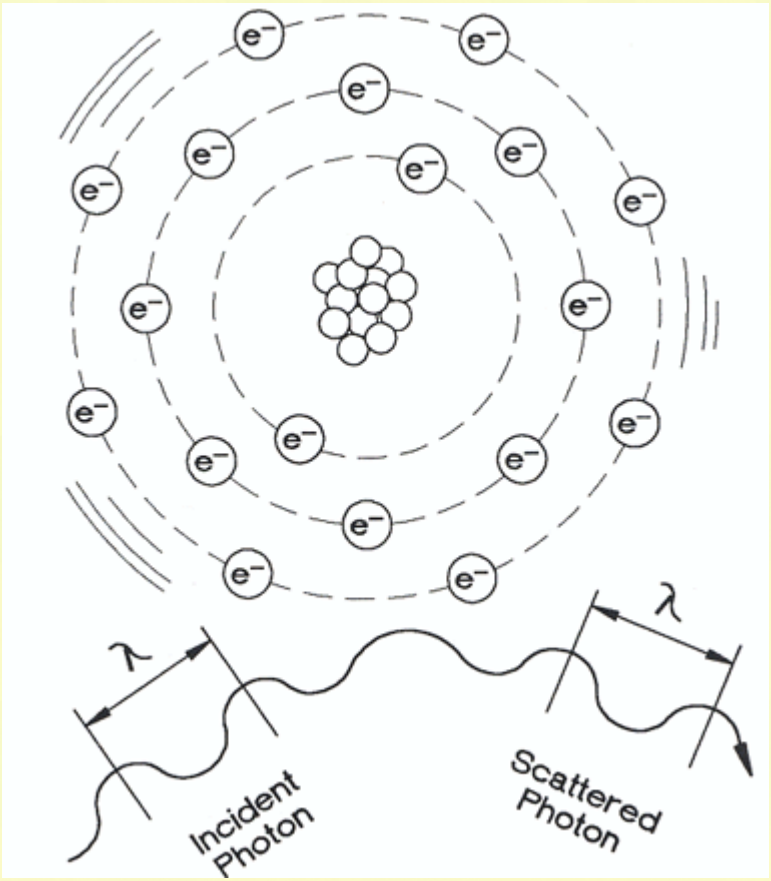


Matter-photon interactions



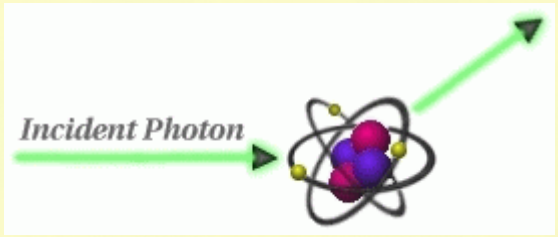
Thomson scattering





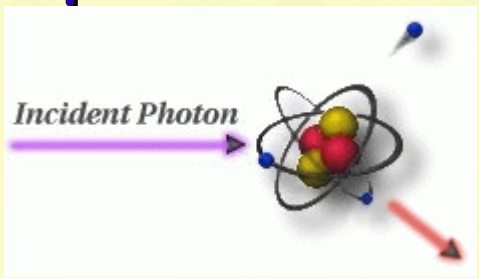
Matter-photon interactions: Thomson or Compton scattering?

Thomson scattering



$$h\nu \ll m_e c^2$$

Compton scattering



$$h\nu \leqslant m_e c^2$$

Thomson scattering

It is the interaction of an electron at rest with a low energy photon:

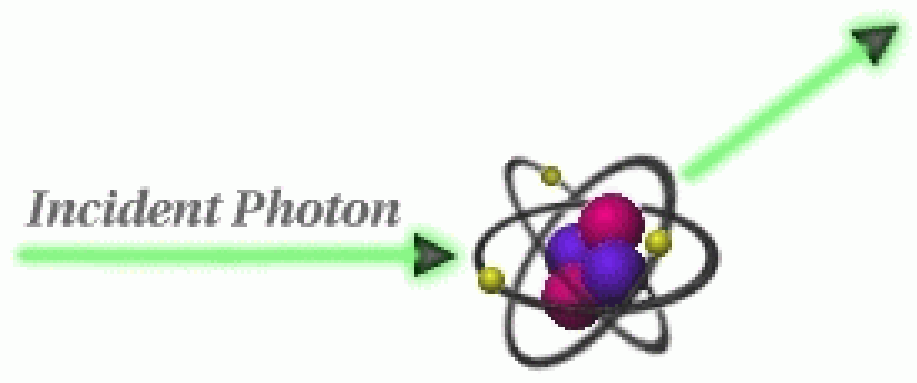
$$h\nu \ll m_e c^2$$

In such a case the incident photons can be approximated as a continuous e.m. wave.

When an e.m wave is incident on a charged particle, the electric and magnetic components of the wave exert a Lorentz force on the particle, setting it into motion.

Since the wave is periodic in time, so is the motion of the particle, that will oscillate.

The particle is accelerated and consequently emits radiation: energy is absorbed from the incident wave by the particle and re-emitted as e.m. radiation. Such a process is clearly equivalent to the scattering of the e.m. wave by the particle.



