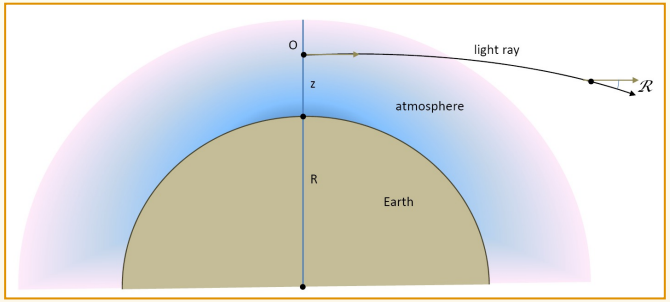
Astronomical Refraction

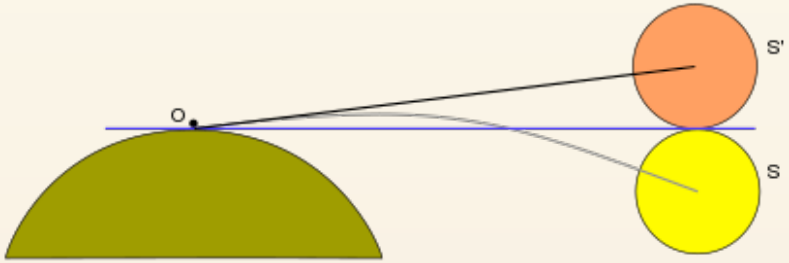


F. Mignard
UCA/CNRS
Observatoire de la Côte d'Azur

Summary



- Opinions about the astronomical refraction
- Refraction in history
  - before Snell law
  - modern approaches
- Numerical refraction
  - basic equations
  - the model of atmosphere
  - numerical solutions and comparisons to tables
- Application to imaging near horizon

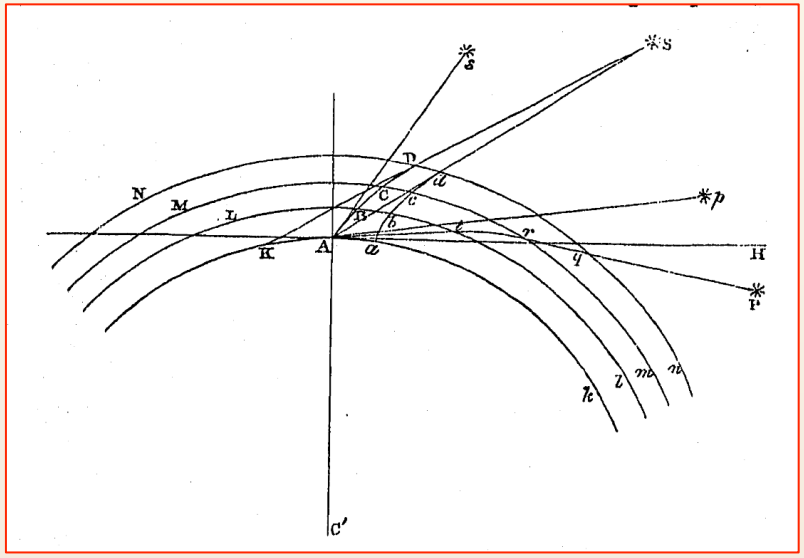


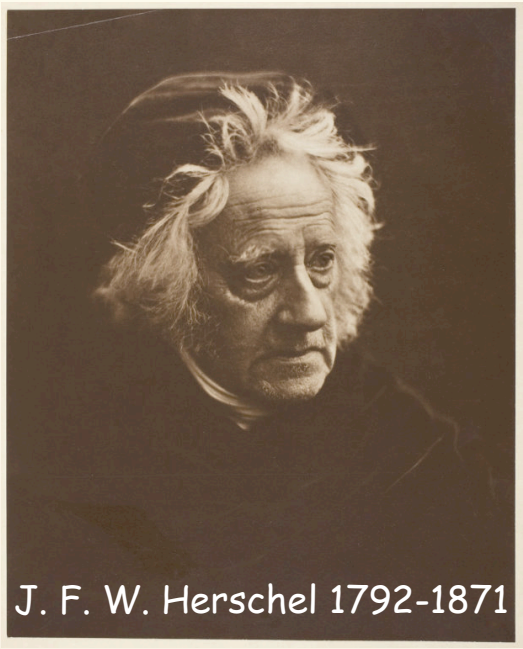


Quotations …

…The exact estimation of the amount of atmospheric refraction is a very difficult subject of physical inquiry, one on which geometers are not yet entirely agreed.

J. F. Herschel, Outline of Astronomy, 1849, Sect. 41.

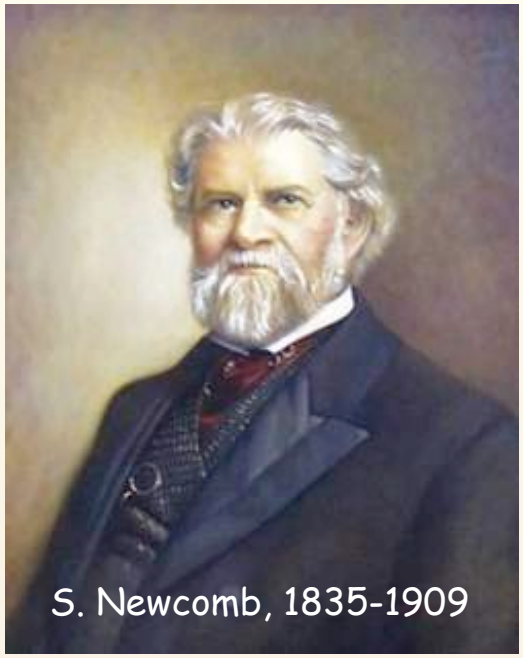



J. F. W. Herschel 1792-1871

Quotations …

There is, perhaps, no branch of practical astronomy on which so much has been written as on this and which is still in so unsatisfactory a state. The difficulties connected with it are both theoretical and practical. The

The list of modern writers on the subject includes many of the greatest names in theoretical and practical astronomy, extending from the time of Laplace to the present. Among those who have most contributed to the advance of the subject are,—Bouguer, Bradley, Laplace, Bessel, Young, Schmidt, Ivory, Gyldén and Radau.


S. Newcomb, 1835-1909

S. Newcomb, *A compendium in spherical astronomy*, 1906, ,Sect. 118.

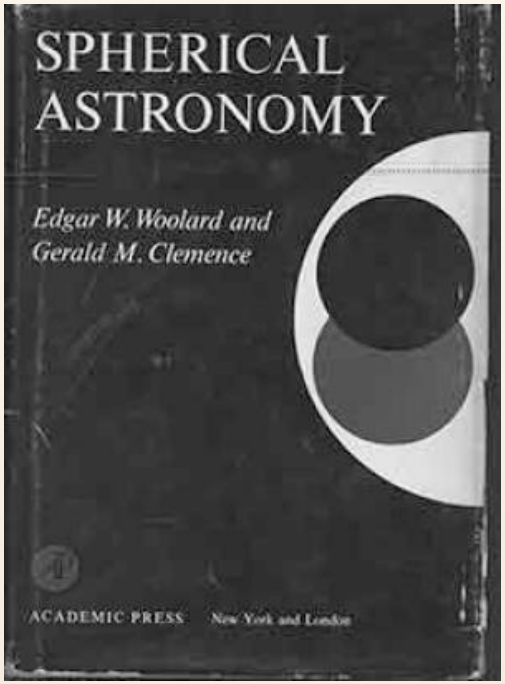
$$R = a \tan z \int_0^1 \frac{dw}{\sqrt{1+2u\sec^2 z}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

$$R = a \sin z \int_0^1 \frac{dw}{\sqrt{2u+\cos^2 z}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (15b)$$

Quotations …

…the indeterminacy is a geometrical one, and the refraction at any instant may differ by ***several minutes of arc*** *from the most accurate value that can be calculated.* G.M. Clemence


G.M. Clemence, 1908-1974



"Remarks on suggested improvements in sunrise-sunset-twilight tables,"
Pop. Astr. 59, 358–361 (1951).

Quotations …

From A. T. Young (San Diego Univ.) authoritative website on refraction and green-flash

Why We Can't Predict Sunset Times Exactly

Introduction

So, if we can calculate astronomical refraction to a gnat's eyelash, and we know the position of the Sun to a fraction of a second of arc, we should be able to predict when it will set tonight, with an accuracy of a fraction of a second of time, right?

Wrong. (Although that's a reasonable question.)

SUNSET SCIENCE. IV. LOW-ALTITUDE REFRACTION

ANDREW T. YOUNG
Department of Astronomy, San Diego State University, 5500 Campanile Drive, San Diego, CA 92182-1221;
aty@mintaka.sdsu.edu

Received 2004 January 13; accepted 2004 February 25

A.J., 127, 2004

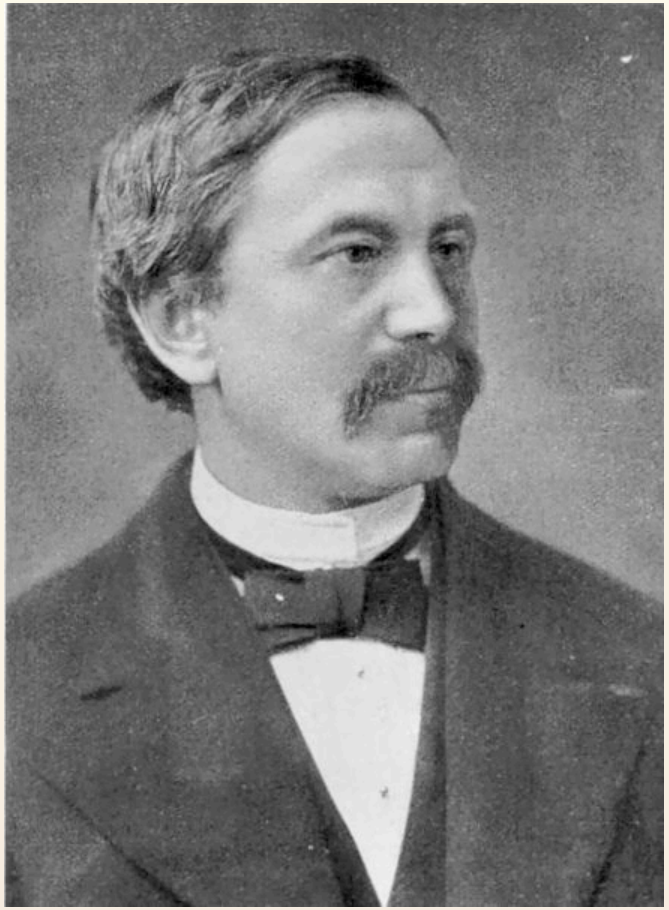
Some historical aspects

An age-old problem of practical astronomy

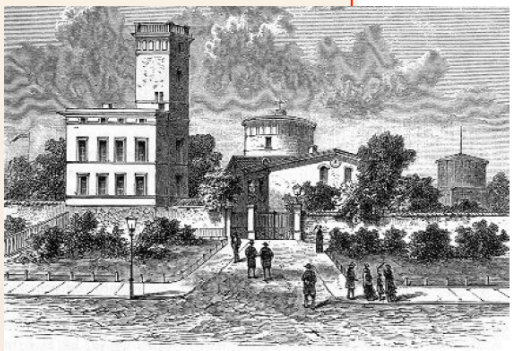


Refraction: History

- A single monograph covering the history of the astronomical refraction
  - up to mid 19th century



K. C. Bruhns, 1830-1881



Sternwarte im Johannistal in Leipzig (eingeweiht 1861)


DIE

ASTRONOMISCHE

STRAHLENBRECHUNG

IN

IHRER HISTORISCHEN ENTWICKELUNG

DARGESTELLT

VON

DR. C. BRUHNS,

ASTRONOM DER NEUEN STERNWARTE UND PROFESSOR AN DER UNIVERSITÄT ZU LEIPZIG.

EINE GEKRONTE PREISSCHRIFT.

LEIPZIG,
VOIGT & GÜNTHER.
1861.

Refraction in Tycho era

- Ty… for refraction
  - …al-Haytham), Vitellion,
  - …sun and for the stars
    …tion

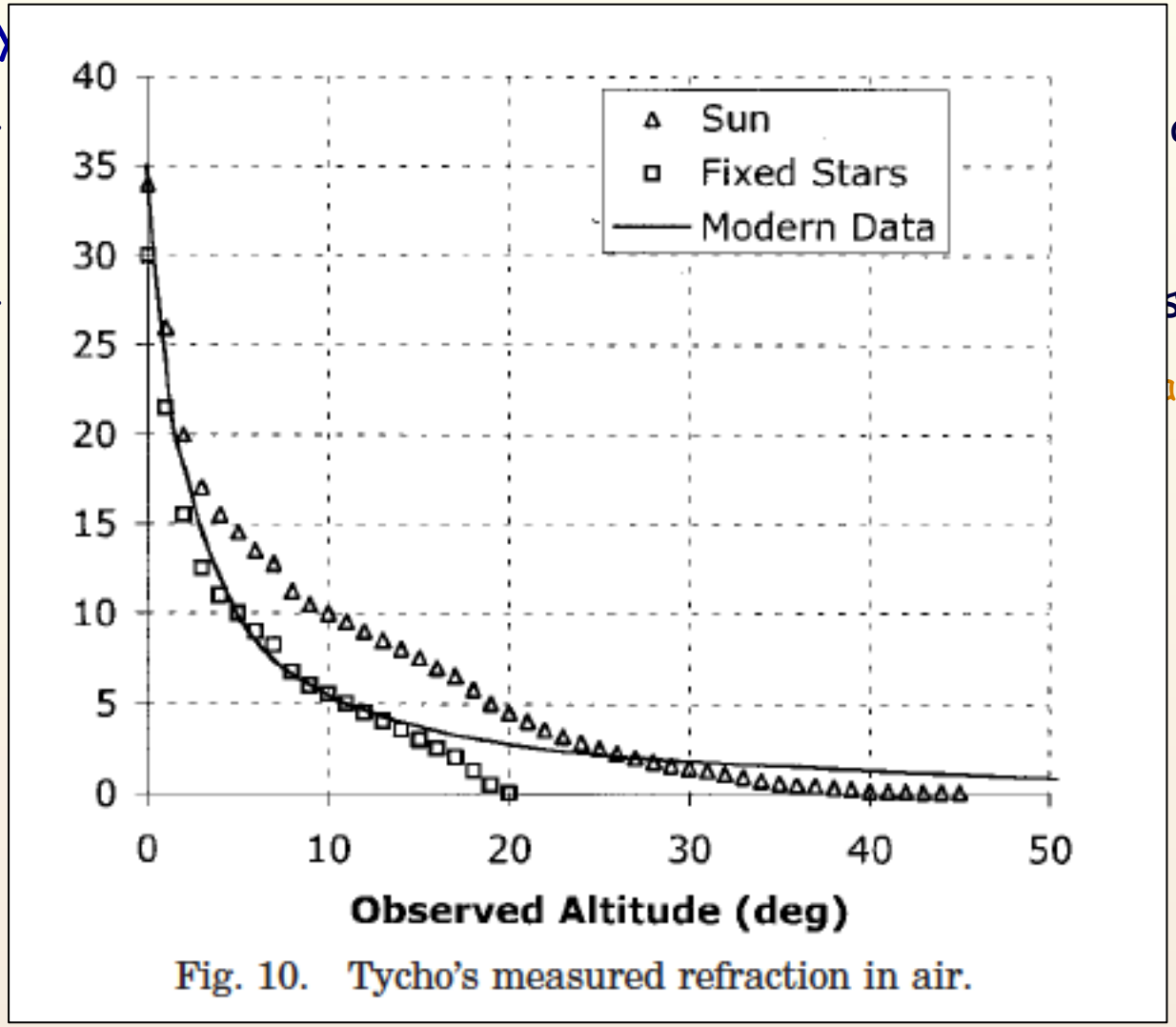
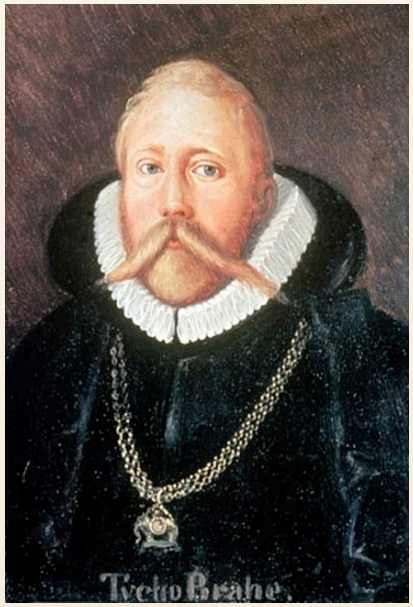


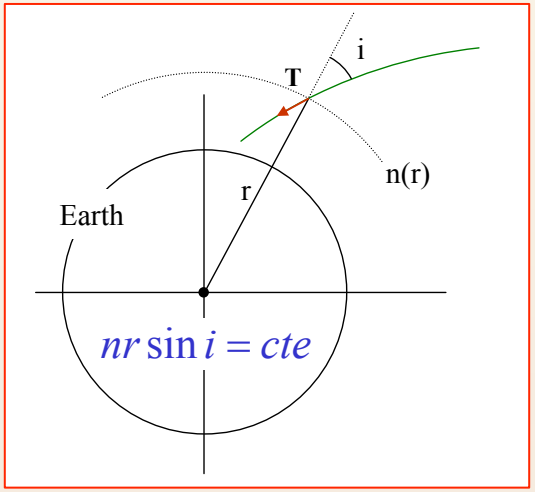
Fig. 10. Tycho's measured refraction in air.

W.H. Lehn, Appl. Opt., 44, 2005



Refraction from Cassini onwards

- Tables from Cassini, Flamsteed, Halley, Bradley and empirical rules
- Models from B. Taylor, P. Bouguer, J. Cassini, L. Euler, J.H. Lambert, J.L. Lagrange, F.W.Bessel, P.S. Laplace, R. Radau, H. Gylden …
  - elaborate mathematical modelling to achieve the arcsec accuracy
  - Laplace (100 p), Biot (100p), Delambre (30 p), Chauvenet (70p), Newcomb (63 p.), Ball (40 p.), Garfinkel (1944, 1967) for extensive analytical theories
  - very complex and extensive tables (eg Pulkovo)
  - numerous practical formulas
  - see Biot(1836), Auer & Standish (2000), Hohenkerk & Sinclair (1985) for numerical methods
- Well defined problem for a spherically symmetric atmosphere



Could be very intricate …

F.W. Bessel

und

$$\left(\frac{d^2Q}{d\beta^2}\right) - \frac{\cot g^2 z}{2}\left(\frac{dQ'}{d\beta}\right) + \frac{\cot g\, z}{2(2\beta)^{\frac{1}{2}}}\,\frac{x}{1-x}$$

$$-\frac{\cot g^4 z}{4}Q'' - \sqrt{\frac{\beta}{2}}\,\frac{\cot g^2 z}{4\beta}\,\frac{x}{(1-x)^2} + \sqrt{\frac{\beta}{2}}\,\frac{\cot g\, z}{4\beta^2}\,\frac{x}{(1-x)} \cdots\cdots (k)$$

Aus (g) ist

$$\left(\frac{dQ}{dx}\right) = \frac{1-x}{x}Q'$$

mithin

$$\left(\frac{d^2Q}{dx\,.\,d\beta}\right) - \frac{1-x}{x}\,\frac{\cot g^2 z}{2}Q'' - \sqrt{\frac{\beta}{2}}\,\frac{\cot g\, z}{2\beta}\,\frac{1}{(1-x)^2} \cdots\cdots (l)$$

Aus

$$\beta = \frac{g-1}{g}\,l,\ l = l_0(1+mr),\ \alpha = \alpha_0\,\frac{b}{B}\,\frac{1}{1+mr}\,^{1}),\ x = \frac{\alpha\beta}{4\sin^2 z}$$

folgt:

$$\left(\frac{d\beta}{dl}\right) = -\frac{\beta}{l},\quad \left(\frac{dl}{dr}\right) = lm,\quad \left(\frac{d\alpha}{dr}\right) = -\alpha m,$$

$$\left(\frac{dx}{d\beta}\right) = \frac{x}{\beta},\quad \left(\frac{dx}{d\alpha}\right) = \frac{x}{\alpha},\quad \left(\frac{d^2\beta}{dl^2}\right) = \frac{2\beta}{l^2},\quad \left(\frac{d^2l}{dr^2}\right) = 0.$$

also:

$$\left.\begin{aligned}&\left(\frac{d\beta}{dr}\right) = \left(\frac{d\beta}{dl}\right)\left(\frac{dl}{dr}\right) = -\frac{\beta m}{l} = -m\beta\,\frac{g}{g-1}\\ &\left(\frac{d^2\beta}{dr^2}\right) = m^2 2\beta\,\frac{g}{g-1},\ \left(\frac{dx}{dr}\right) = \left(\frac{dx}{d\beta}\right)\left(\frac{d\beta}{dr}\right) + \left(\frac{dx}{d\alpha}\right)\left(\frac{d\alpha}{dr}\right) = -mx\left(\frac{2g-1}{g-1}\right)\\ &\left(\frac{d^2x}{dr^2}\right) = 2m^2x\left(\frac{3g-1}{g-1}\right)\end{aligned}\right\}\ (m)$$

Nach den Formeln (f), (g), (i), (l), (m) erhält man

$$\left(\frac{dQ}{dr}\right) = -mQ'\left\{\frac{2g-1}{g-1}(1-x) + \frac{g}{g-1}\,\frac{\beta}{2}\cot g^2 z\right\} + m\sqrt{\frac{\beta}{2}}\,\frac{\cot g\, z}{2}\,\frac{g}{g-1}\,\frac{x}{1-x} \cdots (n)$$

$$\left.\begin{aligned}&\left(\frac{d^2Q}{dr^2}\right) = m^2Q''\left\{\frac{2g-1}{g-1}(1-x) + \frac{g}{g-1}\,\frac{\beta}{2}\cot g^2 z\right\}^2\\ &\quad + m^2Q'\left\{\frac{6g-2}{g-1}(1-x) - \left(\frac{2g-1}{g-1}\right)^2 + \frac{\beta}{2}\cot g^2 z\,\frac{2g}{g-1}\right\}\\ &\quad - m^2\,\frac{x}{1-x}\sqrt{\frac{\beta}{2}}\cot g\, z\,.\,\frac{g}{g-1}\left\{\frac{g}{g-1}\,.\,\frac{\beta}{4}\,.\,\frac{\cot g^2 z}{(1-x)^2} + \frac{2g-1}{g-1}\,\frac{1}{1-x}\right.\\ &\qquad\qquad \left. + \frac{3g-4}{4g-4}\right\}\end{aligned}\right\}\ (o)$$

und

$$\left(\frac{dQ}{db}\right) = \left(\frac{dQ}{dx}\right)\left(\frac{dx}{db}\right) = \frac{1-x}{x}Q'\left(\frac{dx}{db}\right) = \frac{1-x}{x}Q'\,\frac{x}{\alpha}\left(\frac{d\alpha}{db}\right) = \frac{1-x}{b}Q' \cdots\cdots (p)$$

---

[1]) Den Factor $1 + nr'$ kann man sich mit in b enthalten denken.