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$$\vec{F} = \frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \frac{\vec{H}}{c}\right)$$

Non relativistic particles

The main cause of acceleration of the particle will be due to the electric field component, along whose direction the particle will oscillate → dipole radiation polarized along the direction of motion of the particle

Thomson scattering

The electron oscillates with an acceleration a(t) due to the electric field of the e.m. wave:

$$E = E_0 \cos(2\pi\nu t) \qquad a(t) = \frac{e E_0}{m_e} \cos(2\pi\nu t)$$

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The electron radiates photons at the same frequency than the incoming radiation; therefore the energy of the incoming and radiated photons is the same. The mean radiated power is the power of a dipole subject to an acceleration of amplitude:

$$a = \frac{e E}{m_e} \longrightarrow -\left\langle \frac{d W}{d t} \right\rangle = \frac{2}{3} \frac{e^2}{c^3} a^2 = \frac{\cancel{2}}{3} \frac{e^4 E^2}{m_e^2 c^3}$$

Larmor's formula

(<cos^2>=1/2)

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Larmor's formula

$$|\vec{S}| = \frac{c E^2}{8\pi} \longrightarrow \sigma_T = \frac{\langle -dW/dt \rangle}{|\vec{S}|} = \frac{8\pi}{3}\frac{e^4}{m_e^2 c^4} = 6.65 \times 10^{-25}\, cm^2$$

Poynting's vector = flux of e.m. energy in units of time and of surface

Thompson cross section. The inverse of it gives the number of electrons necessary to re-radiate in all directions the power of the incoming e.m. wave.

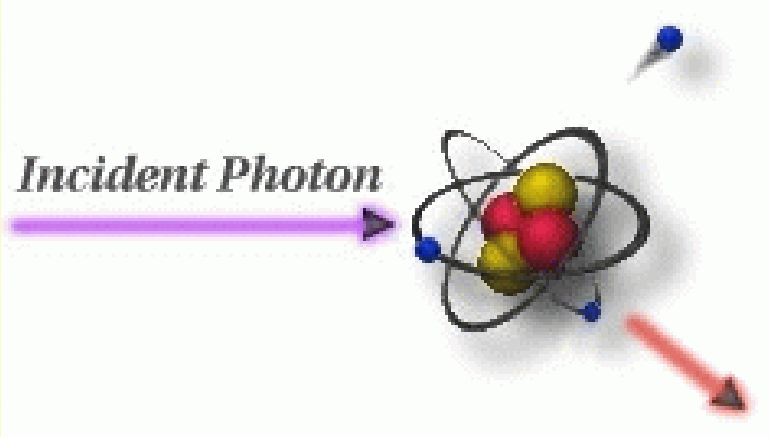
$$-\left\langle \frac{dW}{dt} \right\rangle = 2\frac{E^2}{8\pi} c\,\sigma_t$$

Compton scattering

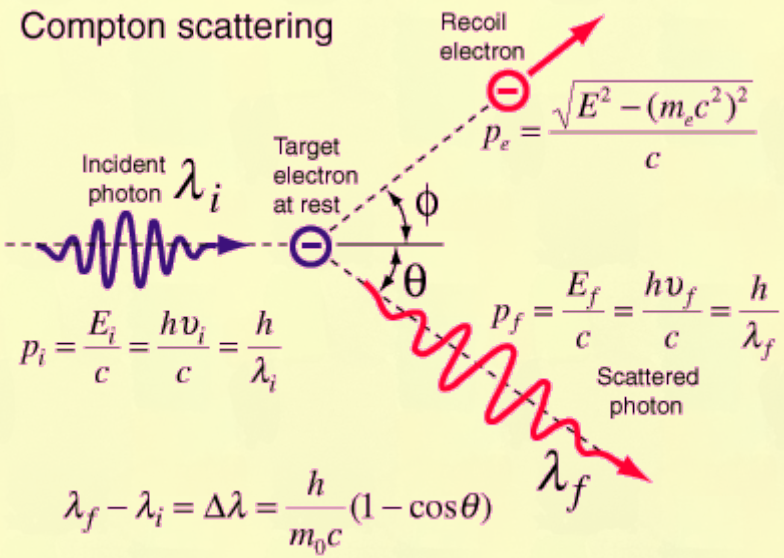
It is the interaction of an electron at rest with a similar energy photon:

$$h\nu \leq m_e c^2$$

In such a case the incident photon is deflected from its original path by an interaction with an electron. The electron gains energy and is ejected from its orbital position. The incident photon loses energy due to the interaction but continues to travel through the material along an altered path. Since the scattered incident photon has less energy, it has a longer wavelength than the incident photon. The interaction can be assimilated to an elastic crash, after which the total energy and quantity of motion of the system is conserved



Compton scattering



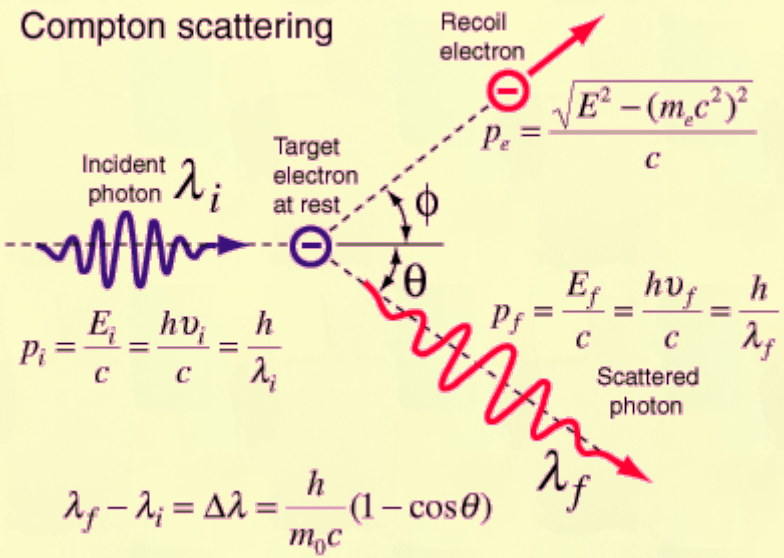
$$h\upsilon_i + m_e c^2 = h\upsilon_f + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$ *Conservation of energy*

$$\vec{p}_i = \vec{p}_f + \vec{p}_e$$ *Conservation of momentum*

$$E_f = \frac{E_i}{1 + E_i(1 - \cos\theta)/m_e c^2}$$

(See: http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/compeq.html for a demonstration)

Compton scattering



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From previous equation it can be derived:

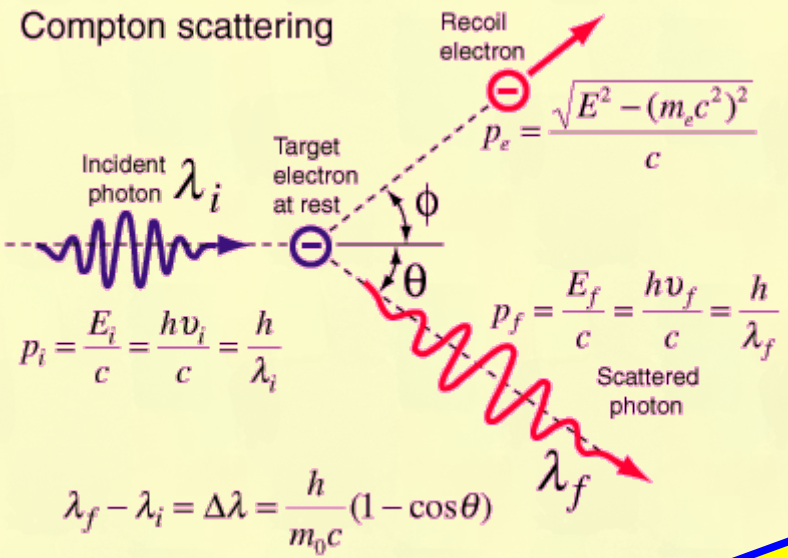
$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\theta) = \lambda_0(1 - \cos\theta)$$

λ_0 = **Compton wavelength of electrons**

(remember: $E=h\nu$ and $\lambda=c/\nu$)

The photon always looses energy, unless $\theta=0$

Compton scattering



$$h\upsilon_i + m_e c^2 = h\upsilon_f + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$ *Conservation of energy*

$$\vec{p}_i = \vec{p}_f + \vec{p}_e$$ *Conservation of momentum*

$E_f = h\upsilon_i / [1 + (h\upsilon_i / m_e c^2)(1 - \cos\theta)]$

(See: http://hyperphysics.phy…rum/compeq.html for a demonstration)

OK when $h\upsilon < m_e c^2$. If not: Klein-Nishina's cross section. It depends on the inverse square of the incident photon energy. Compton effect is less efficient

Fr…quation it can be derived:

(remember: $E=h\nu$ and $\lambda=c/\nu$)

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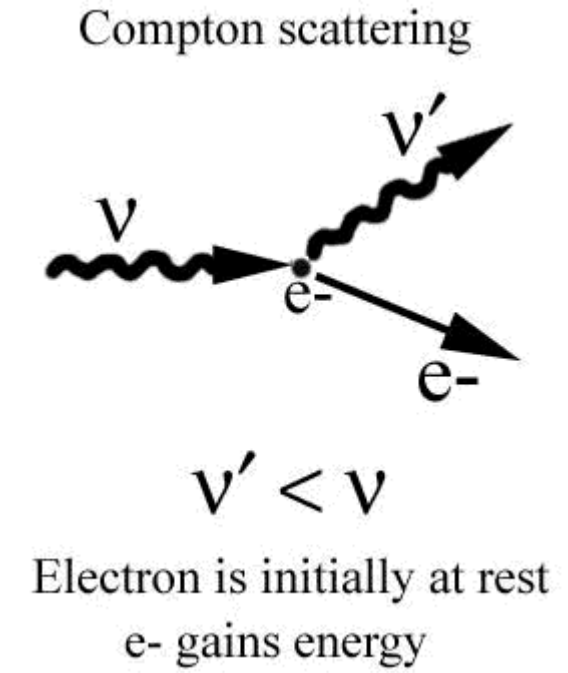
The photon always looses energy, unless θ=0

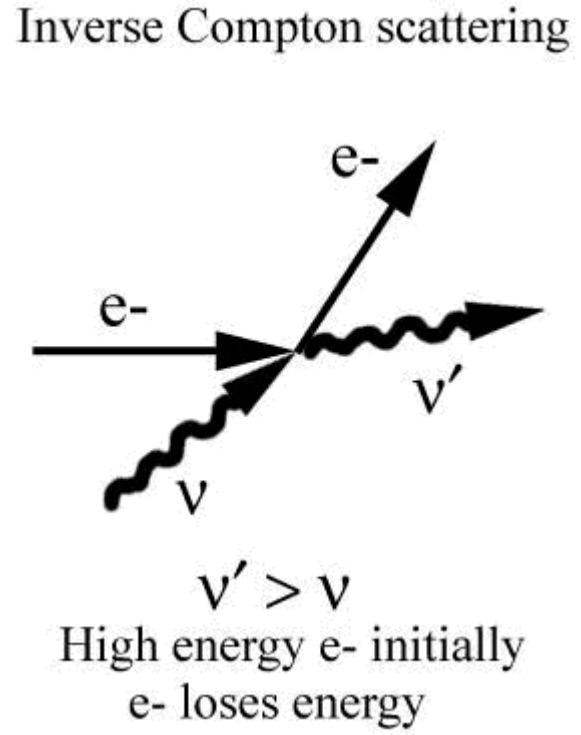
Inverse Compton

It is the interaction of a photon with a moving electron!

$$h\nu \ll \gamma m_e c^2$$

If the moving electron has an energy significantly higher than the incoming photon (in the case of relativistic electrons), energy is transferred from the electron to the photon, i.e. it is the opposite of the Compton scattering. This process does not produce radio emission ("only" from optical to X-ray and γ), but it strongly affects the radiation losses in radio sources!





Inverse Compton

In the reference frame S' of the moving electron: $h\nu \ll m_e c^2$

And the electron is at rest → we are in the Thompson scattering case!

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Based on the relativistic Doppler effect (see a course of Physics) in S' the frequency of the incoming photon is:

$\beta = v/c$

$\gamma = 1/\sqrt{1-\beta^2}$

$$\nu' = \nu\,\gamma(1-\beta\cos\theta) = \frac{\nu}{D}$$

D = relativistic Doppler factor

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$$\downarrow$$

$$\nu'' = \nu'\,\gamma(1+\beta\cos\theta) = \nu\,\gamma(1-\beta\cos\theta)\,\gamma(1+\beta\cos\theta) \sim \nu\,\gamma^2$$

→ Radio photons can be boosted in the hard part of the e.m spectrum by relativistic electrons!!!