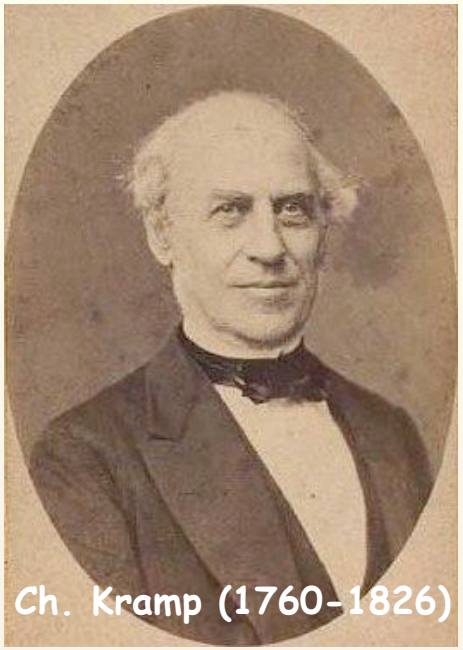


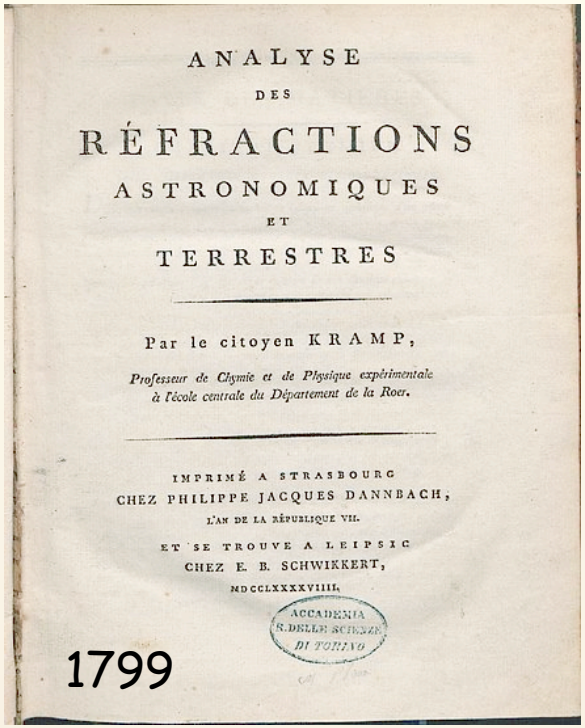
F. W. Bessel 1784-1846



Konigsberg Observatory

Could be very intricate …


Ch. Kramp (1760-1826)


ANALYSE
DES
RÉFRACTIONS
ASTRONOMIQUES
ET
TERRESTRES

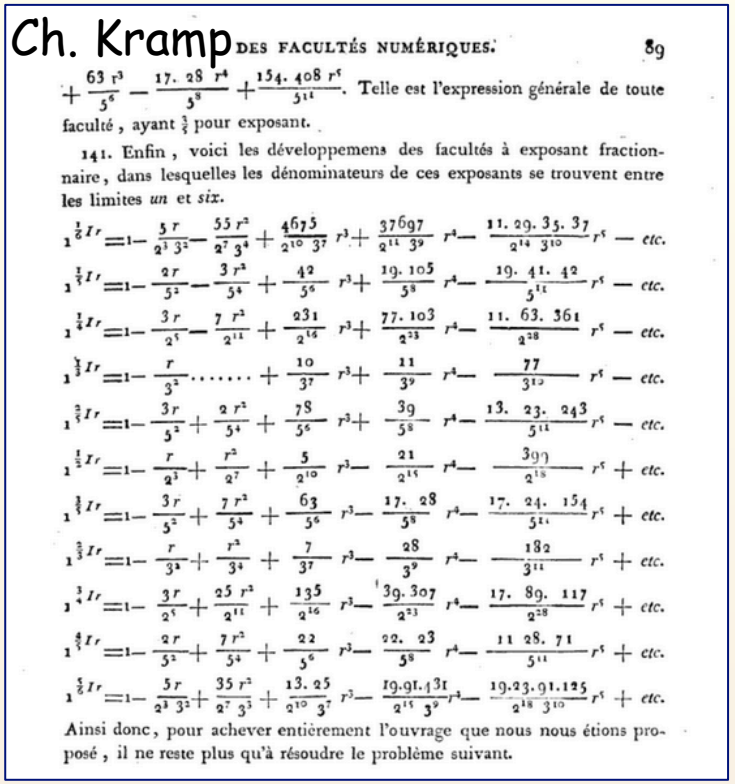
Par le citoyen KRAMP,

Professeur de Chymie et de Physique expérimentale à l'école centrale du Département de la Roer.

IMPRIMÉ A STRASBOURG
CHEZ PHILIPPE JACQUES DANNBACH,
L'AN DE LA RÉPUBLIQUE VII.
ET SE TROUVE A LEIPSIC
CHEZ E. B. SCHWIKKERT,
MDCCLXXXXVIIII.

ACCADEMIA R.DELLE SCIENZE DI TORINO

1799


Ch. Kramp DES FACULTÉS NUMÉRIQUES. 89

$+\frac{63\,r^3}{5^6}-\frac{17.28\,r^4}{5^8}+\frac{154.408\,r^5}{5^{11}}$. Telle est l'expression générale de toute faculté, ayant $\frac{3}{2}$ pour exposant.

141. Enfin, voici les développemens des facultés à exposant fractionnaire, dans lesquelles les dénominateurs de ces exposants se trouvent entre les limites *un* et *six*.

$$1^{\frac{1}{6}}r=1-\frac{5r}{2^3\,3^2}-\frac{55\,r^2}{2^7\,3^4}+\frac{4675}{2^{10}\,3^7}r^3+\frac{37697}{2^{11}\,3^9}r^4-\frac{11.29.35.37}{2^{14}\,3^{10}}r^5-\text{etc.}$$

$$1^{\frac{1}{5}}r=1-\frac{2r}{5^2}-\frac{3\,r^2}{5^4}+\frac{42}{5^6}r^3+\frac{19.105}{5^8}r^4-\frac{19.41.42}{5^{11}}r^5-\text{etc.}$$

$$1^{\frac{1}{4}}r=1-\frac{3r}{2^5}-\frac{7\,r^2}{2^{11}}+\frac{231}{2^{16}}r^3+\frac{77.103}{2^{23}}r^4-\frac{11.63.361}{2^{28}}r^5-\text{etc.}$$

$$1^{\frac{1}{3}}r=1-\frac{r}{3^2}\ldots\ldots+\frac{10}{3^7}r^3+\frac{11}{3^9}r^4-\frac{77}{3^{12}}r^5-\text{etc.}$$

$$1^{\frac{2}{5}}r=1-\frac{3r}{5^2}+\frac{2\,r^2}{5^4}+\frac{78}{5^6}r^3+\frac{39}{5^8}r^4-\frac{13.23.243}{5^{11}}r^5-\text{etc.}$$

$$1^{\frac{1}{2}}r=1-\frac{r}{2^3}+\frac{r^2}{2^7}+\frac{5}{2^{10}}r^3-\frac{21}{2^{15}}r^4-\frac{399}{2^{18}}r^5+\text{etc.}$$

$$1^{\frac{3}{5}}r=1-\frac{3r}{5^2}+\frac{7\,r^2}{5^4}+\frac{63}{5^6}r^3-\frac{17.28}{5^8}r^4-\frac{17.24.154}{5^{11}}r^5+\text{etc.}$$

$$1^{\frac{2}{3}}r=1-\frac{r}{3^2}+\frac{r^2}{3^4}+\frac{7}{3^7}r^3-\frac{28}{3^9}r^4-\frac{182}{3^{11}}r^5+\text{etc.}$$

$$1^{\frac{3}{4}}r=1-\frac{3r}{2^5}+\frac{25\,r^2}{2^{11}}+\frac{135}{2^{16}}r^3-\frac{39.307}{2^{23}}r^4-\frac{17.89.117}{2^{28}}r^5+\text{etc.}$$

$$1^{\frac{4}{5}}r=1-\frac{2r}{5^2}+\frac{7\,r^2}{5^4}+\frac{22}{5^6}r^3-\frac{22.23}{5^8}r^4-\frac{11\,28.71}{5^{11}}r^5+\text{etc.}$$

$$1^{\frac{5}{6}}r=1-\frac{5r}{2^3\,3^2}+\frac{35\,r^2}{2^7\,3^3}+\frac{13.25}{2^{10}\,3^7}r^3-\frac{19.91.131}{2^{15}\,3^9}r^4-\frac{19.23.91.125}{2^{18}\,3^{10}}r^5+\text{etc.}$$

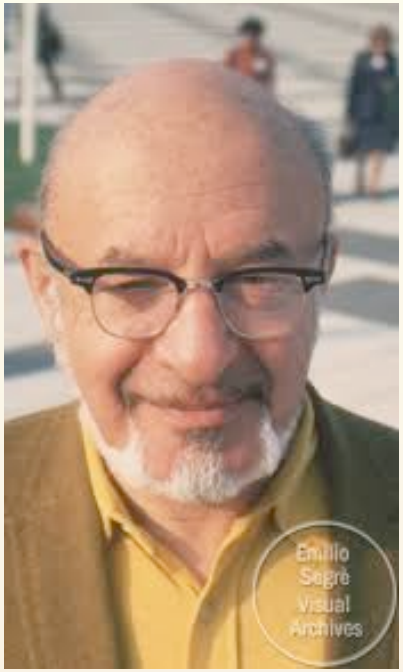
Ainsi donc, pour achever entièrement l'ouvrage que nous nous étions proposé, il ne reste plus qu'à résoudre le problème suivant.

I have given it the name 'faculty'.

I use the very simple notation n! to designate the product of numbers decreasing from n to unity…..

Christian Kramp,Elements d'arithmétique universelle, 1808

Could be very intricate …



B. Garfinkel, 1904-1999

B. Change of Variable

Consider three successive transformations $x \to \xi_1$, $\xi_1 \to \xi_2$, and $\xi_2 \to \xi$ defined by

$$\begin{aligned} \xi_1 &= x+\beta_1(1-\beta_1)^{-1}(x-x^n), \\ \xi_2 &= \xi_1+\alpha(\xi_1-\xi_1^{n+1}), \\ \xi &= \xi_2-\beta_2(1-\beta_2)^{-1}(\xi_2-\xi_2^2). \end{aligned} \tag{17}$$

Then the expressions u and $2u-u^2$, appearing in (14) and (9), can be written to $O(\alpha)$ as

$$\begin{aligned} u &= 2\beta_2(1-\xi_2), \\ 2u-u^2 &= 4\beta_2(1-\beta_2)(1-\xi). \end{aligned} \tag{18}$$

Let us next define Γ and a transform θ of z by

$$\begin{aligned} \Gamma &\equiv [4\beta_2(1-\beta_2)]^{-\frac{1}{2}} \\ &= \gamma[(1-\beta_1)(1-\beta_2)]^{-\frac{1}{2}}, \\ \cot\theta &\equiv \Gamma \cot z. \end{aligned} \tag{19}$$

Then Δ^2 in (7) becomes

$$\Delta^2 = \Gamma^{-2}(\csc^2\theta-\xi), \tag{20}$$

and the integral (16) takes the form

we obtain with the aid of the binomial theorem,

$$x^{p-1}dx = \sum_{i=0}^{\infty}\sum_{j=0}^{i} \epsilon^i(-1)^k \frac{(m-1+i)!}{j!k!(m-1)!} y^{m-1}dy, \tag{26}$$

where m and k are defined by (cf. [1]),

$$\begin{aligned} m &\equiv p+j(q-1), \\ k &\equiv i-j. \end{aligned} \tag{27}$$

If we introduce the parameter κ defined by

$$\epsilon = \kappa(1-\kappa)^{-1}, \tag{28}$$

and change the order of summation in (26), there follows

$$\begin{aligned} x^{p-1}dx &= \sum_{j=0}^{\infty} \epsilon^j \frac{(m-1+j)!}{j!(m-1)!} y^{m-1} \\ &\quad \times \sum_{k=0}^{\infty} (-\epsilon)^k \frac{(m-1+j+k)!}{k!(m-1+j)!} dy \\ &= \sum_{j=0}^{\infty} \epsilon^j (1+\epsilon)^{-m-j} \binom{m-1+j}{j} y^{m-1}dy \\ &= \sum_{i=0}^{\infty} \kappa^i (1-\kappa)^m \binom{m-1+i}{i} y^{m-1}dy, \end{aligned} \tag{29}$$

1967

Astronomical Refraction in a Polytropic Atmosphere

BORIS GARFINKEL
Yale University Observatory, New Haven, Connecticut
(Received 29 November 1966)

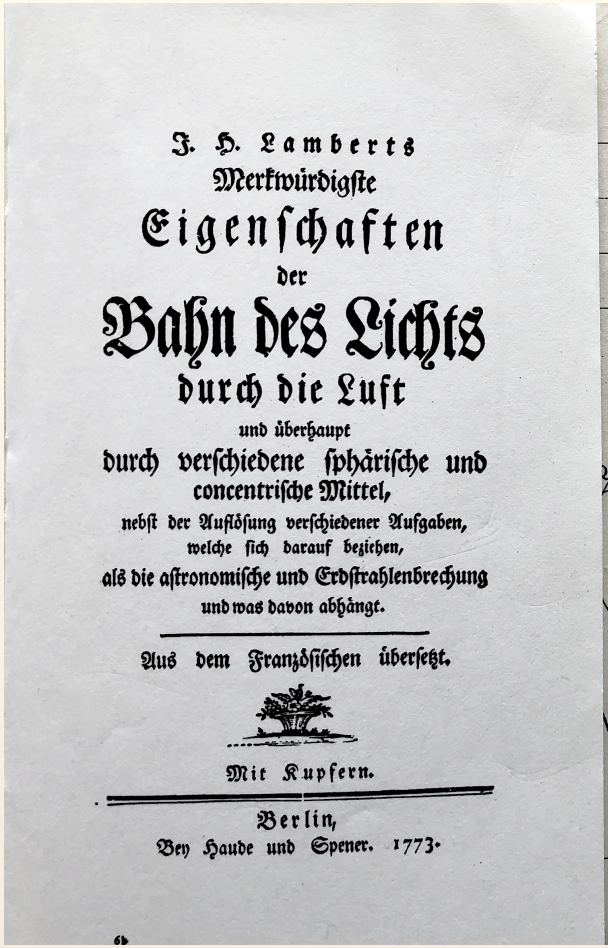
Refraction by Lambert

- J.H. Lambert the first gave the expansion in odd powers of $\tan z_0$

$$\mathcal{R} = A_1 \tan z_0 + A_3 \tan^3 z_0 + A_5 \tan^5 z_0 + \cdots$$



J.H. Lambert 1728- 1777

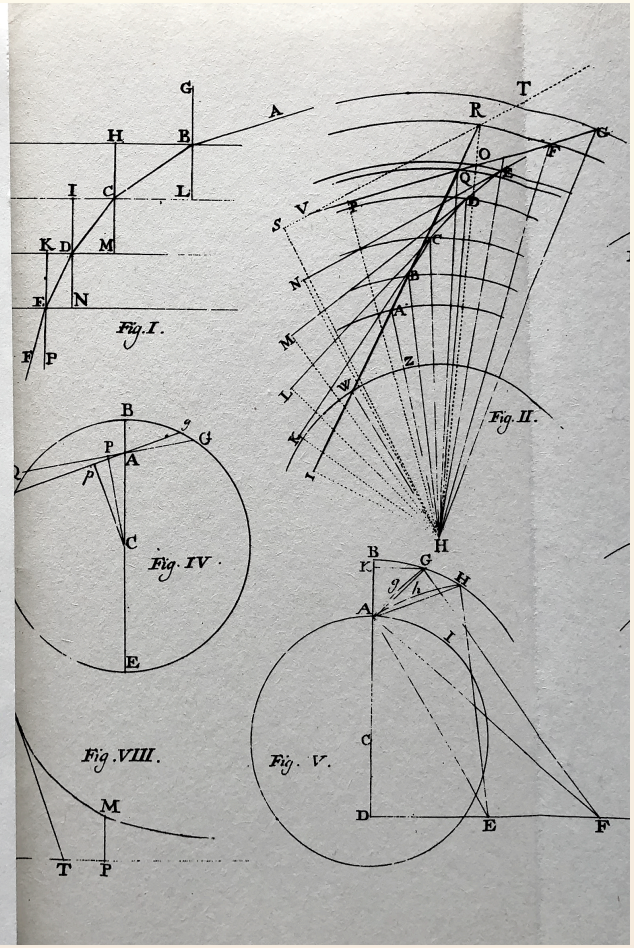


J. H. Lamberts
Merkwürdigste
Eigenschaften
der
Bahn des Lichts
durch die Luft
und überhaupt
durch verschiedene sphärische und concentrische Mittel,
nebst der Auflösung verschiedener Aufgaben,
welche sich darauf beziehen,
als die astronomische und Erdstrahlenbrechung
und was davon abhängt.

Aus dem Französischen übersetzt.

Mit Kupfern.

Berlin,
Bey Haude und Spener. 1773.



Semi-empirical rules

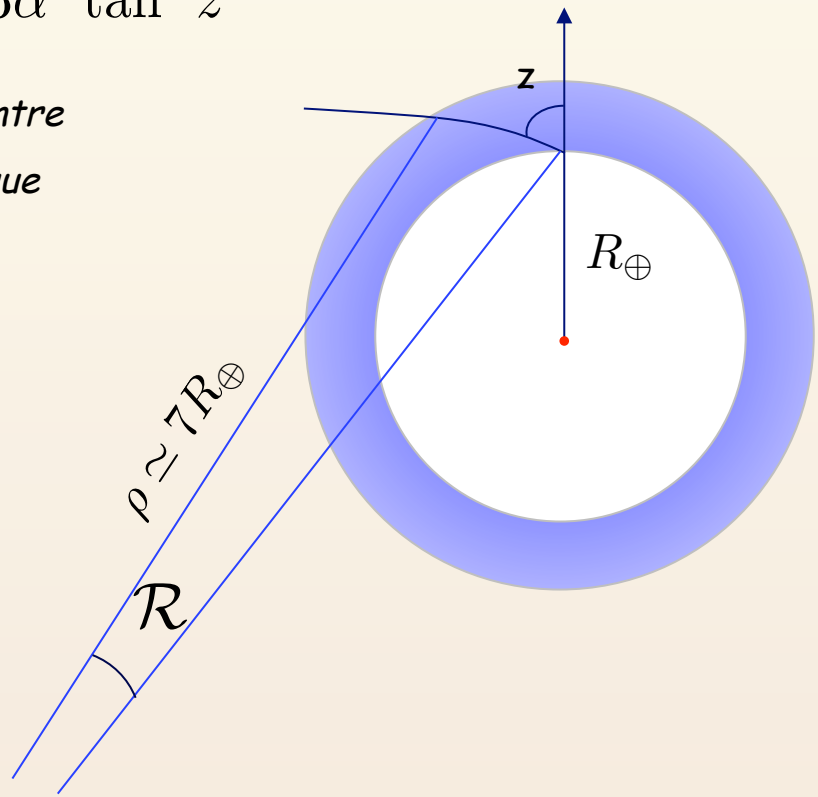
- From Bradley and quoted by Lalande (Astronomie, art. 2202)

"la réfraction est comme la tangente de la distance apparente au zénith diminuée de trois fois la réfraction"

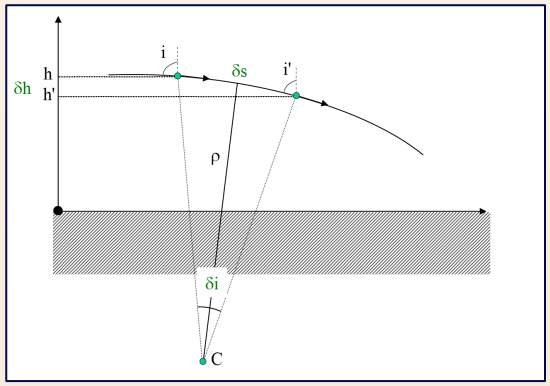
$$\mathcal{R} \sim \tan(z - 3\mathcal{R})$$

➔ very similar to Laplace : $\mathcal{R} = \alpha \tan z - 3\alpha^2 \tan^3 z$

"la réfraction est la septième partie de l'angle au centre de la Terre, dans lequel est renfermé tout l'espace que le rayon a parcouru dans l'atmosphère"



Quite good approximation since



$$\frac{1}{\rho} \sim -\frac{dn}{dh}$$

$$\rightarrow \rho \sim 40,000\,\mathrm{km}$$

First theoretical models of the astronomical refraction

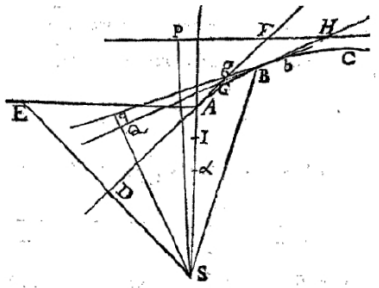
- J.D. Cassini in 1650, with uniform density spherical slab
  - basically wrong, numerically rather accurate to z ~ 75°
- Brook Taylor in 1715 (yes, the one of the Taylor series)

PROP. XXVII. PROB. XXII.

To find the refraction of the light rays passing through the atmosphere of the earth.

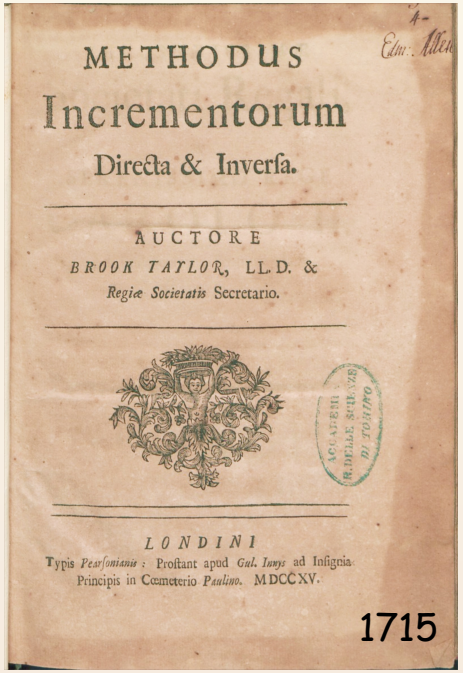


B. Taylor 1685-1731



Let S be the centre of the earth, ABC the radius of a curved light ray, that touches the lines AG; BG in A and B, and these tangents mutually cross each other in G, and the perpendiculars *s*D and SQ are sent to the tangents, and SA and SB are drawn; and at A the normal of SA, AE is drawn crossing SD in E, and A is a given fixed point on the ray, and B is a variable point. Let SA = *a*, SD = *b*, SE = *t* (= $\frac{aa}{b}$,) SB = *x*, *d* is the density at A, *y* the density at B. [p. 109.]

$$\frac{a^2 b y \dot{x}}{2cx^2 \times \overline{1+y} \times \sqrt{\frac{1+d}{1+y}x^2 - b^2}}$$

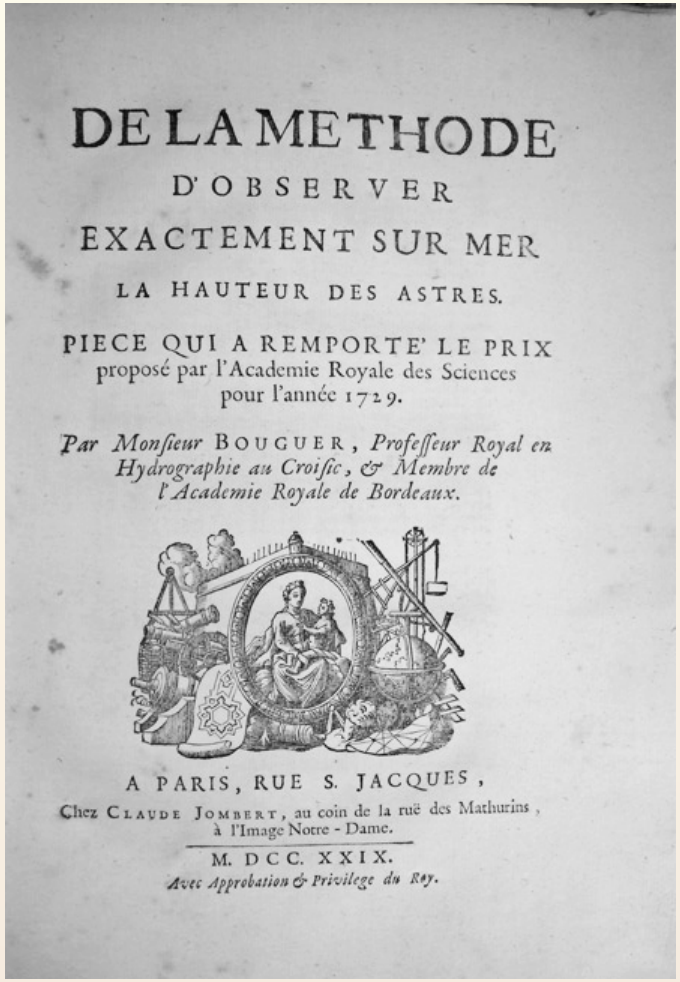


Refraction

- P. Bouguer treatise awarded by the Académie in 1729



P. Bouguer 1698-1758



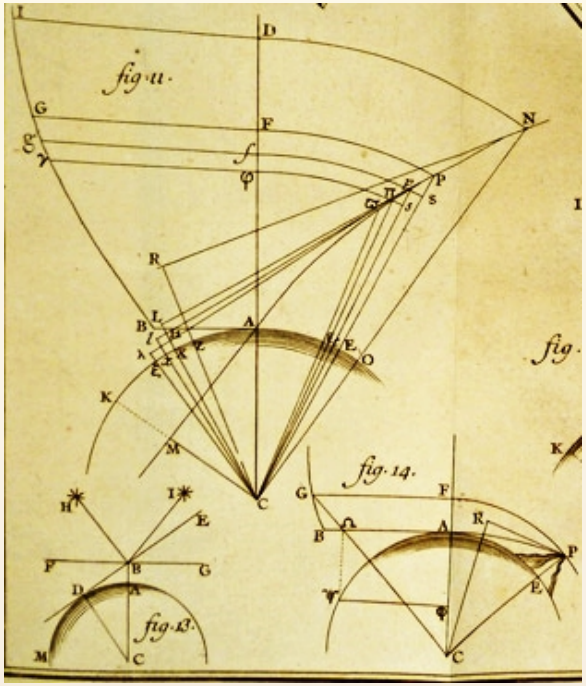
DE LA METHODE
D'OBSERVER
EXACTEMENT SUR MER
LA HAUTEUR DES ASTRES.

PIECE QUI A REMPORTE' LE PRIX
proposé par l'Academie Royale des Sciences
pour l'année 1729.

Par Monſieur BOUGUER, *Profeſſeur Royal en Hydrographie au Croiſic, & Membre de l'Academie Royale de Bordeaux.*

A PARIS, RUE S. JACQUES,
Chez CLAUDE JOMBERT, au coin de la ruë des Mathurins, à l'Image Notre-Dame.

M. DCC. XXIX.
Avec Approbation & Privilege du Roy.



B.Oriani

- Italian astronomer
  - improved the lunar tables of T. Mayer
  - computed one of the first orbit of Uranus
  - was the first to determine a preliminary orbit of Ceres in 1801
  - worked on the theory of refraction : Oriani theorem
    - later extended and simplified by Laplace and Ivory
  - Asteroid (4540) Oriani



B. Oriani 1752-1832

De refractionibus astronomicis

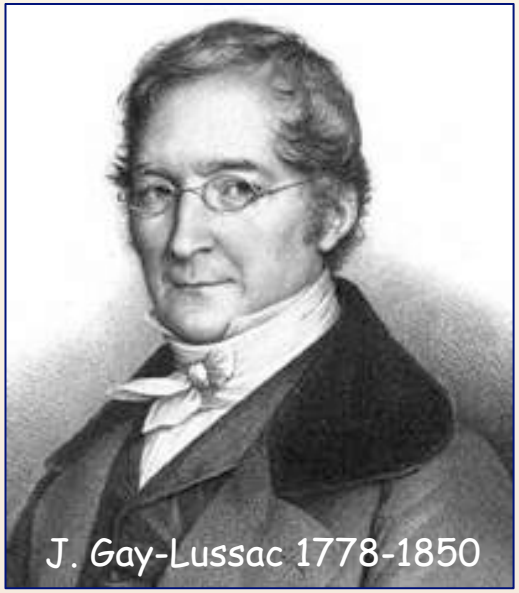
Ephemerides astronomicae anni 1788: Appendix ad ephemerides Anni 1788

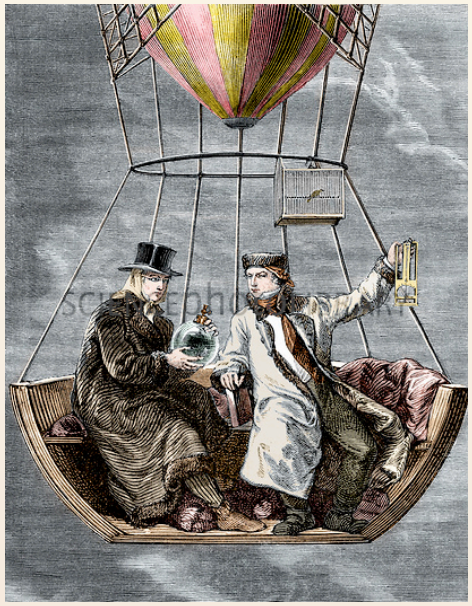
$$\mathcal{R} = A \tan z_0 - B \tan^3 z_0 + \mathcal{O}(\tan^5 z_0)$$

with A, B depending only on the physical conditions at the observer

Gay-Lussac and Biot

- First balloon ascent specifically for scientific purposes
  - main gaol : variation of magnetism with altitude
  - but measurements of pressure, humidity, temperature in upper atmosphere
- Gay-Lussac reached 7016m on 16 Sep 1804 (temperature = -9°C)
  - he noticed the regular decrease of temperature with altitude
  - first empirical study of the upper atmosphere
    - see also Pascal, De Saussure, von Humboldt


J. Gay-Lussac 1778-1850



Numerical Approach

- Pioneered by J.B. Biot and rediscovered by Auer & Standish

THE ASTRONOMICAL JOURNAL, 119:2472–2474, 2000 May
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ASTRONOMICAL REFRACTION: COMPUTATIONAL METHOD FOR ALL ZENITH ANGLES

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AND
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Received 2000 January 19; accepted 2000 January 20

Auer & Standish, 2000

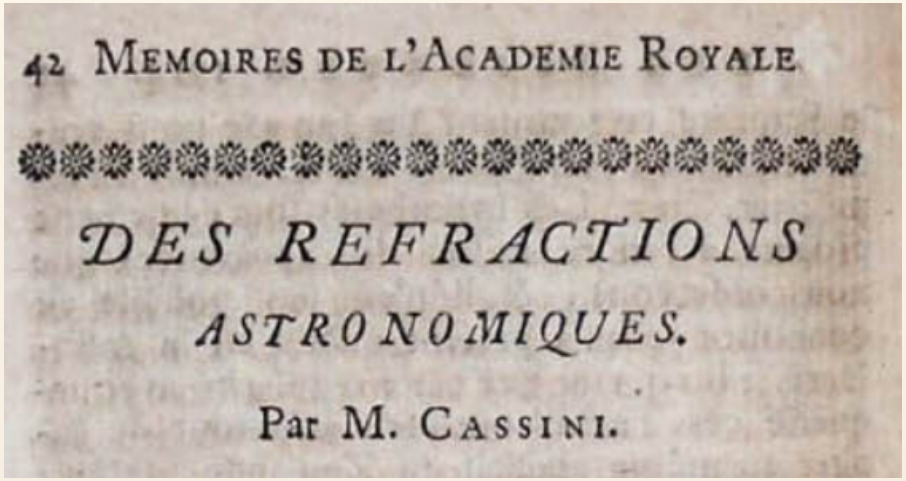
> The analytical formula is, of course, by far the fastest and most simple to use. For zenith angles greater than 75°, however, its accuracy begins to deteriorate seriously. For these higher angles, the method presented here surpasses any other presently known with respect to simplicity, generality, and computational speed.

J.D. Cassini refraction model

- First use of Snell-Descartes law
- Very crude, but surprisingly accurate
- Best first step beyond the plane-parallel model
- Very nice for teaching : simple maths and physics



J.D. Cassini 1625-1712

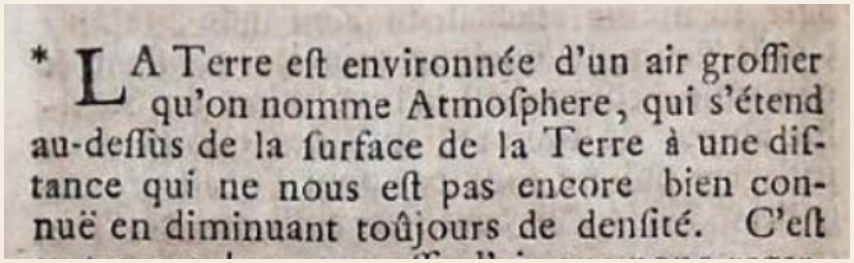


42 MEMOIRES DE L'ACADEMIE ROYALE

DES REFRACTIONS ASTRONOMIQUES.

Par M. CASSINI.

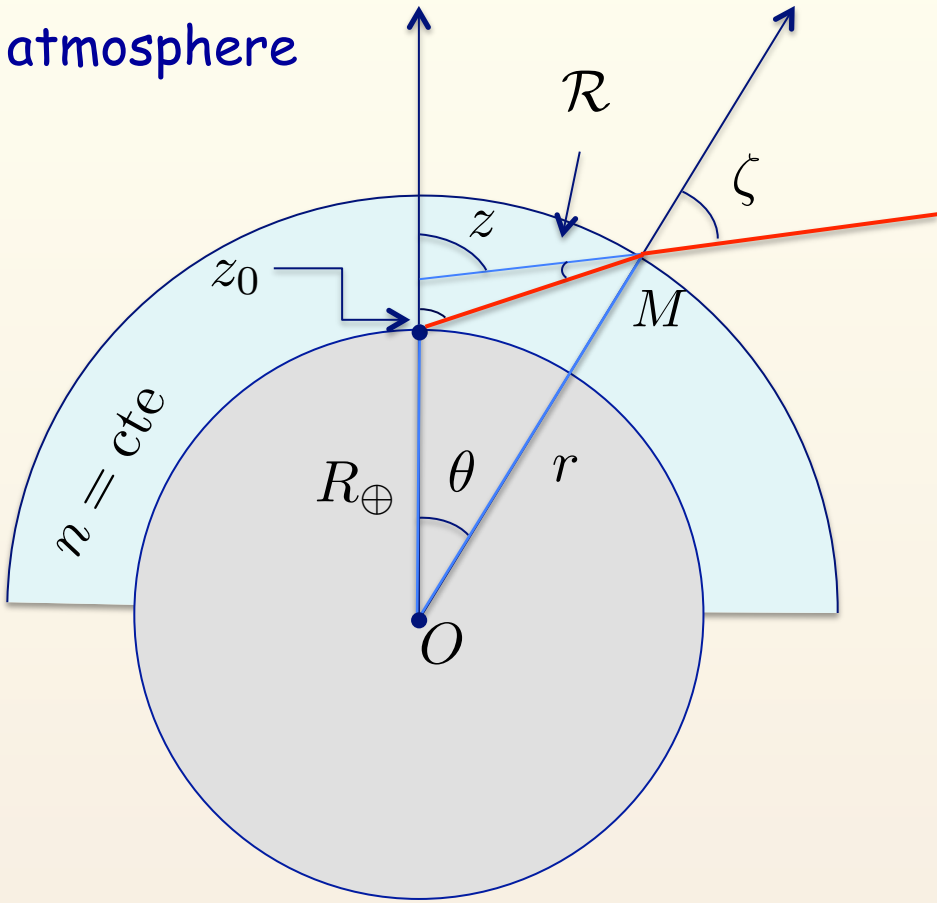
Mémoires de l'Académie Royale des Sciences, 1714



* LA Terre eſt environnée d'un air groſſier qu'on nomme Atmoſphere, qui s'étend au-deſſus de la ſurface de la Terre à une diſtance qui ne nous eſt pas encore bien connuë en diminuant toûjours de denſité. C'eſt

J.D. Cassini refraction model

- Two-parameter model with uniform atmosphere
  - height of the atmosphere
  - refractive index
- Constrained with two refractions
  - at z_0 = 80° and 90°
- Remarkably good to ~ 70°



$$\mathcal{R} = z - z_0$$

$$\sin\zeta = n\sin(\zeta - \mathcal{R})$$

$$r\sin(\zeta - \mathcal{R}) = R_\oplus \sin(z_0)$$

$$r\sin\zeta = nR_\oplus \sin z_0$$

$$\frac{r}{R_\oplus} = 1 + x$$

$$\mathcal{R} = \sin^{-1}\left[\frac{n\sin z_0}{1+x}\right] - \sin^{-1}\left[\frac{\sin z_0}{1+x}\right]$$

J.D. Cassini refraction model

auſſi connu, & par conſequent l'on trouve la valeur du troiſiéme côté, qui eſt la diſtance veritable du Soleil au Zenith. On a donc la hauteur veritable du Soleil au temps de l'Obſervation, qui étant retranchée de la hauteur apparente, reſte la refraction, à laquelle il faut ajoûter la parallaxe pour avoir la refraction veritable qui convient au degré obſervé.

La ſeconde methode ſuppoſe, comme nous l'avons dit ci-deſſus, qu'on a trouvé par Obſervation la refraction qui convient à deux degrés differens de hauteur.

Soit *S*, le Soleil ou une Etoile dont le rayon *SB* rencontrant la ſurface refractive *BHP* en *B*, ſe rompe & vienne à notre œil en *A*; de ſorte que *BA* ſoit perpendiculaire à *AC*, l'objet ſera vû ſuivant la ligne *ABE*, & l'angle *EBS* ou *EAX* ſera la refraction horiſontale que l'on ſuppoſe de 32′ 20″, telle qu'elle eſt marquée dans la *Connoiſſance des Temps*.

Soit un autre rayon *IH* qui rencontrant en *H* la ſurface refractive, vienne ſe rompre en *A*; en ſorte que l'angle *BAH* ſoit de 10 degrés. L'angle *EHI*, ſera la refraction qui convient à 10 degrés que l'on ſuppoſe obſervée de 5′ 28″, telle qu'elle eſt marquée dans la *Connoiſſance des Temps*.

Par la regle des refractions reçûë de tous les Philoſophes, les ſinus d'incidence ſont toûjours proportionnels au ſinus des refractions, & par conſequent le ſinus de l'angle *LBS* eſt au ſinus de l'angle *CBA* comme le ſinus de l'angle *GHI* eſt au ſinus de l'angle *AHC*.

Le

TABLE des Refractions ſuivant l'Ancienne & la Nouvelle Hypotheſe juſques à la hauteur de trente Degrés.