Inverse Compton & Synchrotron radiation

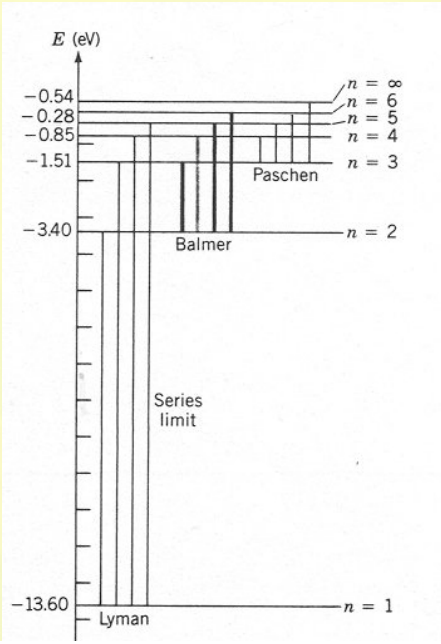
- If a magnetic field is present, the same relativistic electrons radiate via synchrotron and Inverse Compton. The electron lifetime is consequently reduced... **hard time for relativistic electrons!**
- Synchrotron photons may interact with relativistic electrons and be upscattered via I.C. It can be shown that I.C. becomes the dominant cooling process for temperatures above 10^{12} K and a source would radiate its energy in a very short time... **Compton catastrophe!**
- If the spectrum of a source is primarily determined by Compton processes it is termed Comptonised... the hotter the gas, the more chance of **Comptonisation**

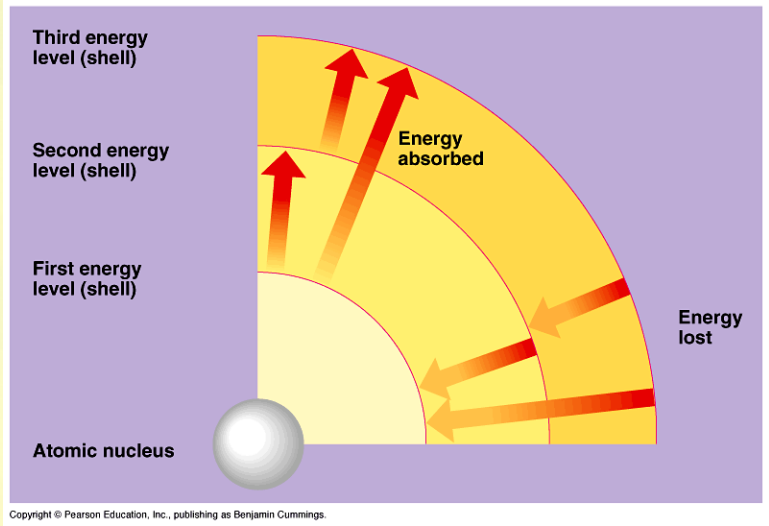
Lecture Outline

- Emission mechanisms
  - Thermal processes: Black Body & Bremsstrahlung
  - Non-thermal processes: Synchrotron & Inverse Compton
  - **Line processes: just very few and basic concepts!!!**

Atomic spectra

- Energy levels as according to quantum mechanics: "orbits" correspond to different energies
- Radiative process (emission / absorption) during quantum transition: it must obey to firm selection rules concerning quantum numbers m, l, m_l, m_s)
- Ground (fundamental) state as minimum electron energy: it also defines the ionization energy (e.g. 13.6 eV for H atom)



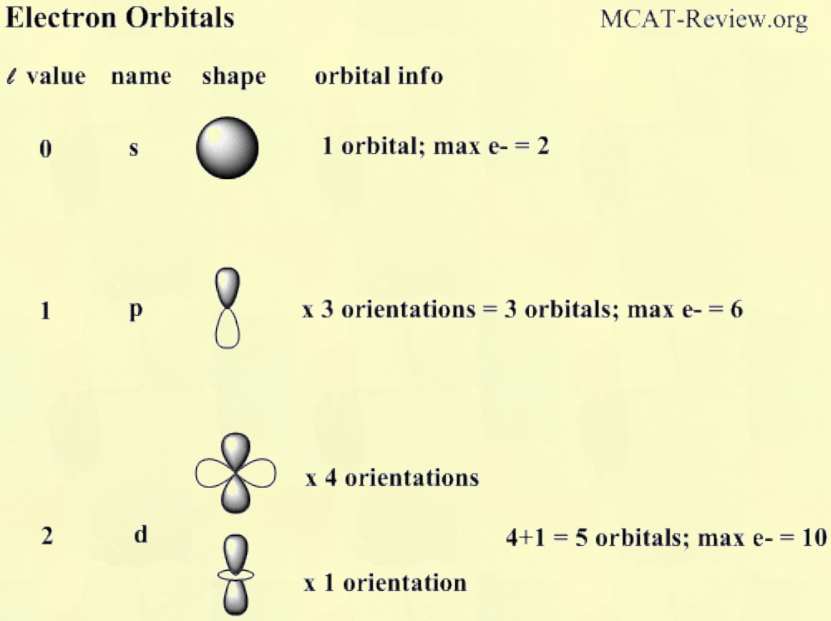


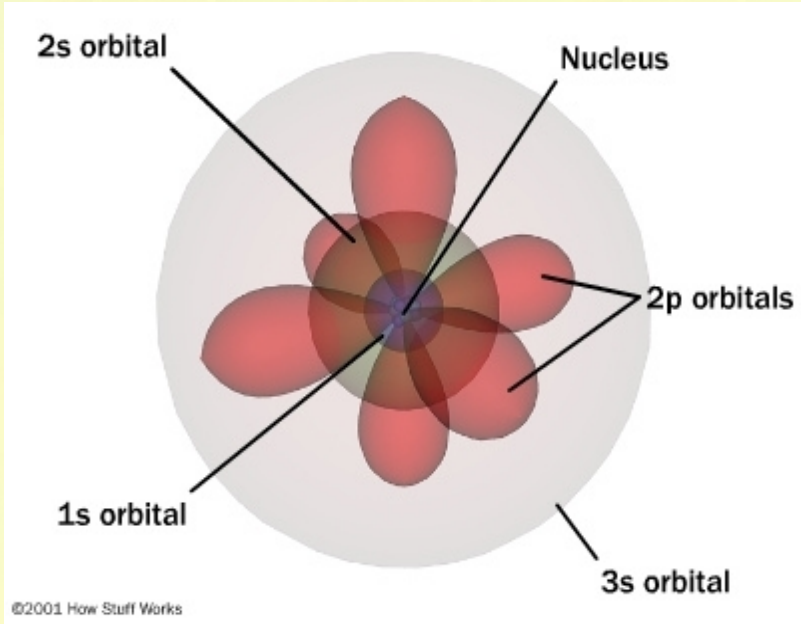
http://spiff.rit.edu/classes/phys301/lectures/spec_lines/Atoms_Nav.swf

Atomic spectra: quantum numbers

- n - principal number (1, 2, 3, ...): it defines the energy and the size of the orbit (i.e. the distance between the electron and the nucleus)
- l – azimuthal (or angular or orbital) number (0, 1, 2, ...n-1): it gives the angular momentum, and is related to the shape – eccentricity – of the orbit
- m_l – magnetic number (-l, ..., 0, ..., l): it defines the orbital projection of the orbital angular momentum along a specific axis
- m_s – spin number (-1/2, 1/2): it defines the intrinsic angular momentum of the electron

At a given main number n, there is a tiny difference in energy between levels with different combinations of (l,m,s)





Atomic spectra: hydrogen

Given the Rydberg constant: $$R_H = \frac{2\pi^2 e^4 m_e}{h^3 c} \simeq 1.1 \times 10^5 cm^{-1}$$

the frequency of a photon emitted/absorbed during a transition between two levels m and n is: $$\nu_{nm} = c R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right) Hz \,; \quad n > m > 0$$

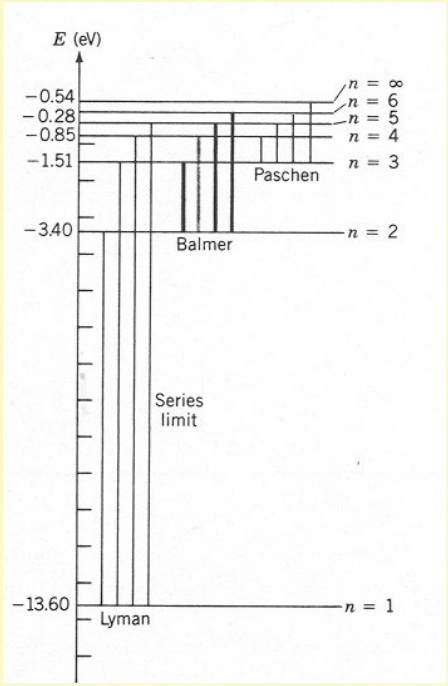
- With increasing n, lines get closer to a limiting frequency: $\nu = \frac{c R_H}{m^2}$
- If m is large, lines of different series start at very close frequencies and produce a spectrum similar to a continuum emission (but it is not!)
- For hydrogen-like atoms (e.g. He^+, Li^{2+}, Be^{3+}, ..., with only one electron):

Atomic number

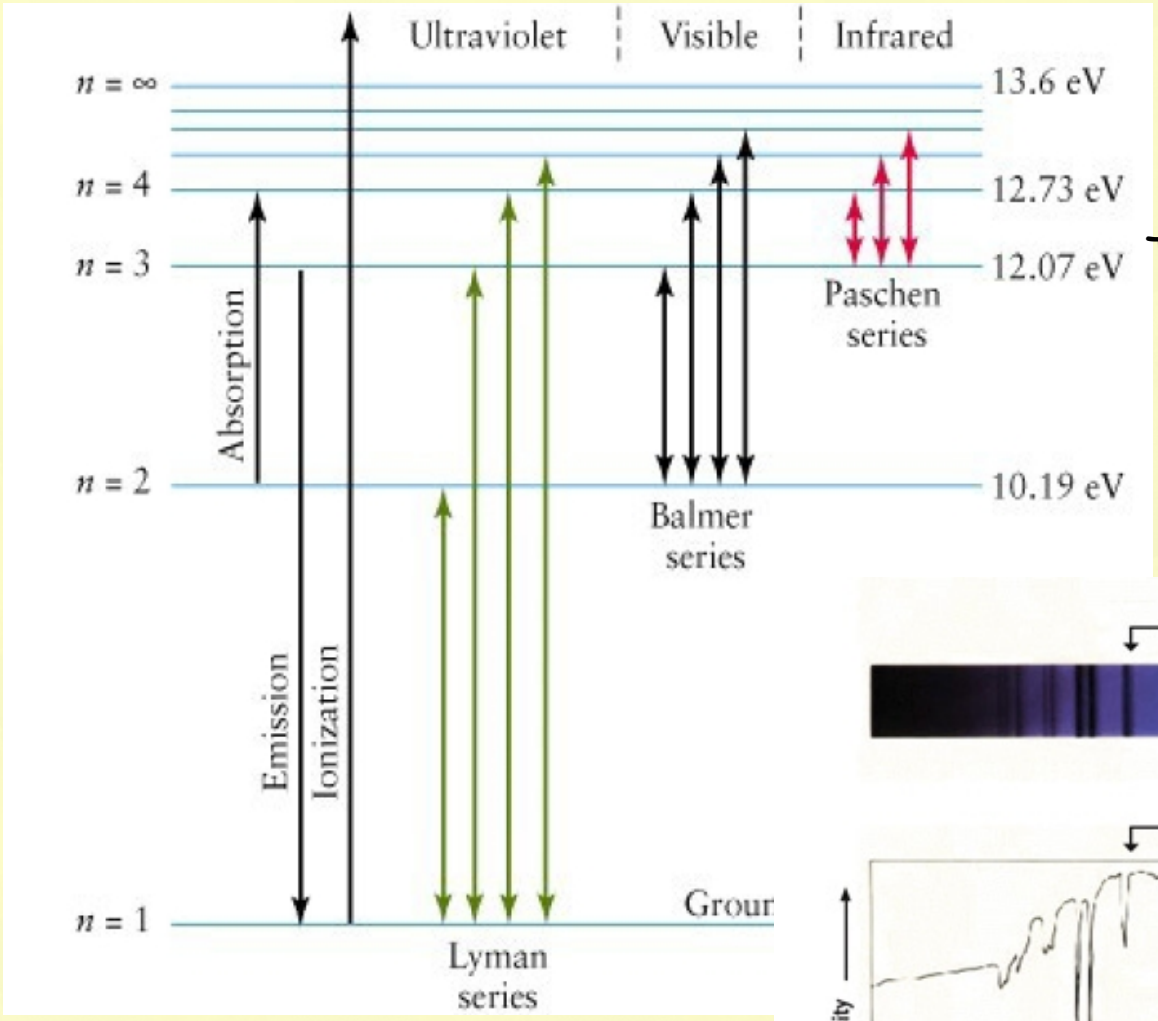
$$\nu_{nm} = c R_M Z^2 \left(\frac{1}{m^2} - \frac{1}{n^2} \right) Hz$$