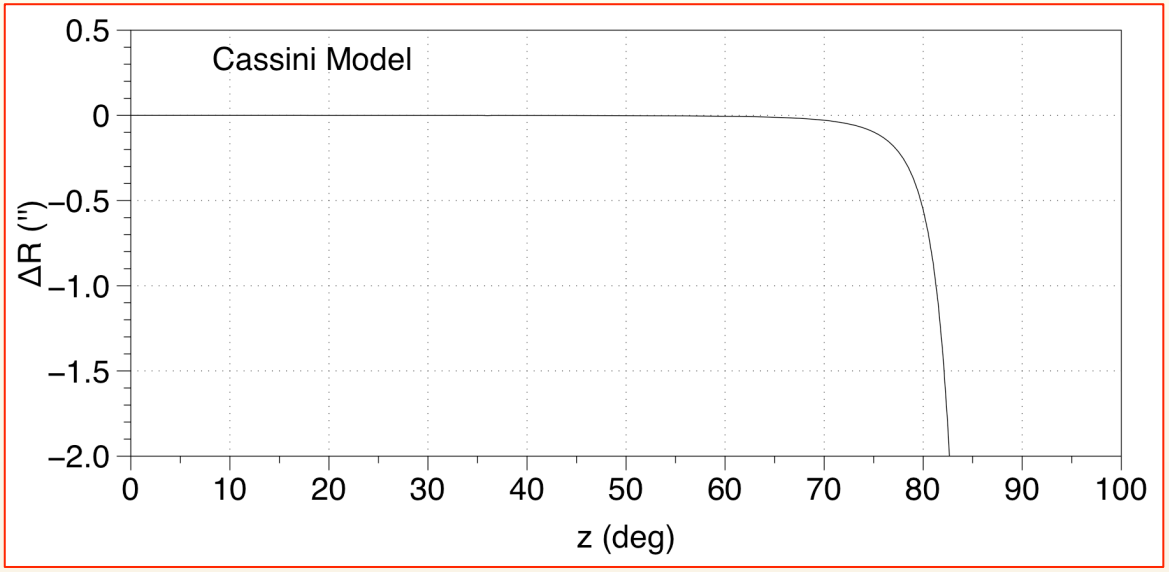
Hauteur ſur l'Horizon. Degres.	Refraction ſuivant l'Hypotheſe rectiligne. M. S.	Refraction ſuivant l'Hypotheſe nouvelle. M. S.	Hauteur ſur l'Horizon. D. M.	Refraction ſuivant l'Hypotheſe rectiligne. M. S.	Refraction ſuivant l'Hypotheſe nouvelle. M. S.	Hauteur ſur l'Horizon. Minutes.	Refraction ſuivant l'Hypotheſe rectiligne. M. S.	Refraction ſuivant l'Hypotheſe nouvelle. M. S.	Hauteur ſur l'Horizon. Minutes.	Refraction ſuivant l'Hypotheſe rectiligne. M. S.	Refraction ſuivant l'Hypotheſe nouvelle. M. S.
0	32 20	32 20	1 0	27 56	24 19	0 0	32 20	32 20	30	31 3	28 1
1	27 56	24 19	10	26 45	23 13	1	32 20	32 9	31	30 57	27 53
2	21 6	18 40	20	25 37	22 12	2	32 19	32 0	32	30 52	27 45
3	16 8	14 46	30	24 22	21 14	3	32 18	31 52	33	30 46	27 37
4	12 49	12 2	40	23 14	20 21	4	32 18	31 43	34	30 41	27 29
5	10 33	10 6	50	22 8	19 28	5	32 18	31 33	35	30 35	27 21
6	8 55	8 38	2 0	21 6	18 40	6	32 16	31 24	36	30 30	27 12
7	7 44	7 32	10	20 8	17 56	7	32 15	31 14	37	30 24	27 5
8	6 48	6 40	20	19 13	17 13	8	32 14	31 5	38	30 19	26 57
9	6 4	5 57	30	18 22	16 32	9	32 12	30 56	39	30 13	26 49
10	5 28	5 23	40	17 34	15 55	10	32 10	30 47	40	30 8	26 42
11	4 58	4 55	50	16 49	15 19	11	32 9	30 38	41	30 2	26 34
12	4 33	4 31	3 0	16 8	14 46	12	32 7	30 30	42	29 56	26 27
13	4 12	4 10	10	15 29	14 15	13	32 4	30 21	43	29 50	26 19
14	3 54	3 52	20	14 53	13 45	14	32 2	30 13	44	29 44	26 12
15	3 36	3 36	30	14 19	13 17	15	32 0	30 4	45	29 37	26 4
16	3 24	3 22	40	13 47	12 50	16	31 57	29 55	46	29 31	25 57
17	3 11	3 10	50	13 17	12 25	17	31 54	29 47	47	29 25	25 50
18	3 0	2 59	4 0	12 49	12 2	18	31 50	29 39	48	29 19	25 43
19	2 49	2 49	10	12 23	11 40	19	31 47	29 30	49	29 12	25 36
20	2 39	2 39	20	11 58	11 19	20	31 44	29 22	50	29 5	25 29
21	2 31	2 32	30	11 35	10 59	21	31 40	29 14	51	28 58	25 22
22	2 25	2 25	40	11 13	10 40	22	31 36	29 5	52	28 51	25 15
23	2 18	2 18	50	10 52	10 22	23	31 33	28 57	53	28 45	25 8
24	2 12	2 12	5 0	10 33	10 6	24	31 29	28 49	54	28 38	25 1
25	2 6	2 6	10	10 15	9 50	25	31 25	28 41	55	28 32	24 54
26	2 0	2 0	20	9 58	9 34	26	31 21	28 33	56	28 25	24 47
27	1 55	1 55	30	9 42	9 19	27	31 16	28 25	57	28 18	24 40
28	1 51	1 51	40	9 25	9 5	28	31 11	28 19	58	28 11	24 33
29	1 46	1 46	50	9 10	8 51	29	31 7	28 9	59	28 4	24 26
30	1 42	1 42	6 0	8 55	8 38	30	31 3	28 1	60	27 57	24 19

from : Histoire de l'Académie Royale des Sciences

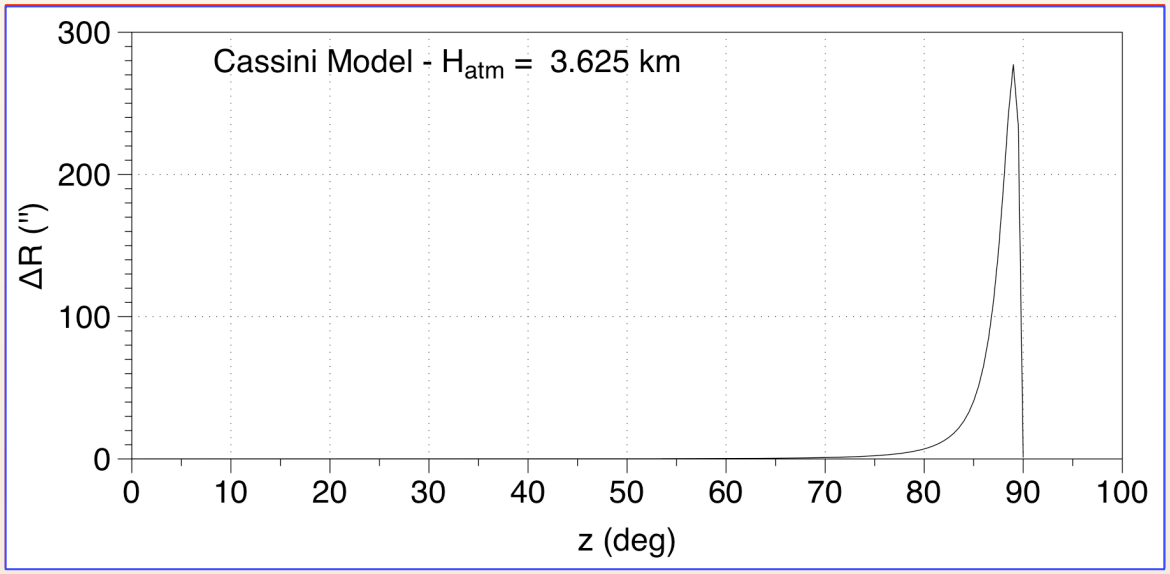
F. Mignard, OCA

Comparison to full model

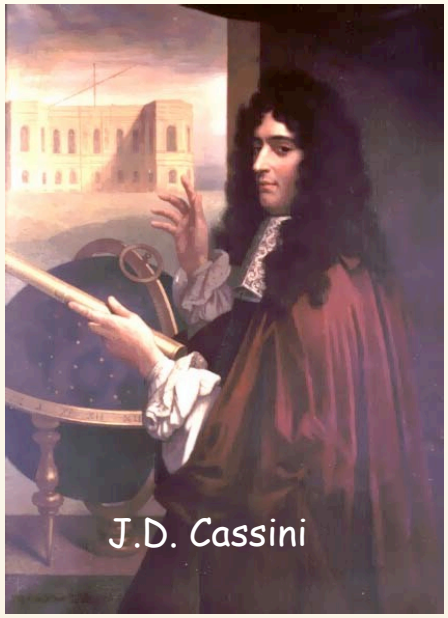
- From 0° to 80°

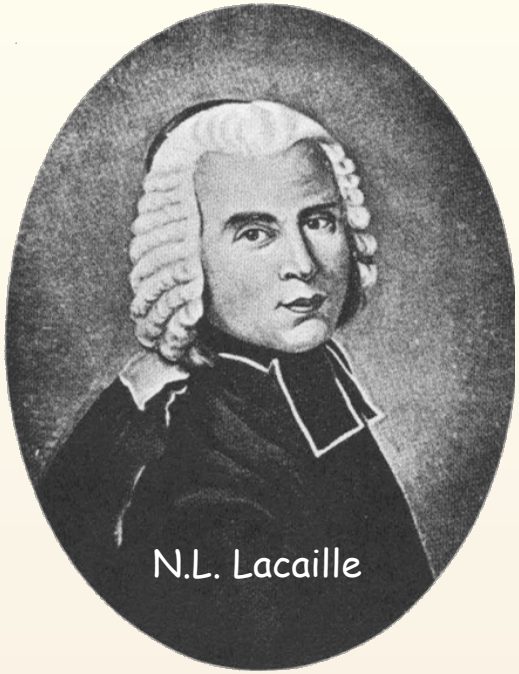


- From 0° to 90°



J.D. Cassini : improved tables





TRAITÉ

D'OPTIQUE,

PAR LACAILLE;

Revu, corrigé et augmenté particulièrement de la Mesure de la Réfraction, de son explication physique, de la Marche des Images dans les instrumens d'Optique, des Lunettes achromatiques, de l'Iris, et de quelques problêmes intéressans;

81

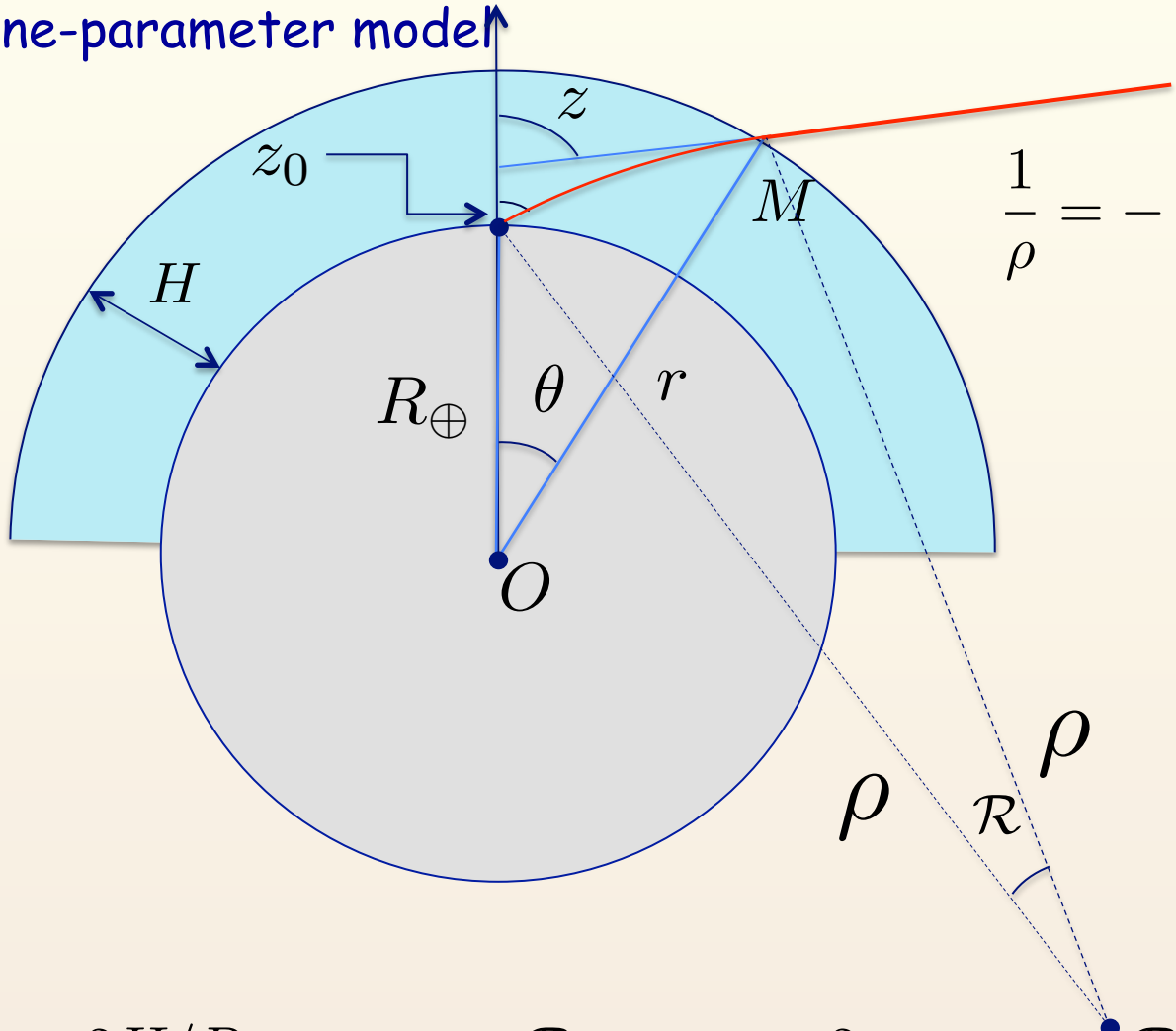
TABLE DES RÉFRACTIONS,
suivant M. Cassini & M. l'Abbé de la Caille.

Les six derniers nombres sont tirés de M. Halley.

Dist au zén.	Réfr de MC	Réfrac de M de la C.	Hauteur	Dist au zén.	Réfr de MC	Réfract. de M. de la C.	Hauteur	Dist au zén.	Réfract. de M. Cass.	Réfraction de M. de la Caille	Hauteur
D.	S.	S.	D.	D.	S.	S.	D.	D.	M. S	M. S	D
1.	1.	1,1	89.	31.	35	40,0	59.	61.	1. 46	1. 59,1	29.
2.	2.	2,3	88.	32.	37	41,6	58.	62.	1. 51	2. 4,0	28.
3.	3.	3,5	87.	33.	38	43,2	57.	63.	1. 55	2. 9,2	27.
4.	4.	4,6	86.	34.	40	44,9	56.	64.	2. 0	2. 14,7	26.
5.	5.	5,8	85.	35.	41	46,6	55.	65.	2. 6	2. 20,5	25.
6.	6.	7,0	84.	36.	43	48,3	54.	66.	2. 12	2. 26,6	24.
7.	7.	8,2	83.	37.	45	50,1	53.	67.	2. 18	2. 33,0	23.
8.	8.	9,3	82.	38.	47	51,9	52.	68.	2. 25	2. 39,8	22.
9.	9.	10,5	81.	39.	49	53,8	51.	69.	2. 31	2. 47,0	21.
10.	10.	11,7	80.	40.	50	55,8	50.	70.	2. 39	2. 54,7	20.
11.	11.	12,9	79.	41.	52	57,9	49.	71.	2. 49	3. 3	19.
12.	12.	14,1	78.	42.	54	60,0	48.	72.	3. 0	3. 12	18.
13.	13.	15,4	77.	43.	56	62,1	47.	73.	3. 11	3. 23	17.
14.	14.	16,6	76.	44.	58	64,3	46.	74.	3. 24	3. 35	16.
15.	16.	17,8	75.	45.	59	66,5	45.	75.	3. 38	3. 49	15.
16.	17.	19,1	74.	46.	61	68,8	44.	76.	3. 54	4. 5	14.
17.	18.	20,3	73.	47.	63	71,2	43.	77.	4. 12	4. 24	13.
18.	19.	21,6	72.	48.	65	73,7	42.	78.	4. 32	4. 45	12.
19.	20.	22,9	71.	49.	67	76,3	41.	79.	4. 58	5. 9	11.
20.	21.	24,2	70.	50.	70	79,0	40.	80.	5. 28	5. 37	10.
21.	22.	25,5	69.	51.	72	81,9	39.	81.	6. 4	6. 10	9.
22.	24.	26,8	68.	52.	75	84,9	38.	82.	6. 47	6. 51	8.
23.	25.	28,2	67.	53.	78	88,0	37.	83.	7. 44	7. 41	7.
24.	26.	29,6	66.	54.	80	91,2	36.	84.	8. 55	8. 42	6.
25.	27.	31,0	65.	55.	83	94,6	35.	85.	10. 32	9. 2	5.
26.	28.	32,4	64.	56.	87	98,1	34.	86.	12. 48	10. 48	4.
27.	30.	33,9	63.	57.	90	101,8	33.	87.	16. 6	15. 2	3.
28.	31.	35,4	62.	58.	94	105,8	32.	88.	21. 4	17. 8	2.
29.	33.	36,9	61.	59.	98	110,0	31.	89.	27. 56	23. 7	1.
30.	34.	38,4	60.	60.	102	114,4	30.	90.	32. 20	33. 45	0.

Super-Cassini refraction model

one-parameter model



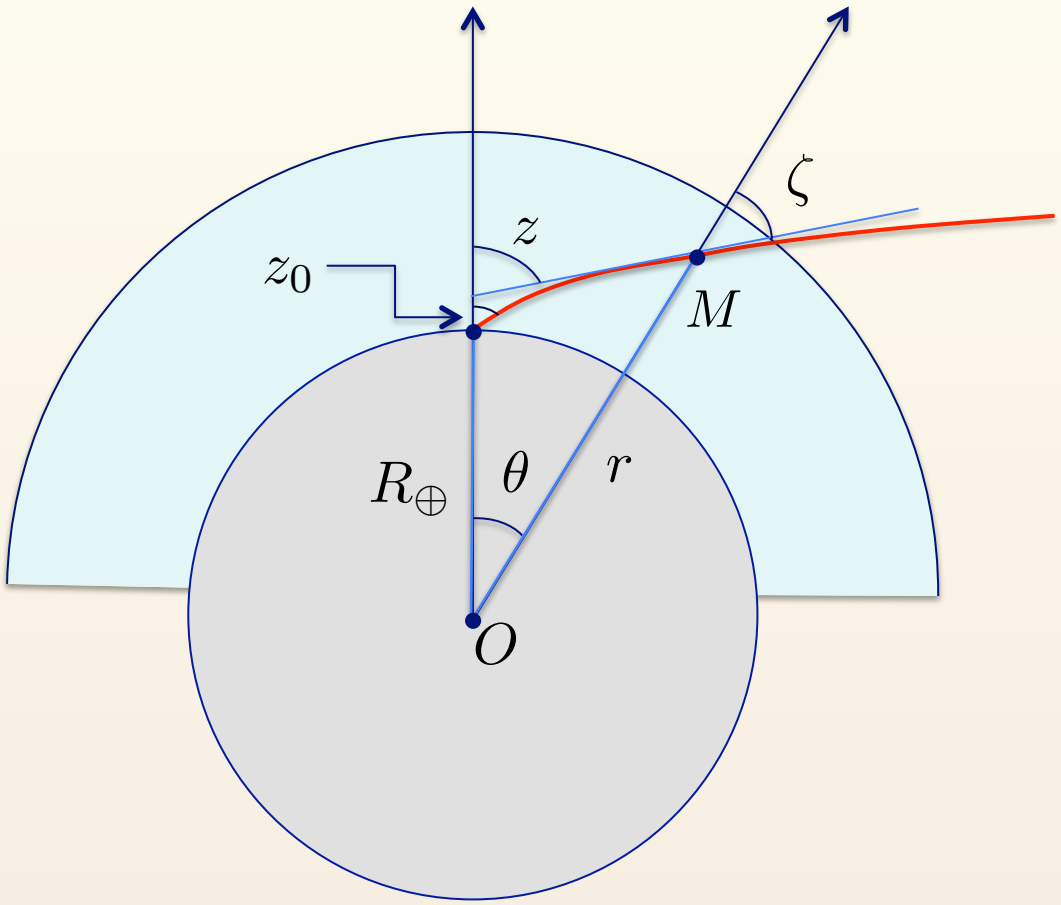
$$\frac{1}{\rho} = -\sin z_0 \frac{dn}{dh} = \sin z_0 \frac{n_0 - 1}{H}$$

$$\frac{2H/R_\oplus}{(n_0-1)^2 \sin^2 z_0} \sin^2 \frac{\mathcal{R}}{2} + \frac{2}{(n_0-1)\sin z_0} \sin \frac{\mathcal{R}}{2} \cos\left(z_0 + \frac{\mathcal{R}}{2}\right) - 1 = 0$$