$$R_M = \frac{R_\infty}{1 + \frac{m_e}{M}}$$

Mass of the atomic nucleus



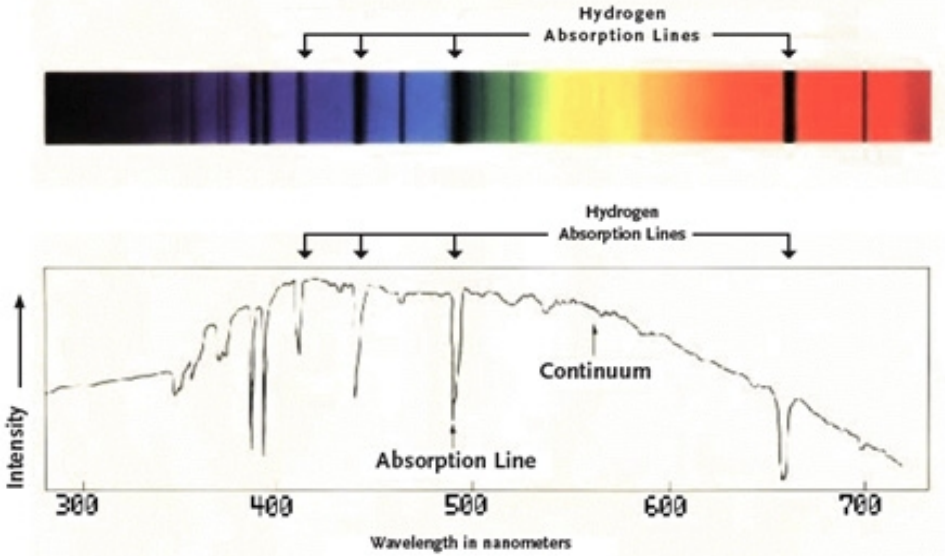
Atomic spectra: Hydrogen



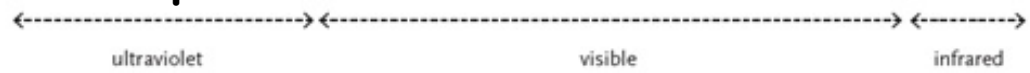
Transitions obey to well known selections rules:

Δn, Δl, Δm

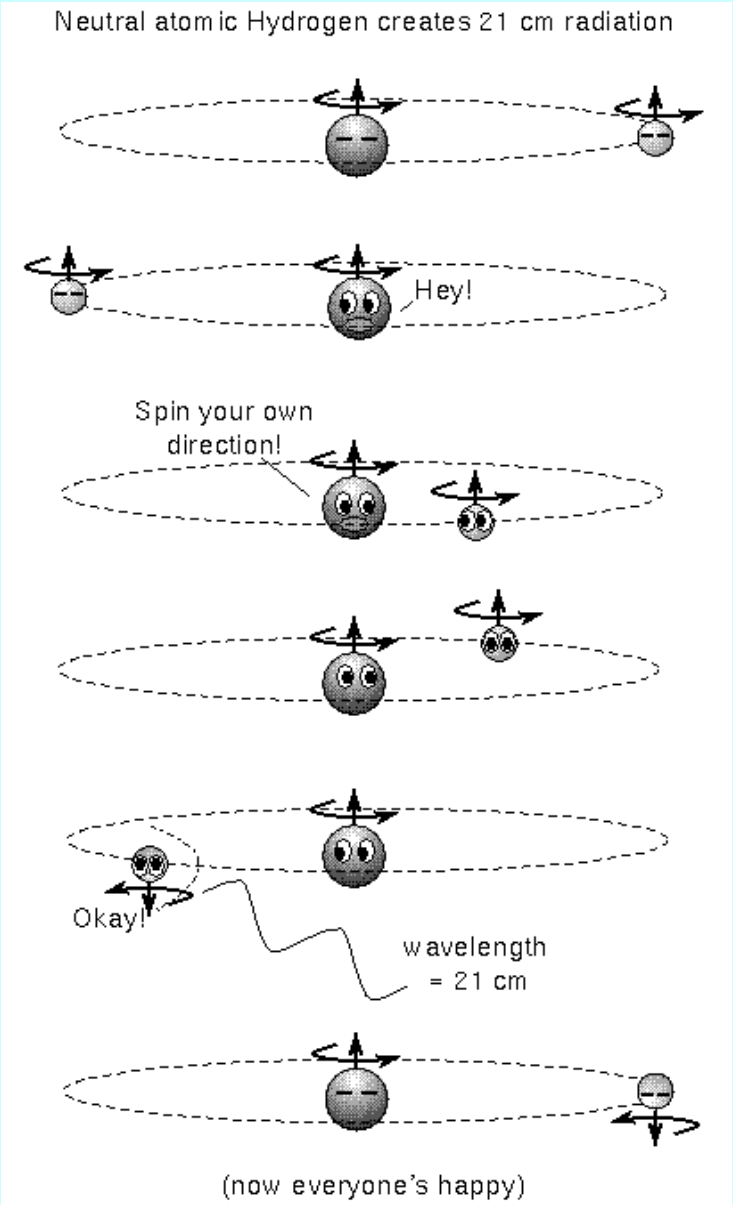
The forbidden transitions are actually "high improbable"... but they can take place in extreme (astrophysical) conditions!