Theoretical developments



Refraction : main equations



Geometry

$$\mathcal{R} = z_\infty - z_0$$

$$z = \theta + \zeta$$

$$z_0 = \zeta_0$$

$$\tan\zeta = \frac{r}{r'}$$

$$r' = \frac{dr}{d\theta}$$

Optics

$$nr\sin\zeta = \text{cte}$$

Refraction integral

$$\mathcal{R} = z_\infty - z_0$$

$$z = \theta + \zeta$$

$$z_0 = \zeta_0$$

$$\tan\zeta = \frac{r}{r'}$$

$$r' = \frac{dr}{d\theta}$$

$$nr\sin\zeta = \text{cte} = C$$

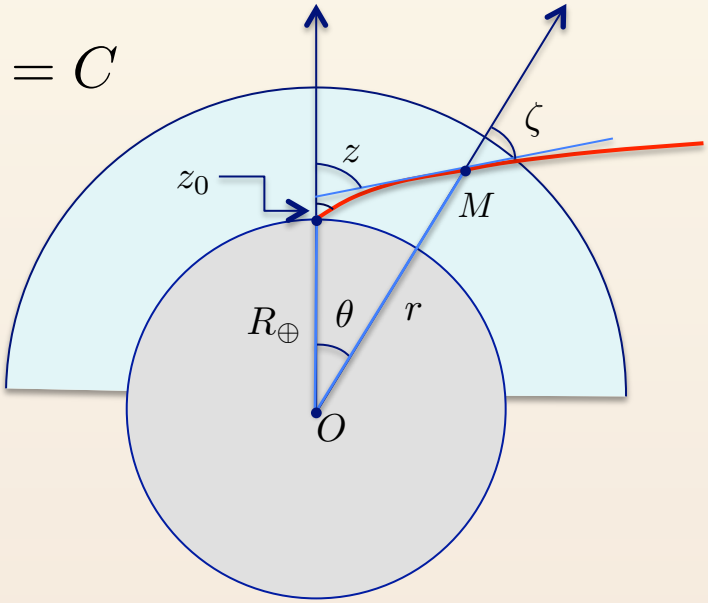
$$d\mathcal{R} = dz = d\theta + d\zeta$$

$$= \frac{\tan\zeta}{r}dr + d\zeta$$

$$d(nr\sin\zeta) = 0 \Longrightarrow$$

$$\tan\zeta\frac{dn}{n} + \tan\zeta\frac{dr}{r} + d\zeta = 0$$

$$d\mathcal{R} = -\frac{\tan\zeta}{n}dn$$



$$\mathcal{R} = \int_1^{n_0} \frac{\tan\zeta}{n}dn$$

Refraction integral

$$\mathcal{R} = \int_1^{n_0} \frac{\tan\zeta}{n} dn$$

$$\mathcal{R} = \int_1^{n_0} \frac{dn}{n\left(\frac{n^2 r^2}{C^2} - 1\right)^{1/2}}$$

$$nr\sin\zeta = \text{cte} = C$$

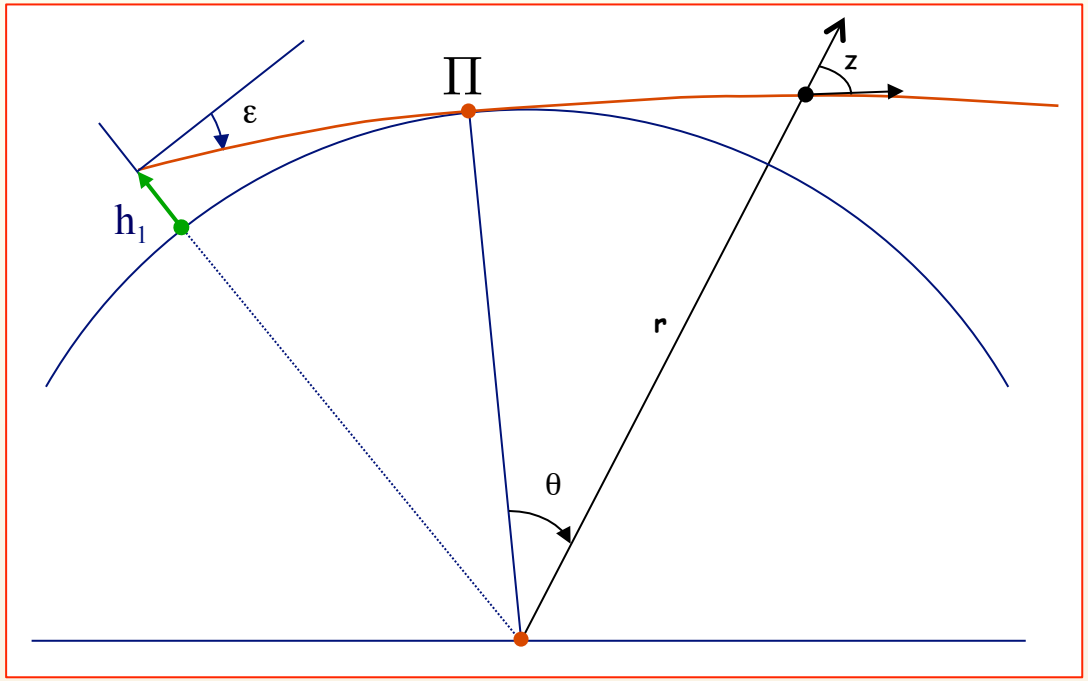
$$n_0 R_\oplus \sin z_0 = C$$

$$\mathcal{R} = -\int_0^{z_0} \frac{r\frac{dn}{dr}}{n + r\frac{dn}{dr}} d\zeta$$

$$\mathcal{R}(z_0),\ \ z_0 \in 0, \frac{\pi}{2}$$

Auer & Standish, 2000

Problems with the integral form



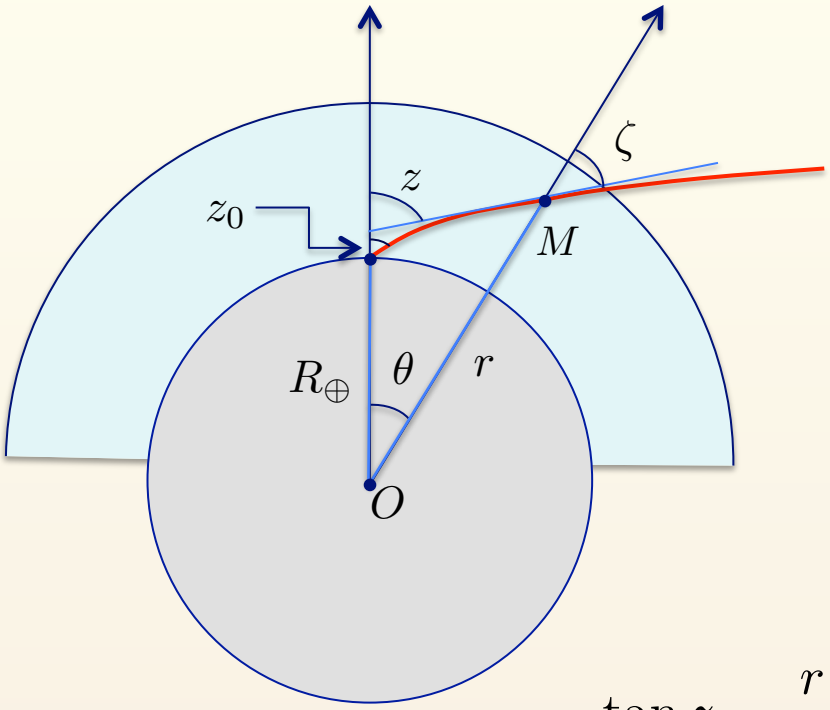
$$z_0 > \pi/2$$

$$nr\sin z = cte = C$$

$$\int d\theta = \eta \int_{r_0}^{r} \frac{dr}{r\left(\frac{n^2 r^2}{C^2} - 1\right)^{1/2}}$$

- One needs to integrate on two branches (η = +/- 1)
  - but one can use the symmetry with respect to perigee to go round
- The initial direction and the contact point are not given
  - the solution for the grazing ray is closer to a boundary conditions problem
- More generally I am interested also by the trajectory

Back to basics



- General equation

$$\frac{d(n\mathbf{T})}{ds} = \nabla n(\mathbf{r})$$

- Spherical symmetry

$$\frac{d(n\mathbf{r}\times\mathbf{T})}{ds} = 0$$

$$nr\sin z = cte = C$$

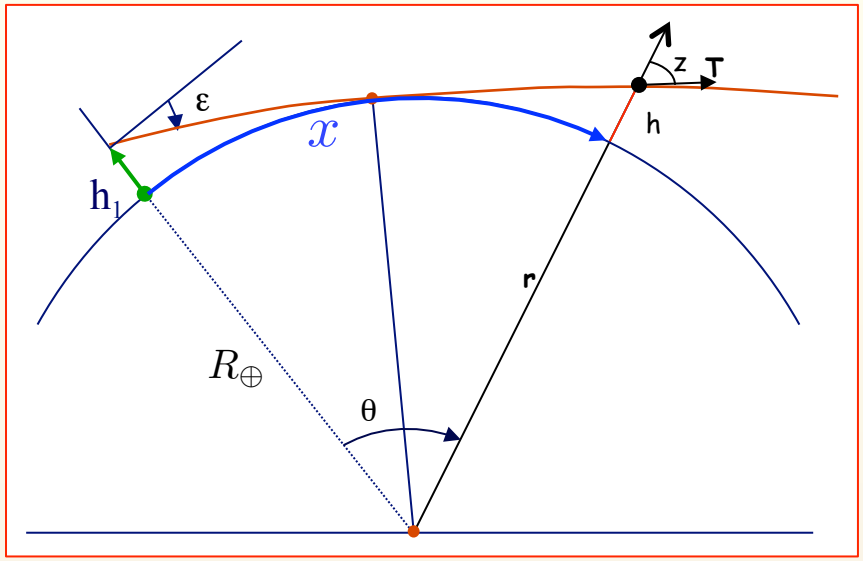
$$\tan z = \frac{r}{r'} \quad\Longrightarrow\quad \frac{dr}{d\theta} = \pm\, r\left[\frac{n^2\, r^2}{C^2} - 1\right]^{1/2}$$

- Regular ODE for all θ with :

$$\frac{d^2r}{d\theta^2} = \frac{1}{2}\frac{d}{dr}\left(\frac{dr}{d\theta}\right)^2 = r\left[\frac{2n^2\, r^2}{C^2} + \frac{n\, r^3}{C^2}\frac{dn}{dr} - 1\right]$$

$$u = \frac{1}{r} \Longrightarrow \qquad \frac{d^2u}{d\theta^2} = -u + \frac{n}{C^2}\frac{dn}{du}$$

Back to basics



$$\frac{d^2r}{d\theta^2} = \frac{1}{2}\frac{d}{dr}\left(\frac{dr}{d\theta}\right)^2 = r\left[\frac{2n^2\,r^2}{C^2} + \frac{n\,r^3}{C^2}\frac{dn}{dr} - 1\right]$$

$$r = R_\oplus + h$$

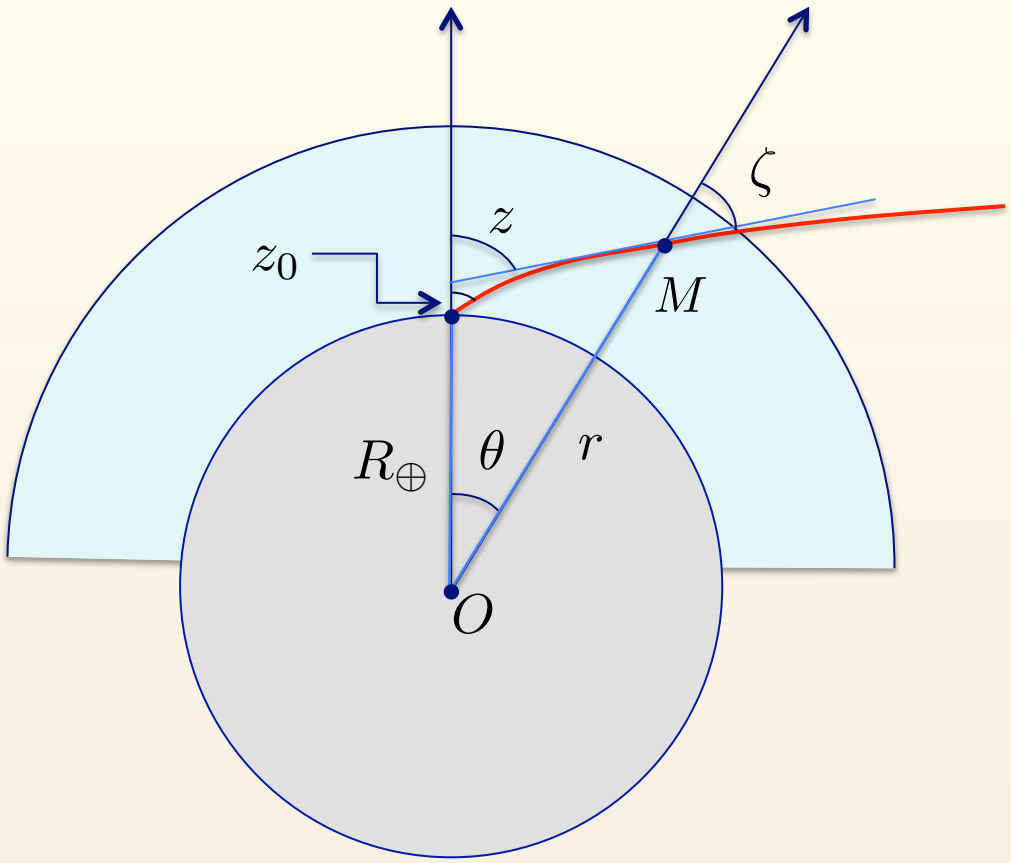
$$h \geq 0$$

$$\boxed{\frac{d^2h}{d\theta^2} = (R_\oplus + h)\left[\frac{2n^2\,(R_\oplus + h)^2}{C^2} + \frac{n\,(R_\oplus + h)^3}{C^2}\frac{dn}{dh} - 1\right]}$$

$$x = R_\oplus\theta$$

$$\boxed{\frac{d^2h}{dx^2} = \frac{(R_\oplus + h)}{R_\oplus^2}\left[\frac{2n^2\,(R_\oplus + h)^2}{C^2} + \frac{n\,(R_\oplus + h)^3}{C^2}\frac{dn}{dh} - 1\right]}$$

Value of the Refraction



Geometry

$$\mathcal{R} = z_\infty - z_0$$

$$z = \theta + \zeta \qquad z_0 = \zeta_0$$

$$\tan\zeta = \frac{r}{r'} \qquad r' = \frac{dr}{d\theta}$$

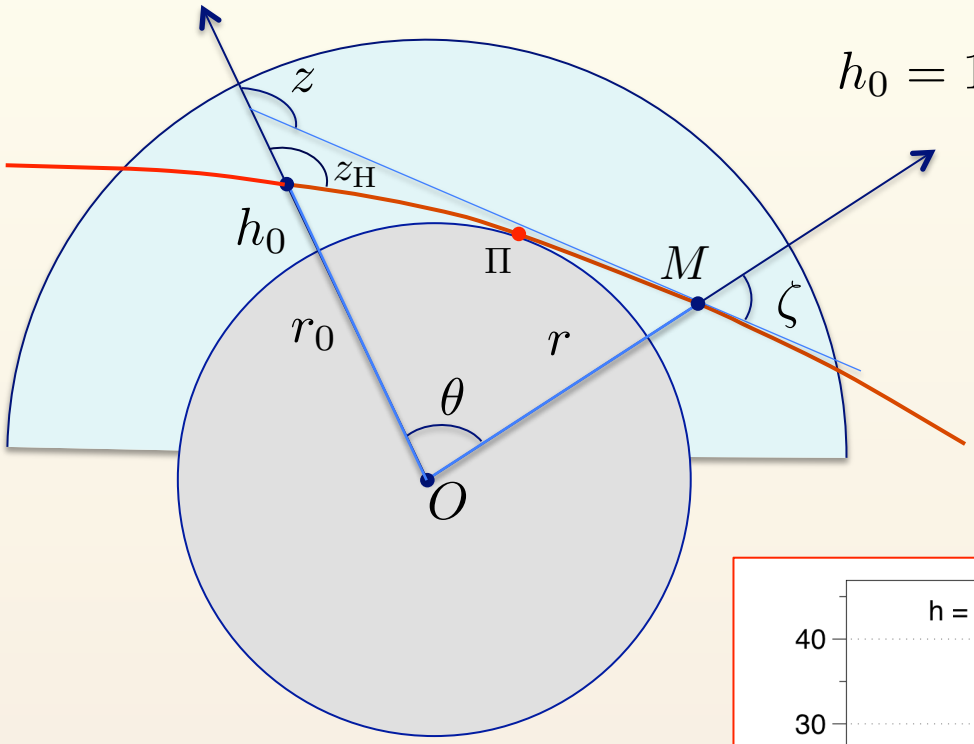
Integration

$$z_\infty = z_{\rm h} \approx 100 \text{ km}$$

$$\zeta = \tan^{-1}\left(\frac{R_\oplus + h}{dh/d\theta}\right)$$

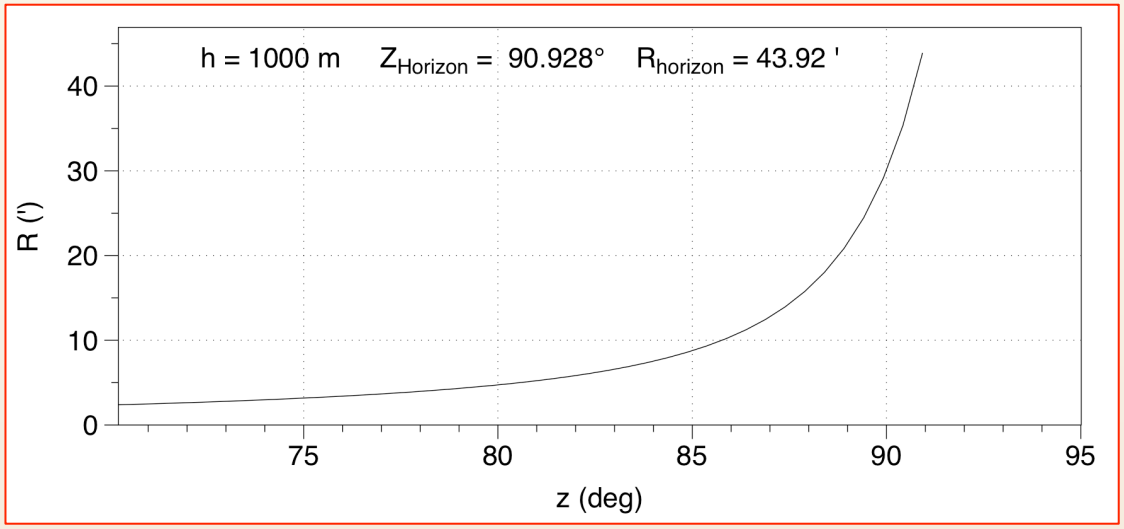
$$\mathcal{R} = (\theta + \zeta)_{\rm h \approx 100\ km} - z_0$$

Example



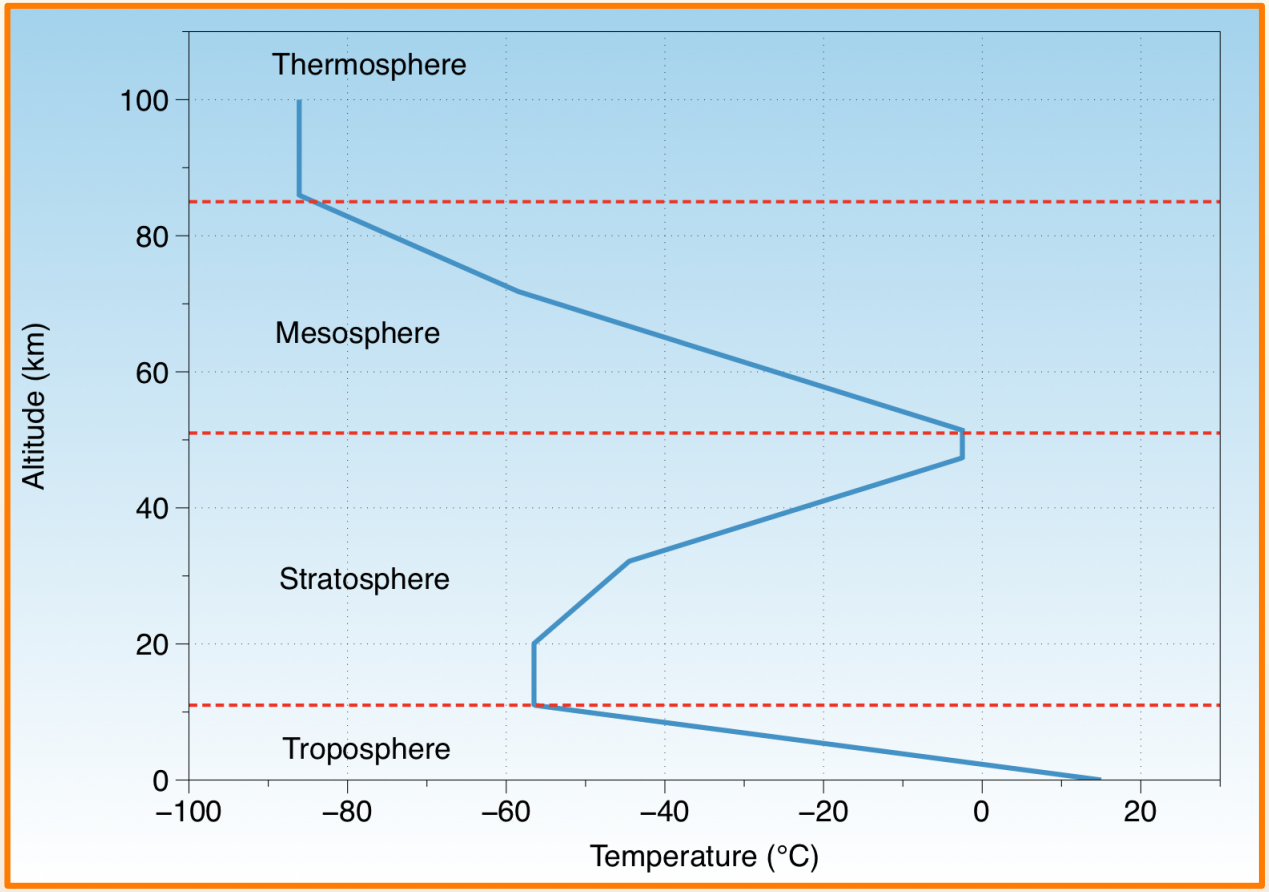
- Refraction for any height

$$h_0 = 1000\mathrm{m} \quad z_{\mathrm{H}} = 90.928^\circ \quad \mathcal{R}_{\max} = 43'.92$$