Solar spectrum



The HI emission at 21 cm: just few ideas!!!

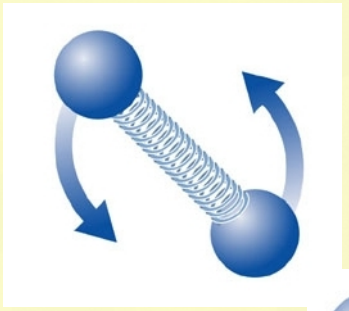


Most of the hydrogen in space is in the ground state. The electron moving around the proton can have a spin in the same direction as the proton's spin (i.e., parallel) or be in the direct opposite direction as the proton's spin (i.e., anti-parallel). The energy state of an electron spinning anti-parallel is slightly lower than the energy state of a parallel-spin electron. Based on quantistic theory, the ratio of electrons populating the upper (parallel) and lower (anti-parallel) levels is 3:1. But remember that the atom always wants to be in the lowest energy state possible, so the electron will eventually flip to the anti-parallel spin direction. The mean life time of the upper level is however ~ 10^7 yrs!!! The collisions between atoms due to the high density of hydrogen in astrophysical conditions lower the mean life time of the upper level to ~400 yrs. Moreover, even though this is a rare transition, the large amount of hydrogen gas means that enough hydrogen atoms are emitting the 21 cm radiation at any one given time to be easily detected with our radio telescopes.

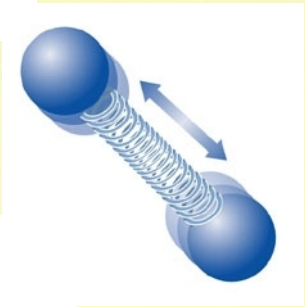
Molecular spectra

Molecular energy is quantized: molecules can exist only in specific quantum states with each quantum state having a set amount of energy. The quantized energy stored in a molecule can be thought of as the sum of energy stored in three distinct modes: (1) rotation, (2) vibration, and (3) electronic:

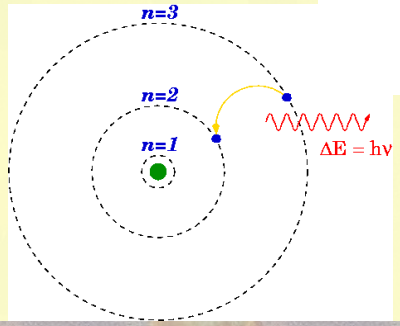
$$E_{tot} = E_{rot} + E_{vib} + E_{elec}$$