Atmospheric model

- Baseline is the International Standard Atmosphere
  - a multi-layers atmosphere with dT/dh = 0 or cte
  - but P(h=0), T(h=0) can be different from standard conditions
  - includes a dry and a humid component



Solutions for each layer

$$\frac{dP}{dh} = -\rho g \qquad P = \frac{\rho RT}{M_{\text{air}}} \qquad \Longrightarrow \frac{1}{P}\frac{dP}{dh} = -\frac{M_{\text{air}} g}{RT(h)}$$

$$\ln \frac{P}{P_0} = -\frac{M_{\text{air}} g}{RT_0} \int_{h_0}^{h} \frac{T_0}{T(h)} dh$$

Isotherm layer
$T = \text{cte}$

$$\frac{P}{P_0} = \exp\left[-\frac{M_{\text{air}} g h}{RT_0}\right]$$

Polytropic layer
$T = T_0 + \gamma h$

$$\frac{P}{P_0} = \left[\frac{T(h)}{T_0}\right]^{\beta} \qquad \beta = -\frac{g M_{\text{air}}}{\gamma R}$$

Atmospheric model: humidity

- Serious complications for little at the end
  - only significant in the troposphere
  - vapour content decreases quickly with altitude
- The model includes a dry and a humid component in the troposphere
  - mixture of perfect gas
  - water vapour pressure determined by temperature

$$P = P_\mathrm{d} + P_\mathrm{w} \qquad \rho = \rho_\mathrm{d} + \rho_\mathrm{w} \qquad T = T_0 + \gamma h \quad \gamma \simeq -6.5\mathrm{K/km}$$

$$P_\mathrm{d} = \frac{\rho_\mathrm{d} RT}{M_\mathrm{d}} \qquad P_\mathrm{w} = \frac{\rho_\mathrm{w} RT}{M_\mathrm{w}} \qquad M_\mathrm{d} = 28.963\ \mathrm{g/mol} \quad M_\mathrm{w} = 18.015\ \mathrm{g/mol}$$

$$\frac{dP}{dh} = -\rho g = -(\rho_\mathrm{d} + \rho_\mathrm{w}) g$$

$$\boxed{\frac{dP}{dT} = \beta \frac{P}{T} - \beta \left[1 - \frac{M_\mathrm{w}}{M_\mathrm{d}}\right] \frac{P_\mathrm{w}}{T}}$$

$$\beta = -\frac{M_\mathrm{d} g}{\gamma R} \simeq 5.3$$

$$P_\mathrm{w} = f P_\mathrm{ws}(T)$$

Air refractive index

- Model from Ciddor (Applied Optics, 35, 9, 1996)
  - Designed for metrological purposes in laboratory conditions
    - much too accurate given the uncertainty in the atmospheric conditions
  - Determined by temperature, pressure and water vapour content
    - computed from humidity and saturation vapour pressure at T
    - humidity is kept constant in the troposphere
  - Depends also on the wavelength
    - smaller λ, larger n
- Discontinuities
  - n(h) is a continuous function
  - dn/dh has discontinuities at atmospheric layer boundaries
    - care needed in the numerical integration

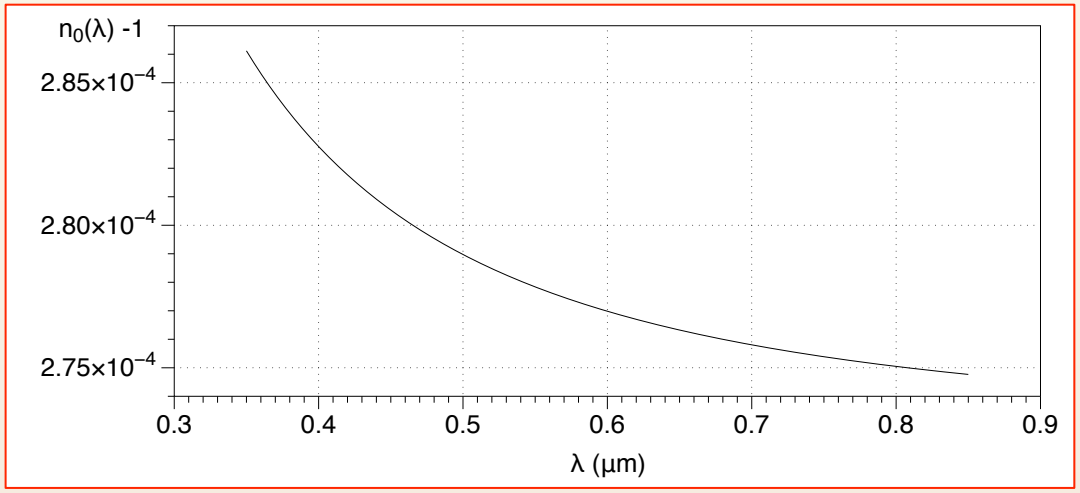
Effect of wavelength (dry air)

$$n_0(\lambda) = n(\lambda), P = P_{\text{standard}}, t = 15°C$$

$$10^8 \left[n_0(\lambda) - 1\right] = \frac{a_1}{w_1 - 1/\lambda^2} + \frac{a_2}{w_2 - 1/\lambda^2} \qquad \lambda \text{ in } \mu\text{m}$$

$$a_1 = 5792105\ \mu\text{m}^{-2}, \quad w_1 = 238.0185\ \mu\text{m}^{-2}$$

$$a_2 = 167917\ \mu\text{m}^{-2}, \quad w_2 = 57.3620\ \mu\text{m}^{-2}$$



$$\frac{n-1}{\rho} = \text{cte}$$

P. E. Ciddor Appl. Optics, 35, 9, p.1566, 1996

Validation of numerical refraction model

- Comparison to the table in "Introduction aux Ephémérides"

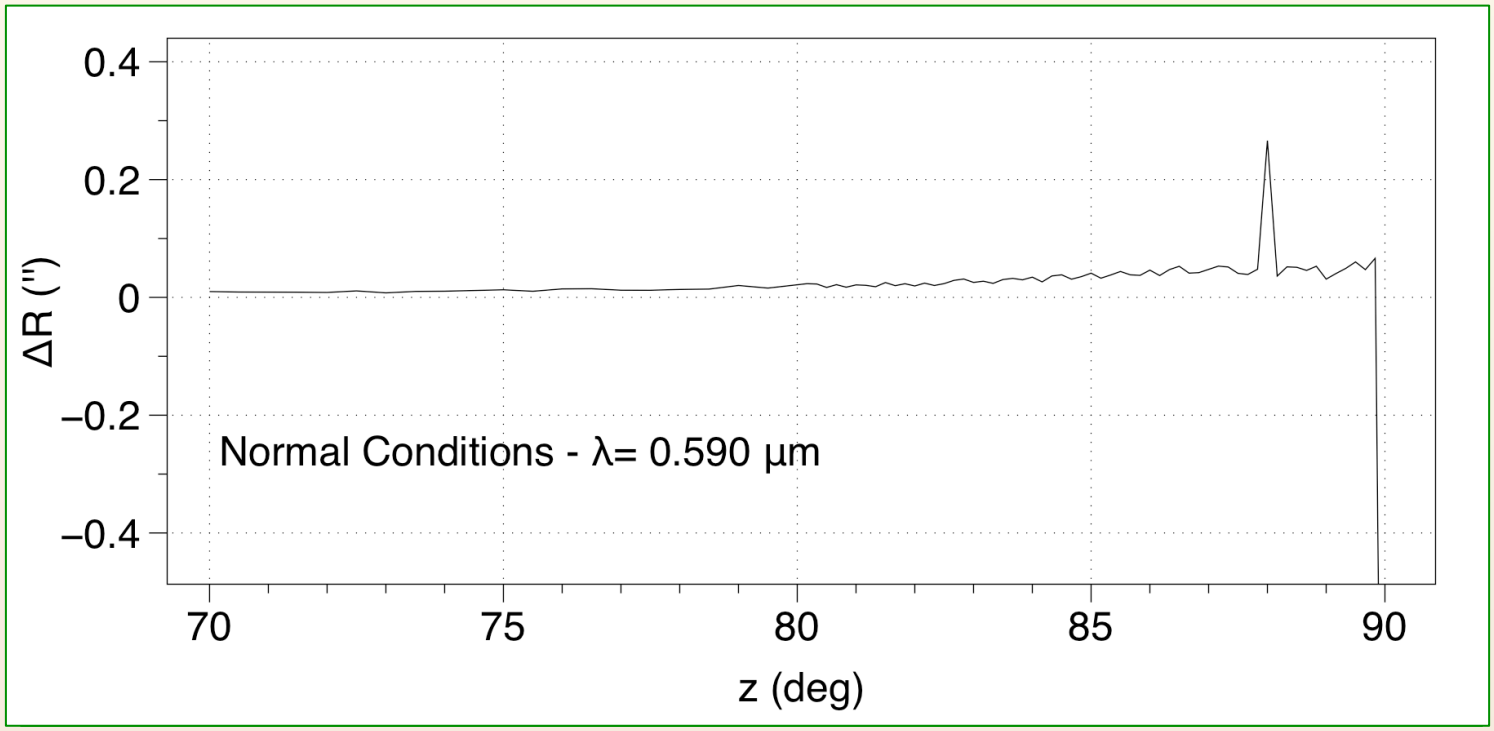
Table 7.3.3. Réfraction normale pour des distances zénithales comprises entre 70° et 90°.

z		Réfraction		z		Réfraction		z		Réfraction	
°	′	′	″	°	′	′	″	°	′	′	″
70	0	2	35.471	77	0	4	2.081	84	0	8	16.924
70	10	2	36.866	77	10	4	5.200	84	10	8	29.003
70	20	2	38.281	77	20	4	8.393	84	20	8	41.615
70	30	2	39.720	77	30	4	11.669	84	30	8	54.823
70	40	2	41.183	77	40	4	15.023	84	40	9	8.668
70	50	2	42.667	77	50	4	18.458	84	50	9	23.171
71	0	2	44.176	78	0	4	21.987	85	0	9	38.389
71	10	2	45.710	78	10	4	25.601	85	10	9	54.388
71	20	2	47.271	78	20	4	29.315	85	20	10	11.195
71	30	2	48.857	78	30	4	33.124	85	30	10	28.883
71	40	2	50.471	78	40	4	37.032	85	40	10	47.530
71	50	2	52.112	78	50	4	41.048	85	50	11	7.191
72	0	2	53.780	79	0	4	45.174	86	0	11	27.939
72	10	2	55.477	79	10	4	49.421	86	10	11	49.888
72	20	2	57.203	79	20	4	53.778	86	20	12	13.095
72	30	2	58.962	79	30	4	58.270	86	30	12	37.687
72	40	3	0.755	79	40	5	2.887	86	40	13	3.791
72	50	3	2.574	79	50	5	7.638	86	50	13	31.503
73	0	3	4.434	80	0	5	12.529	87	0	14	0.969
73	10	3	6.320	80	10	5	17.570	87	10	14	32.348
73	20	3	8.248	80	20	5	22.768	87	20	15	5.816
73	30	3	10.209	80	30	5	28.133	87	30	15	41.564
73	40	3	12.207	80	40	5	33.657	87	40	16	19.783
73	50	3	14.245	80	50	5	39.367	87	50	17	0.704
74	0	3	16.322	81	0	5	45.255	88	0	17	44.388
74	10	3	18.440	81	10	5	51.342	88	10	18	31.783
74	20	3	20.600	81	20	5	57.635	88	20	19	22.519
74	30	3	22.801	81	30	6	4.133	88	30	20	17.223
74	40	3	25.050	81	40	6	10.869	88	40	21	16.292
74	50	3	27.343	81	50	6	17.833	88	50	22	20.170
75	0	3	29.681	82	0	6	25.053	89	0	23	29.419
75	10	3	32.070	82	10	6	32.527	89	10	24	44.568
75	20	3	34.511	82	20	6	40.286	89	20	26	6.318
75	30	3	37.005	82	30	6	48.329	89	30	27	35.427
75	40	3	39.548	82	40	6	56.677	89	40	29	12.790
75	50	3	42.152	82	50	7	5.353	89	50	30	59.345
76	0	3	44.807	83	0	7	14.380	90	0	32	57.971
76	10	3	47.525	83	10	7	23.762				
76	20	3	50.303	83	20	7	33.534				
76	30	3	53.147	83	30	7	43.703				
76	40	3	56.059	83	40	7	54.308				
76	50	3	59.034	83	50	8	5.376				

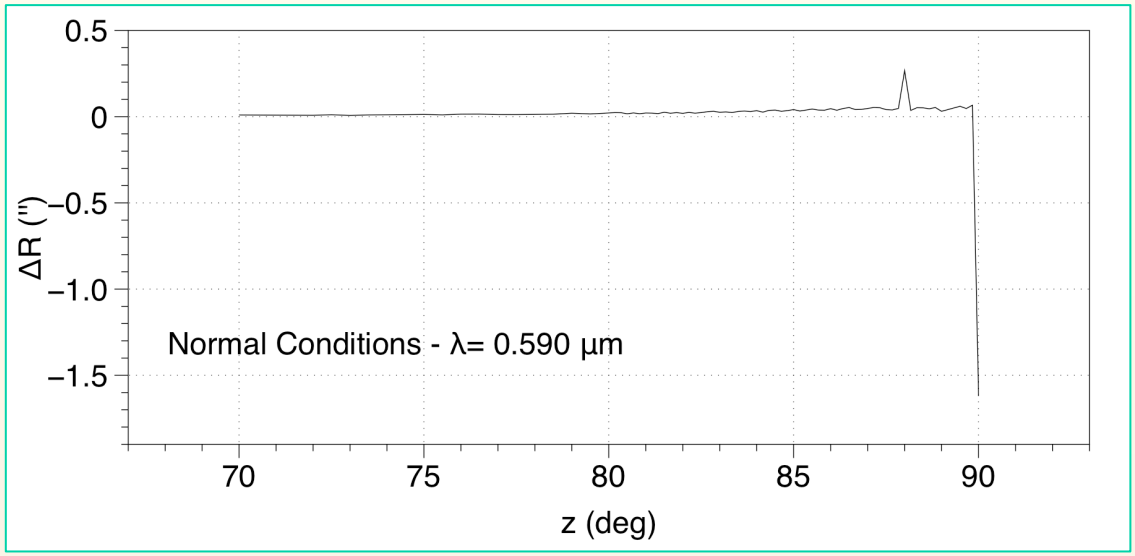
- Table 7.3.3 p. 198
- Normal refraction for z_0 between 70° and 90°
- Every 10'
- Standard conditions
  - t = 15 °C
  - p = 1013.25 hPa
  - H = 0 %

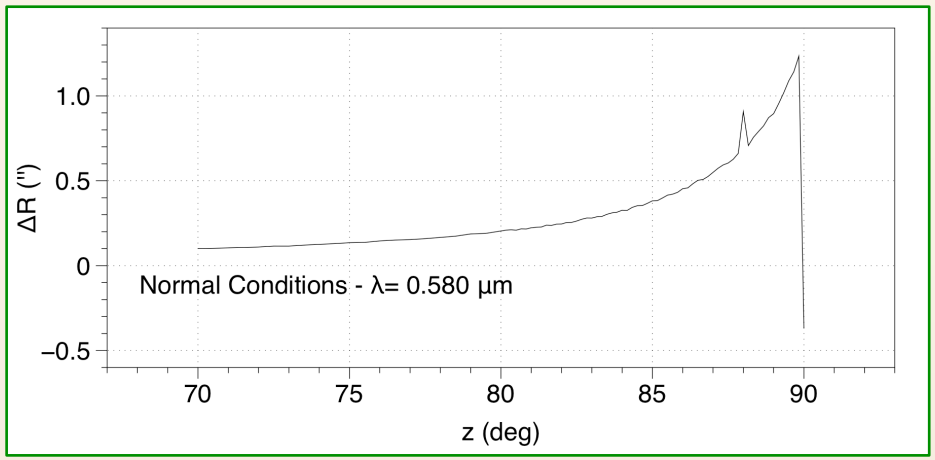
Validation of refraction model

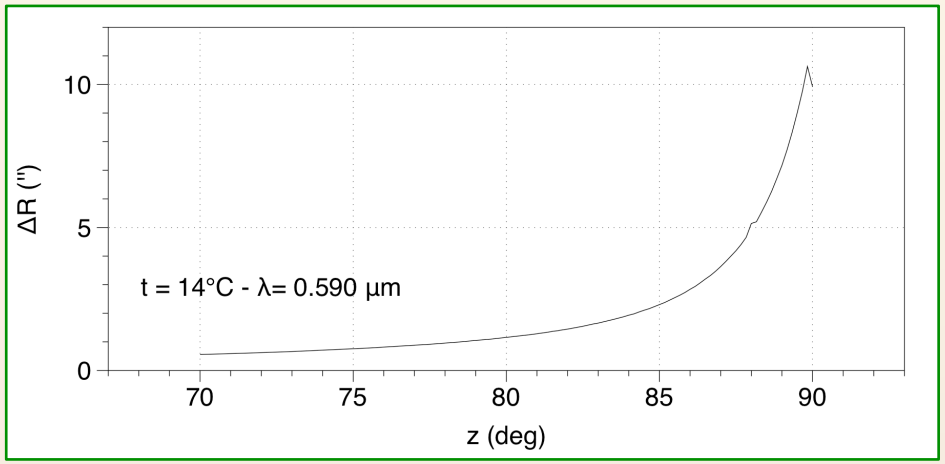
- Comparison to the table in "Introduction aux Ephémérides"
  - P = 1013.25 hPa , t = 15 °C
  - humidity = 0
  - altitude = 0
- The plot gives ΔR(z) = My_value - CdT_value in arcsec
  - R(85) ~ 10' ~ 600 " R(90) ~ 33' ~ 2000 "



Sensitivity of the model



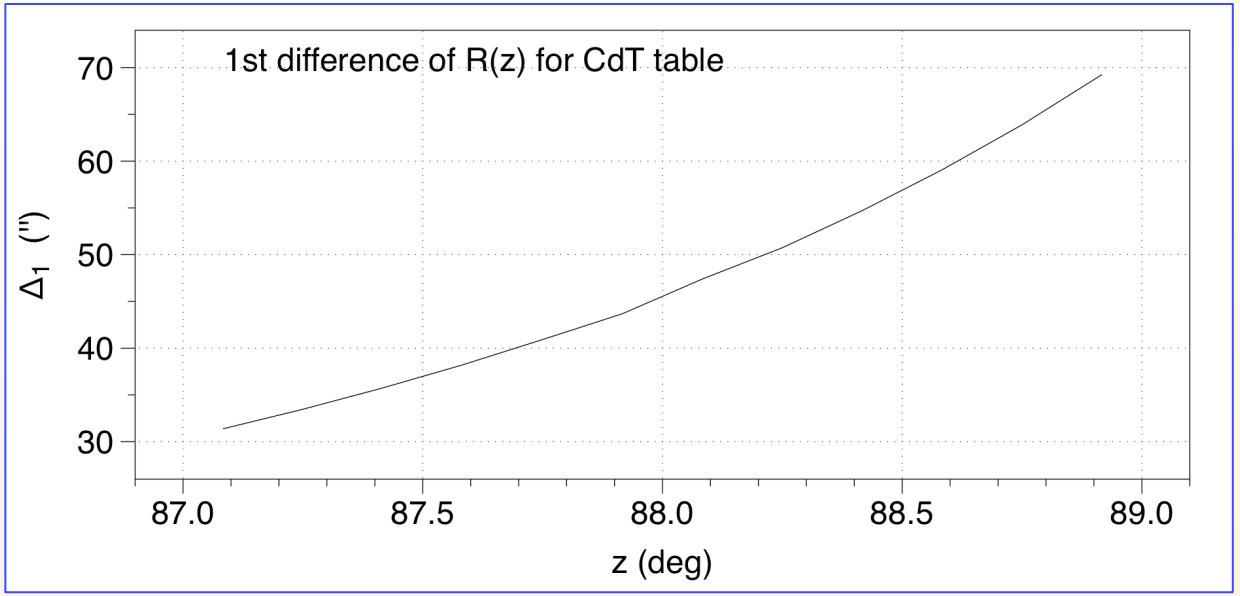


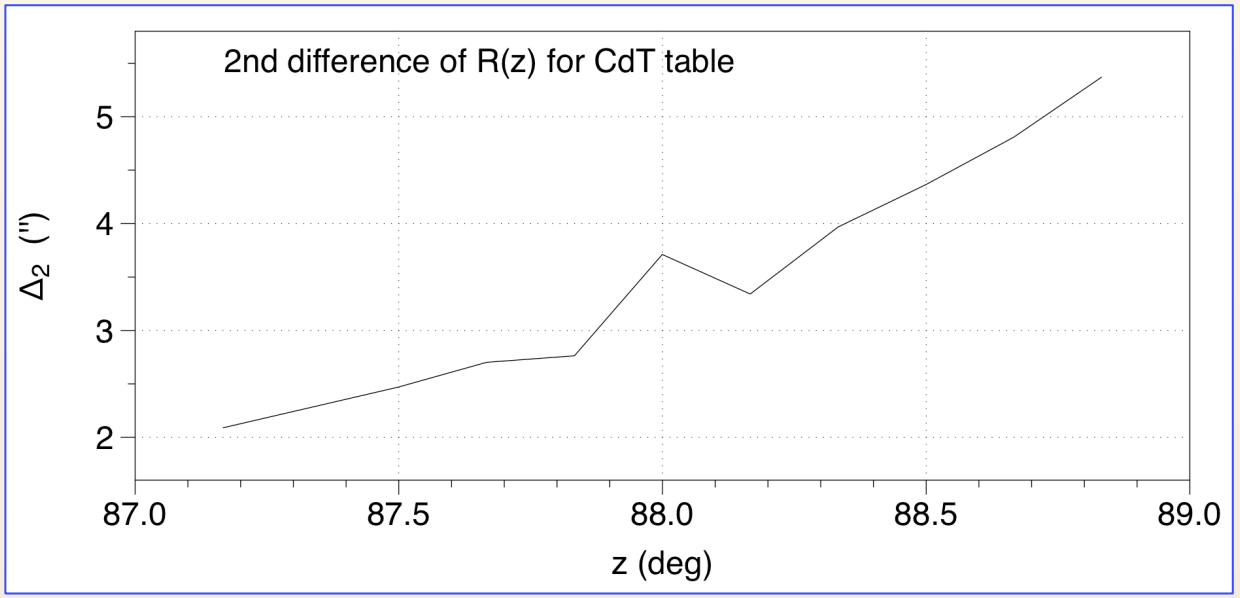


88° anomaly

$\mathrm{CdT} = 17'44\overset{''}{.}388$

$\mathrm{FM} = 17'44\overset{''}{.}608$

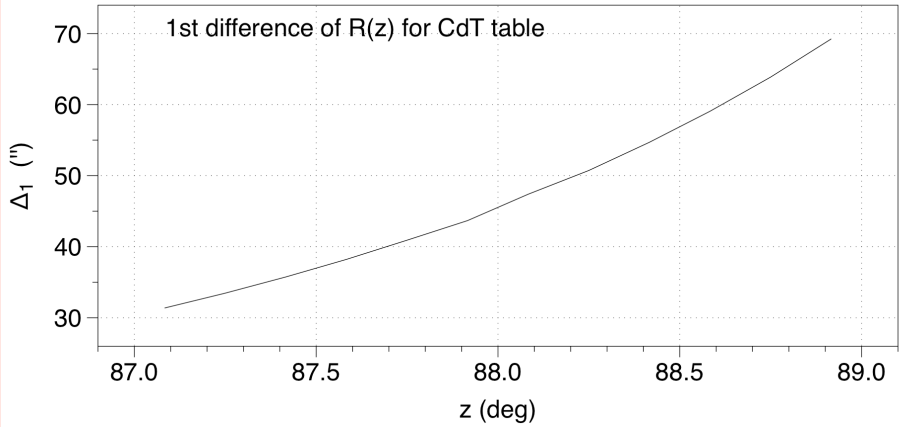


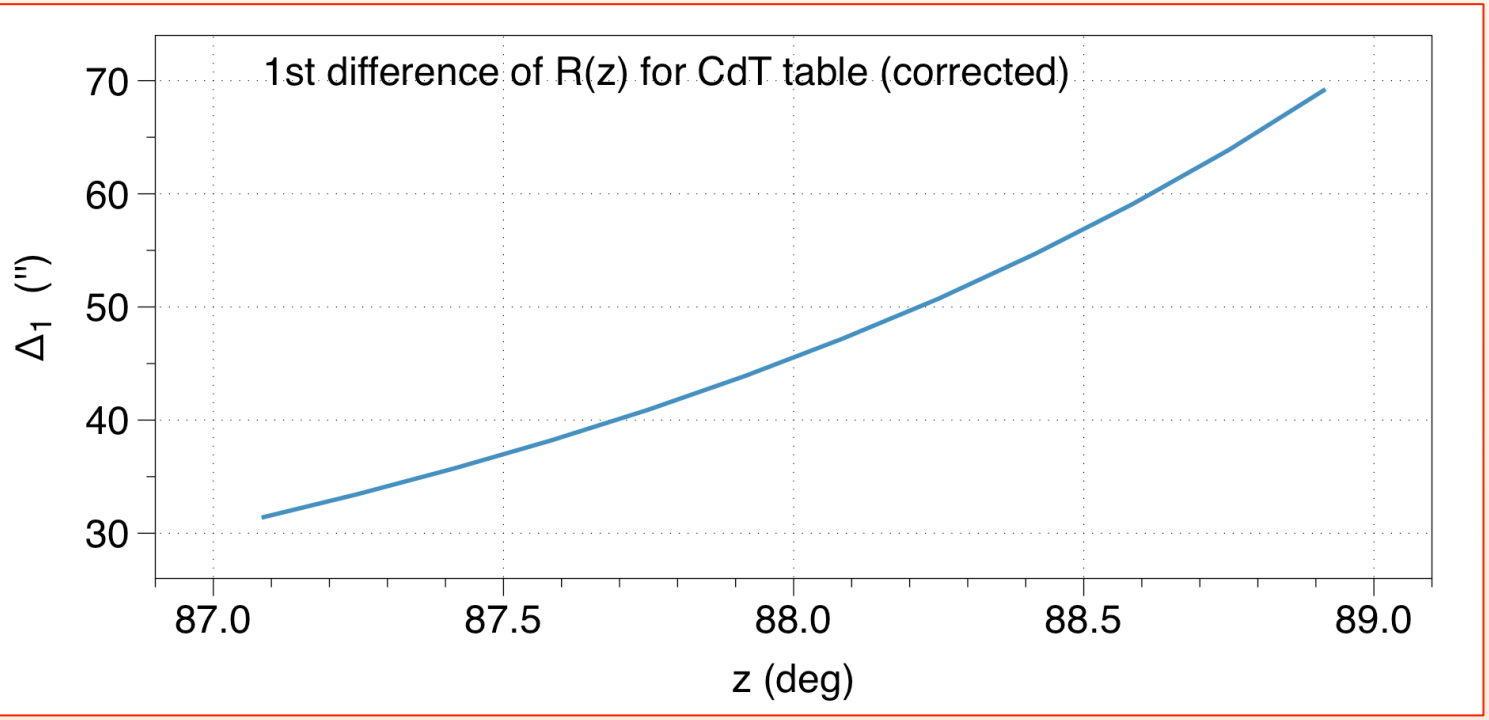


88° anomaly

$$\mathrm{CdT} = 17'44\overset{''}{.}388$$

$$\mathrm{FM} = 17'44\overset{''}{.}608$$

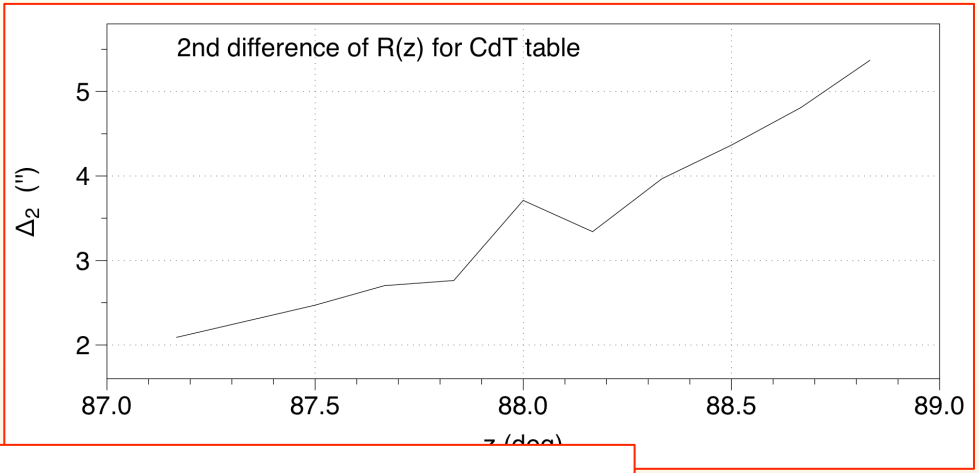


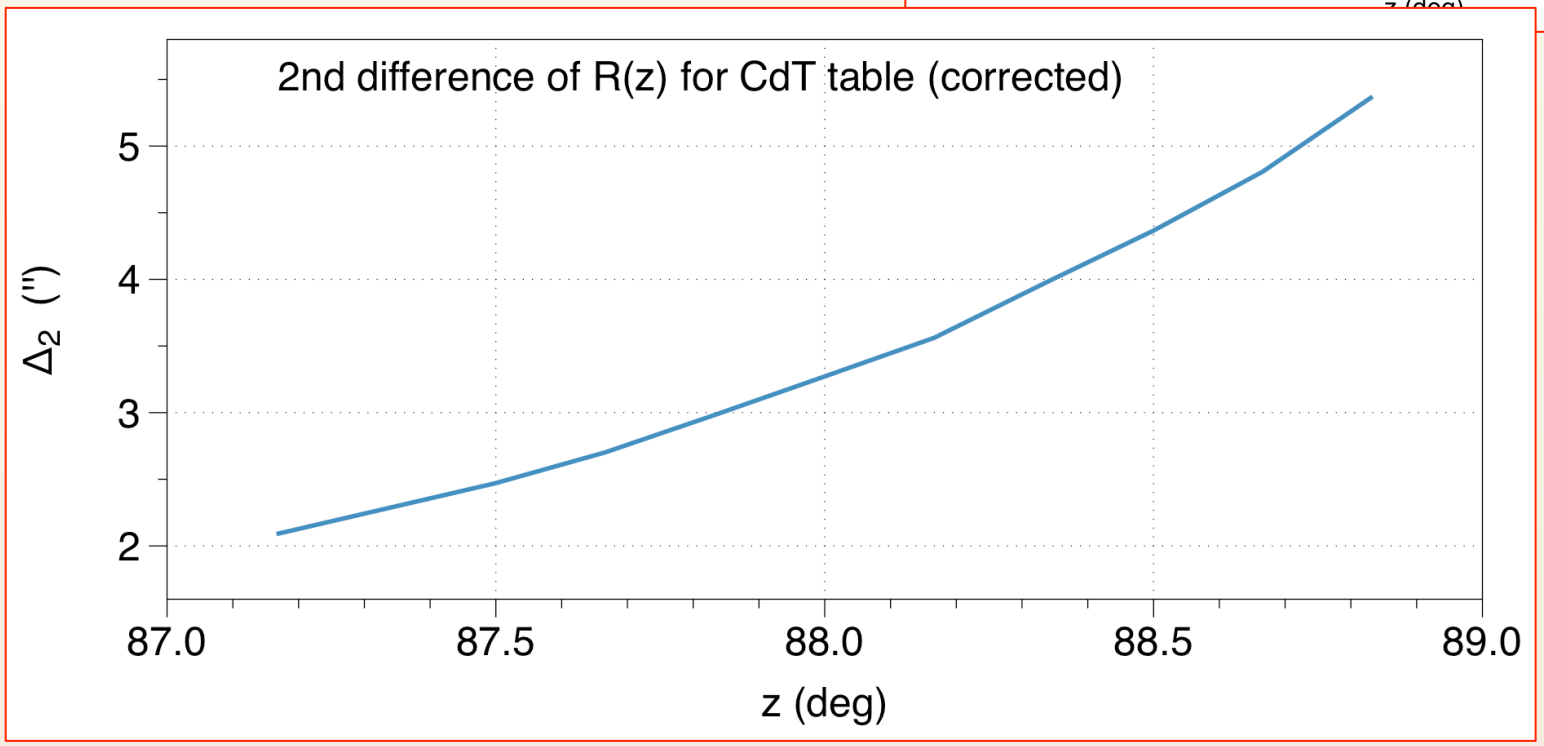


88° anomaly

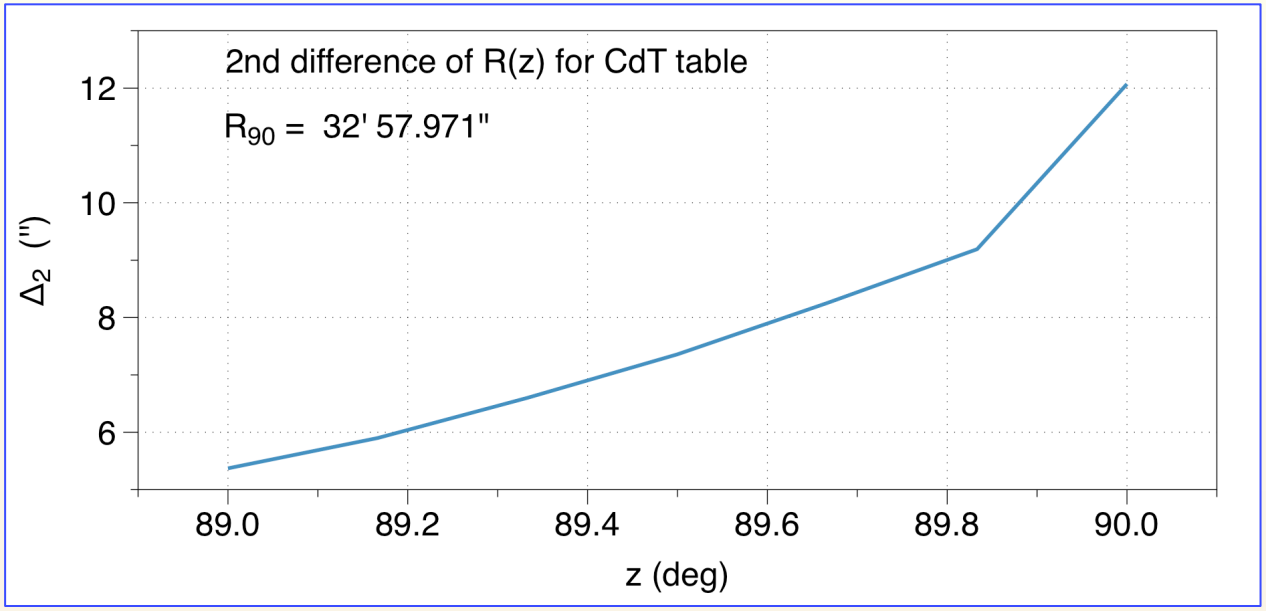
$$\mathrm{CdT} = 17'44\overset{''}{.}388$$

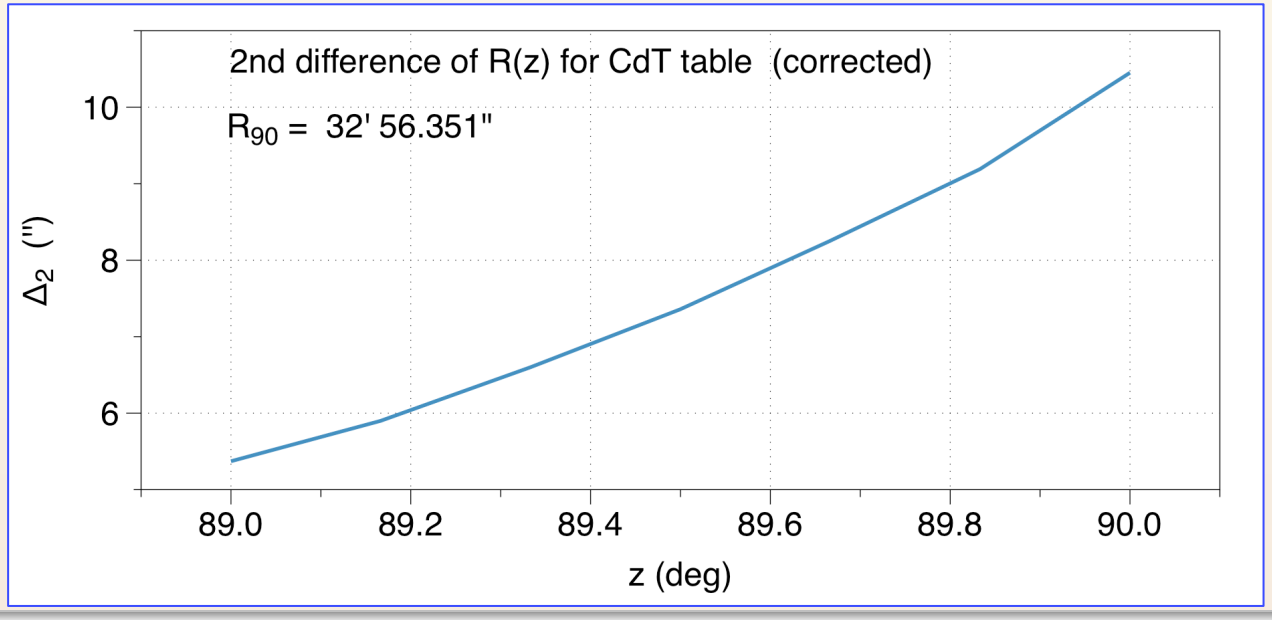
$$\mathrm{FM} = 17'44\overset{''}{.}608$$





90° anomaly

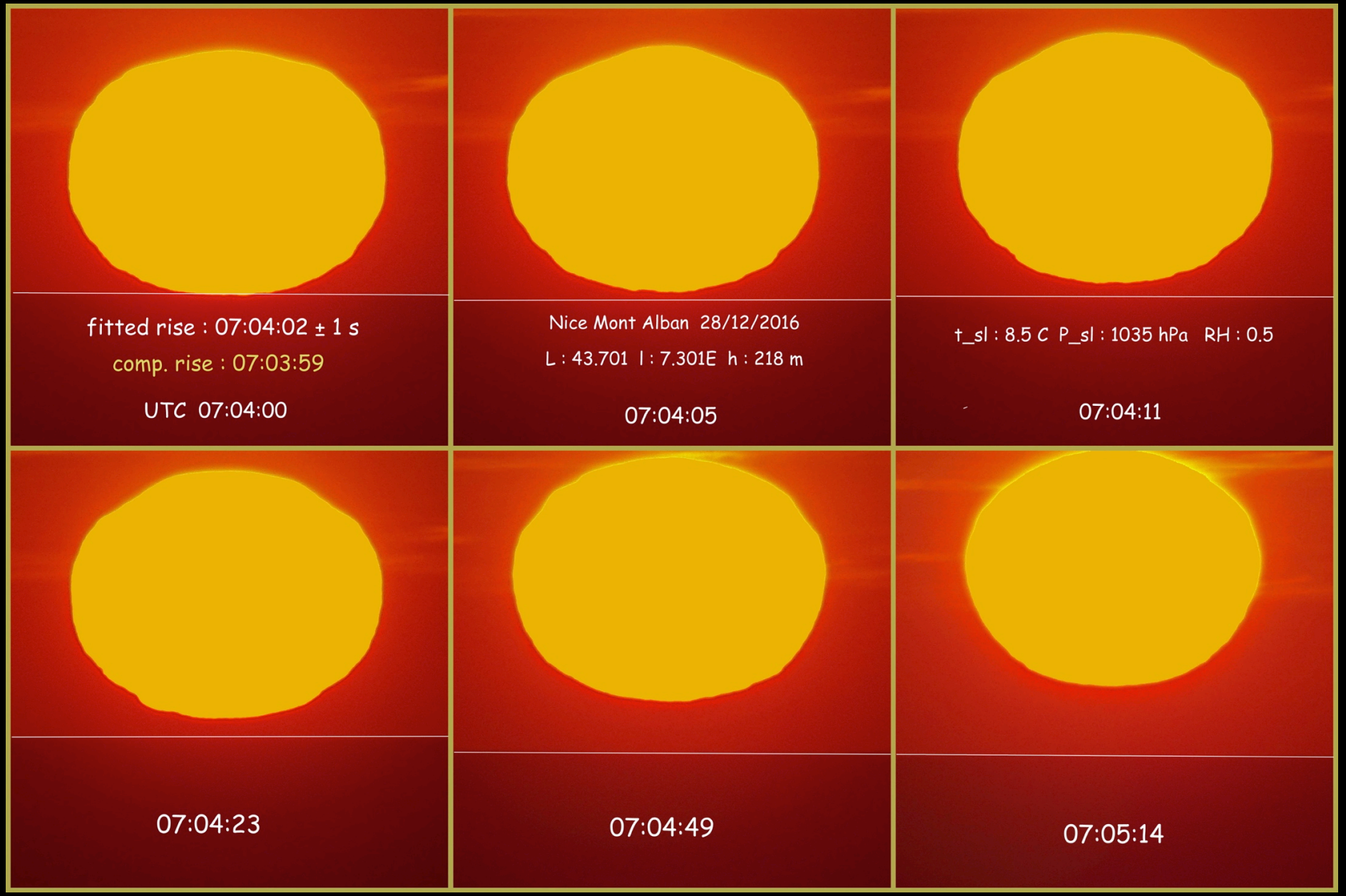

2nd difference of R(z) for CdT table
R_{90} = 32' 57.971"
Δ_2 (")
12
10
8
6
89.0
89.2
89.4
89.6
89.8
90.0
z (deg)


2nd difference of R(z) for CdT table (corrected)
R_{90} = 32' 56.351"
Δ_2 (")
10
8
6
89.0
89.2
89.4
89.6
89.8
90.0
z (deg)

External validation :Time of sunrise



External validation :Time of sunrise



Model sensitivity

- Change of refraction (R) in horizon for a change Δ in physical parameter

Parameter	Δ	δR1	δR2	δt (rise/set)
		h = 5m	h= 500m	
T	1°C	-11 "	-15"	1.5 - 2s
P	10 hPa	+20''	+27''	3s
h	10m	50''(*)	6'' (*)	6s – 1s
λ	0.1 μm	-9''	-12''	1s
humidity	50%	-5 ''	-8''	< 1s
dT/dh(**)	0.5 °C/km	15''	24''	3 s