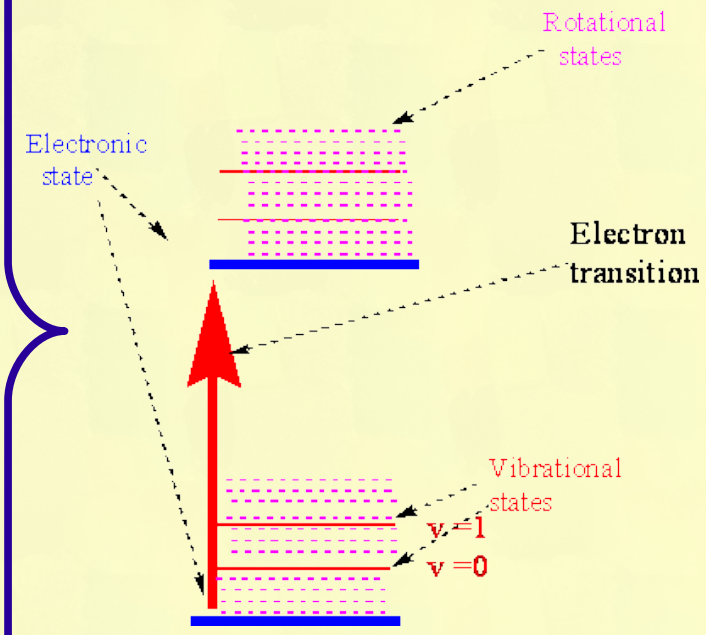
Rotational transitions



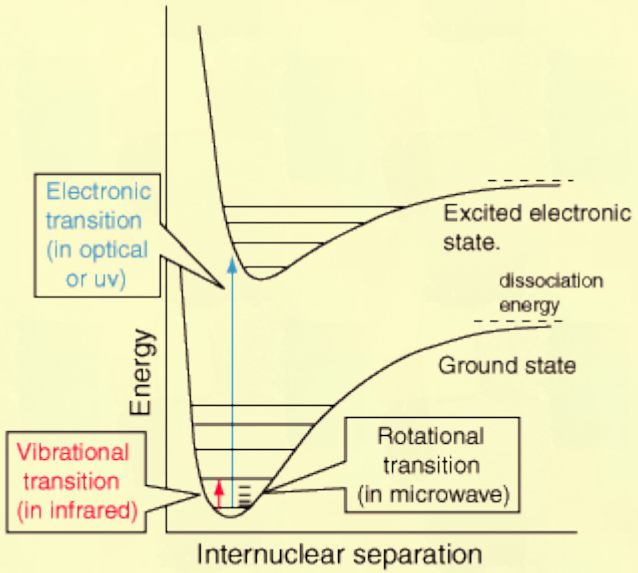
Vibrational transitions

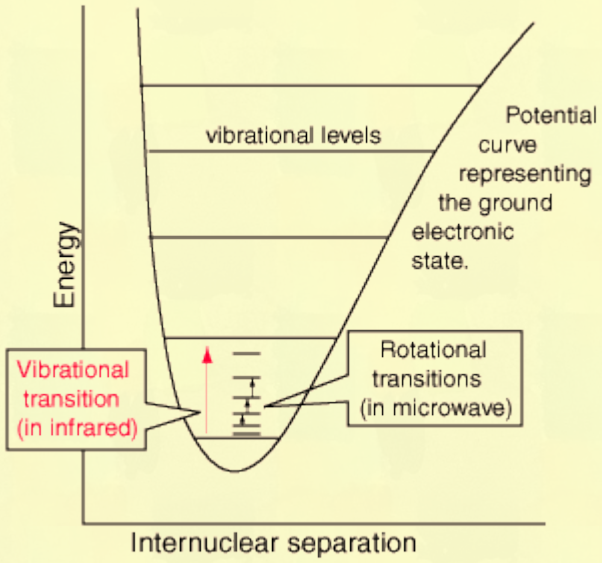


Electronic transitions



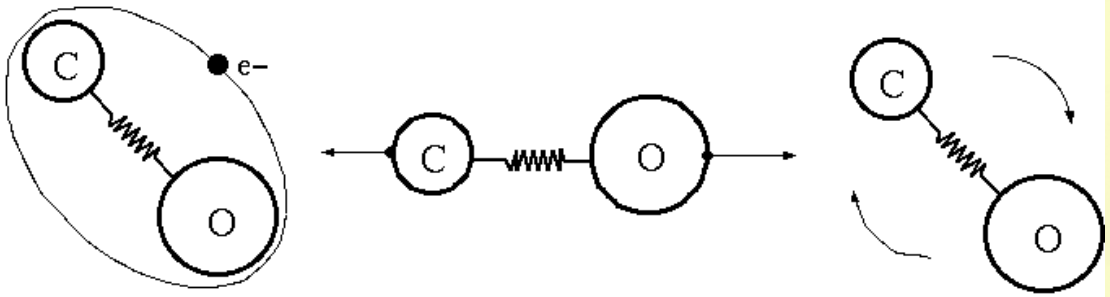
Molecular spectra



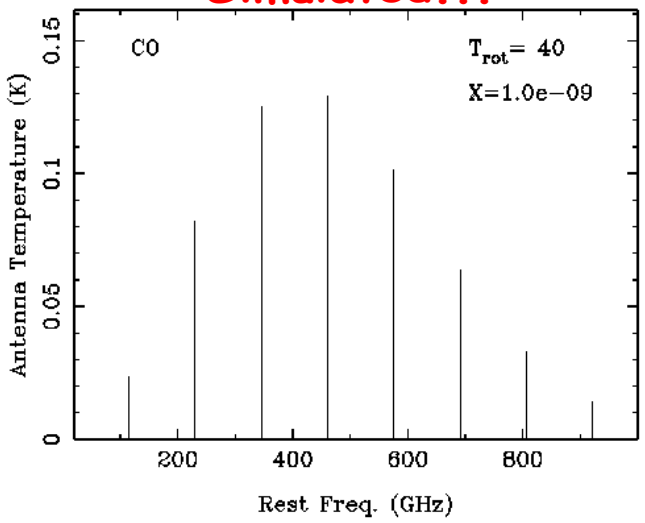


- **Electronic transitions** are vertical lines on such a plot since the electronic transition occurs so rapidly that the internuclear distance can't change much in the process (visual or UV)
- **Vibrational transitions** occur between different vibrational levels of the same electronic state (IR)
- **Rotational transitions** occur mostly between rotational levels of the same vibrational state (although there are examples of combined vibrational-rotational transitions for light molecules) – (cm and mm wavebands)

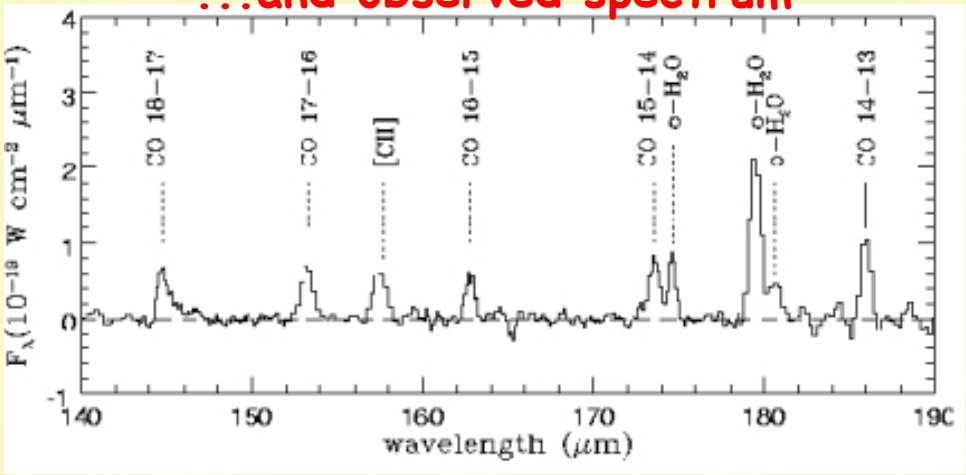
Molecular spectra: emission...



Simulated...



...and observed spectrum



Molecular spectra: ...and absorption lines