* Refraction to optical horizon, but the change of dip is much larger

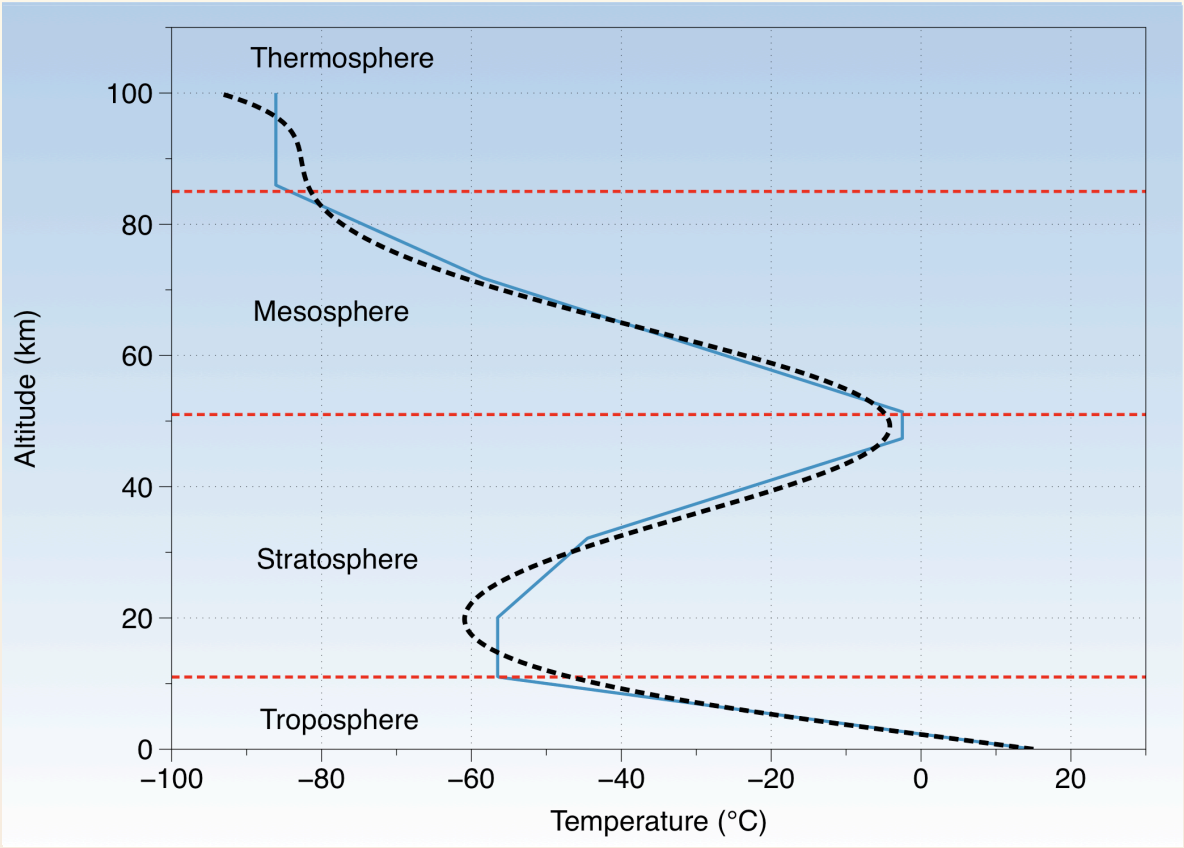
** Nominal value is -6.5 °C/km

Smooth atmospheric model

- Goal : avoid the discontinuities at layer boundaries
- Method : polynomial approximation of the temperature profile
  - constraint : $T(h=0) = T_0$ & $(dT/dh)_0$ = slope of the ISA
  - weight in the fit in $1/(h+a)^{3/2}$, a = 1km ➔ favors troposphere

ISA

Fit



Smooth atmospheric model

- Pressure : from integration of temperature profile

- In fact my polynomial fit is for $T_0/T(h)$
  - trivial integration of a polynomial

$$\ln \frac{P}{P_0} = -\frac{M_{\mathrm{air}} g}{R T_0} \int_{h_0}^{h} \frac{T_0}{T(h)} dh$$

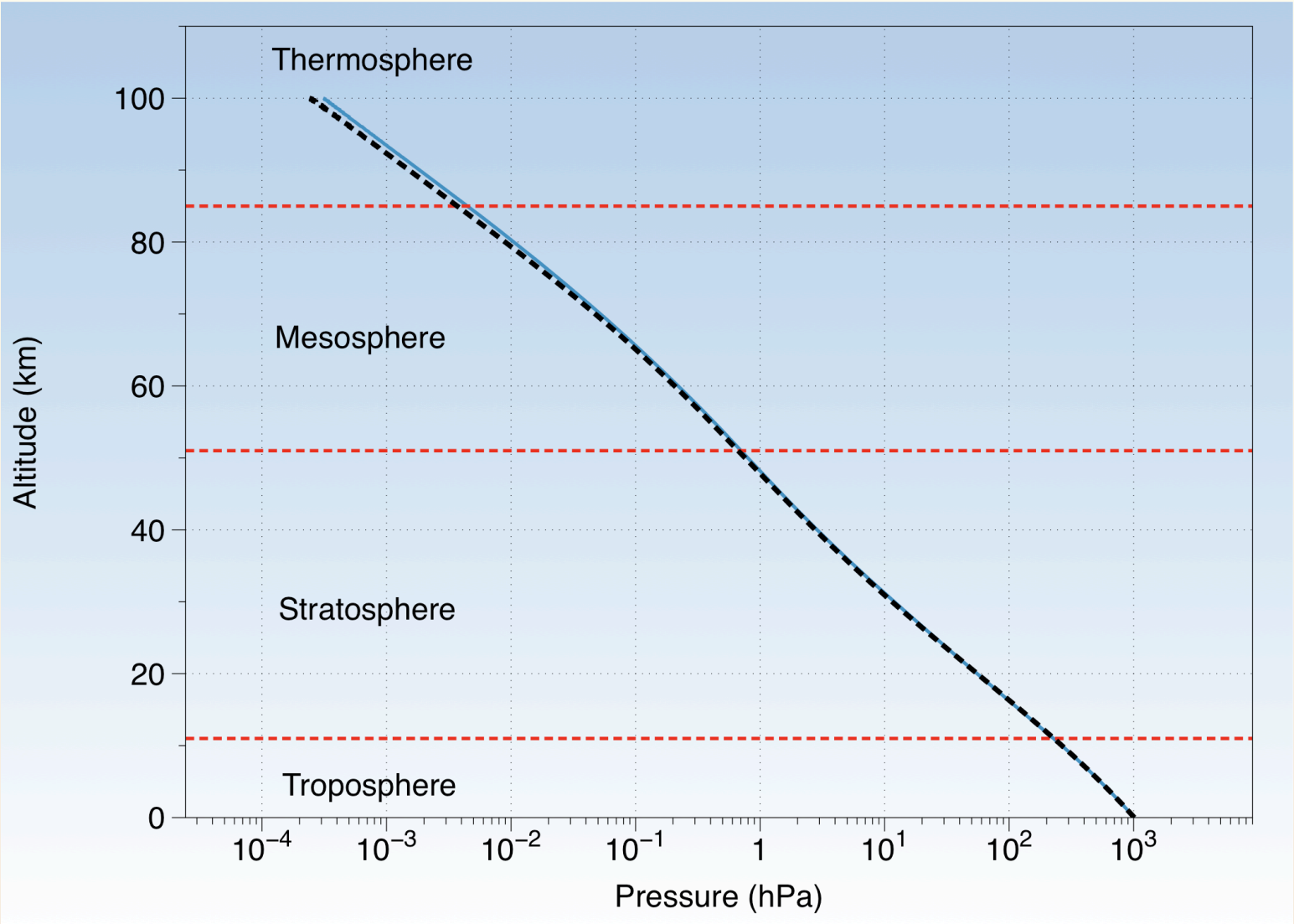
- Then density with perfect gas
- Refraction index with Gladstone law

Smooth atmospheric model

- Pressure

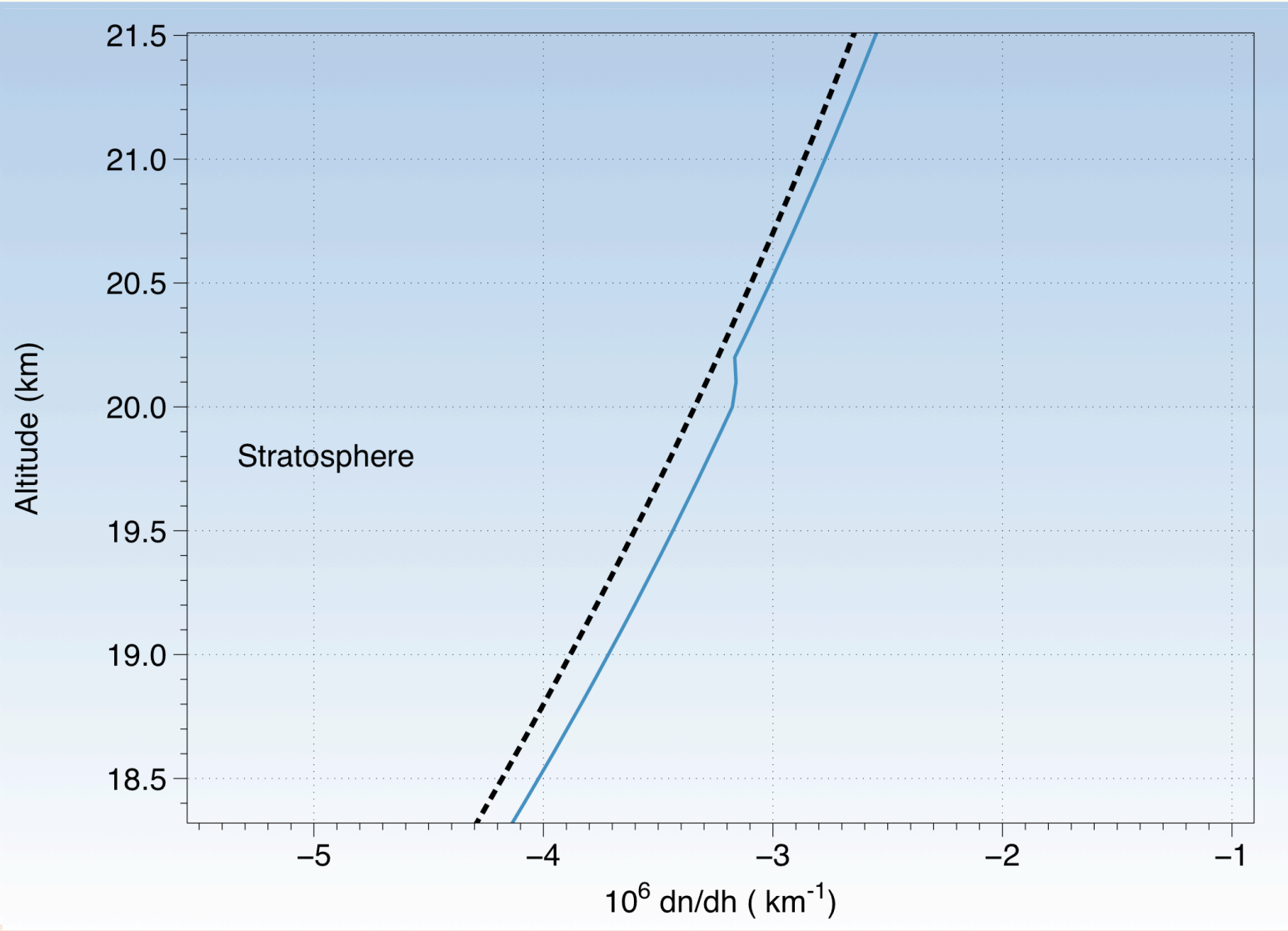
ISA

T-Fit



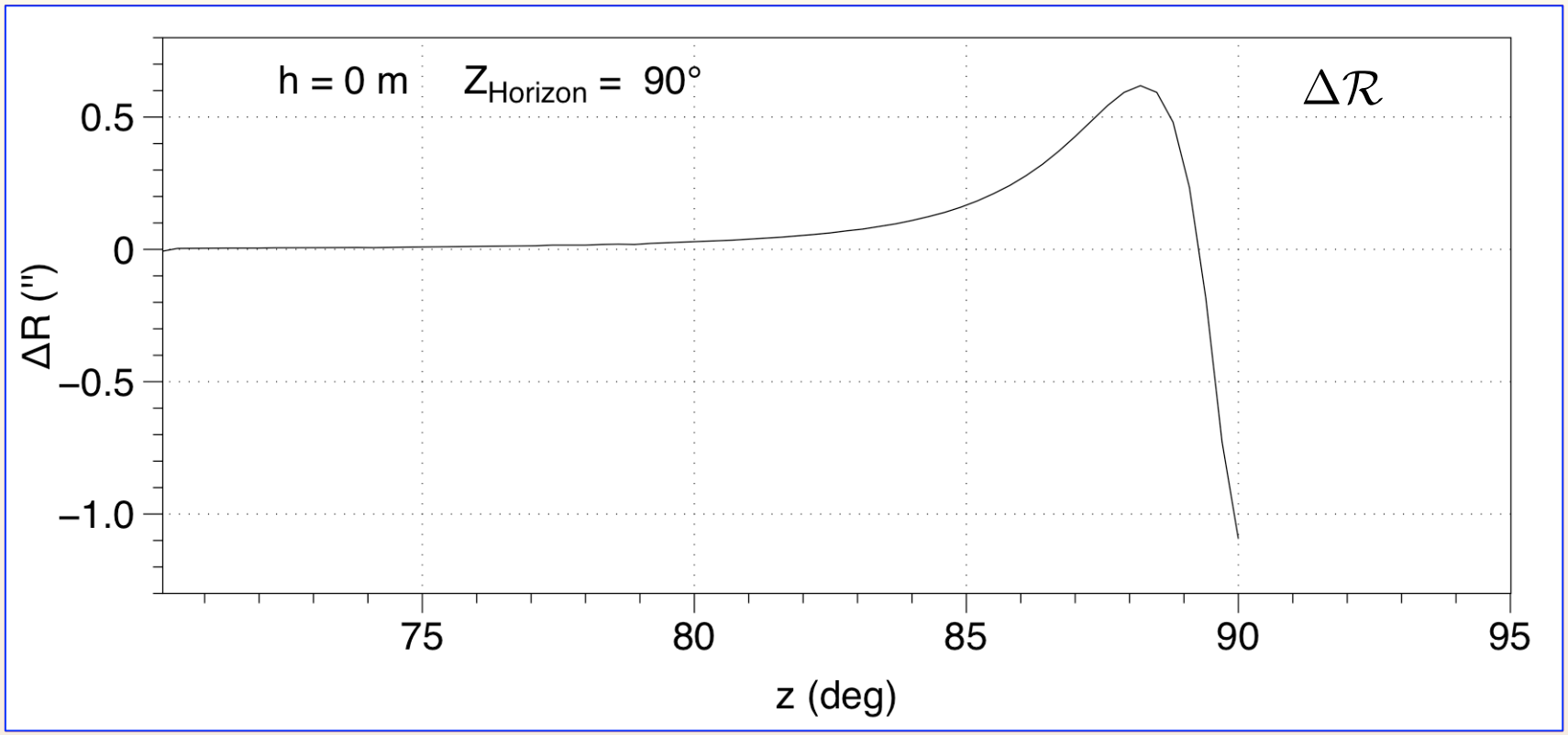
Smooth atmospheric model

- Gradient of refractive index



Smooth atmospheric model

- Normal refraction
  - Difference between the two models (layers vs. smoothed)



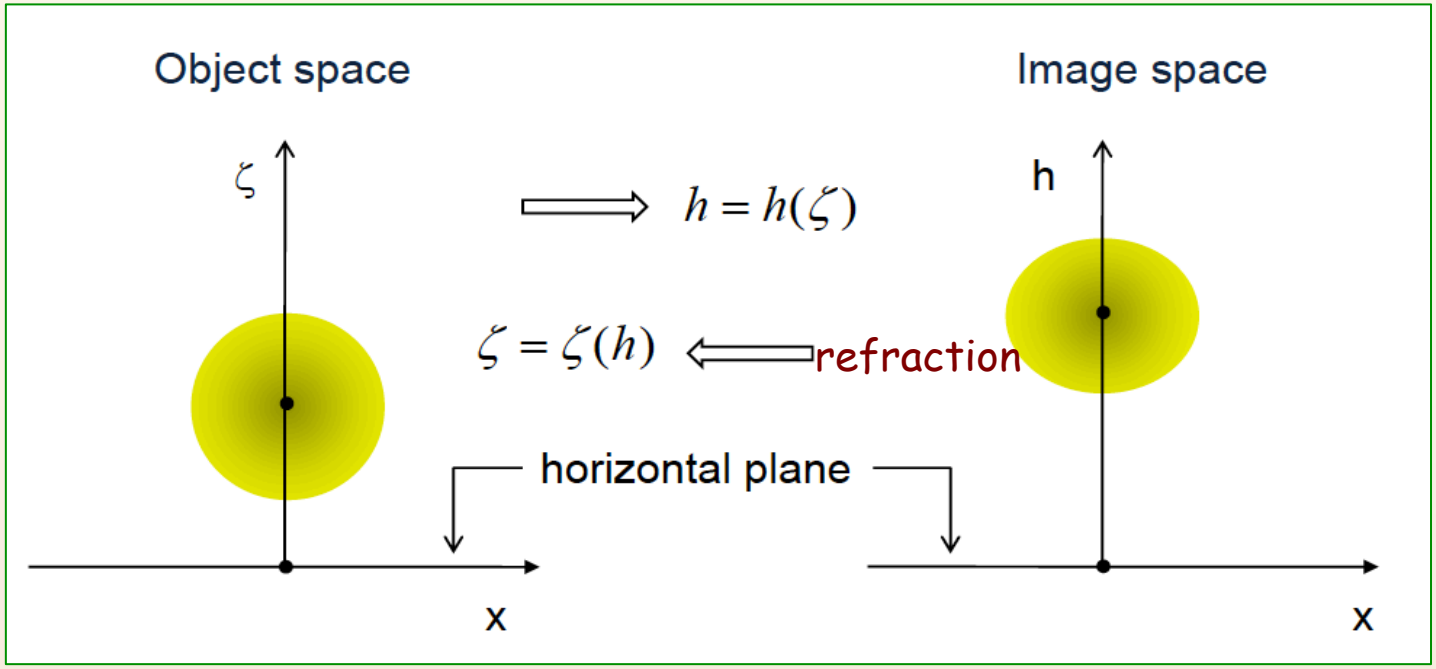
$$1'' = 0.05\%$$

Imaging near horizon

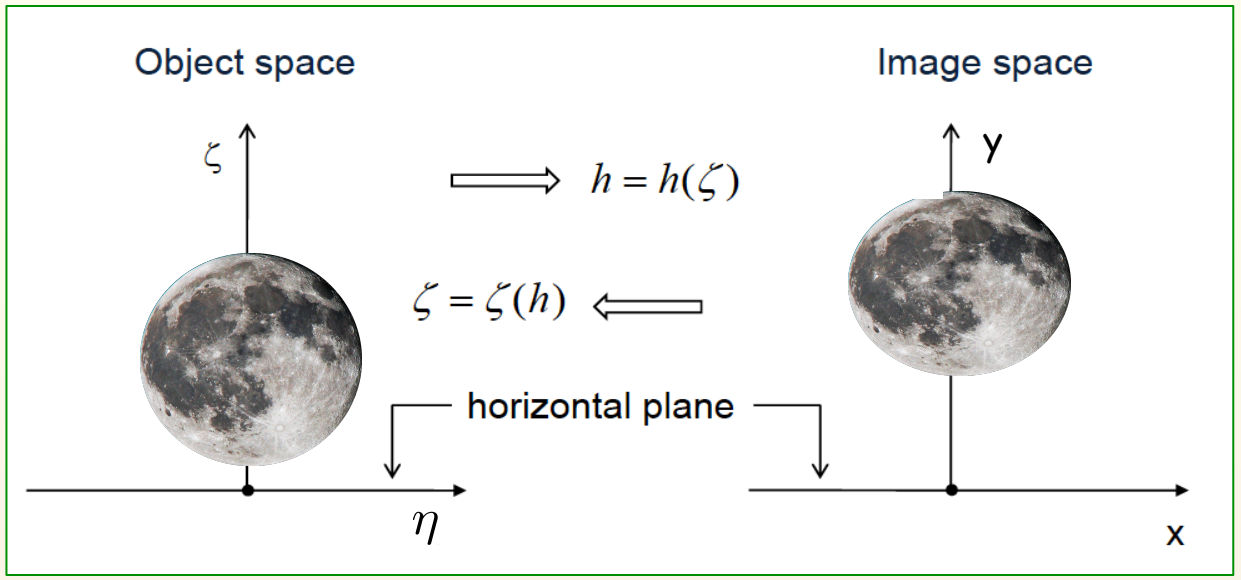


Image of extended sources

- The atmosphere seen as an optical system
  - the refraction is a single value function
  - it gives explicitly $\zeta(h)$
  - the inverse function $h(\zeta)$ is needed to map sources in the image space
    - it is not necessarily single valued (multiple images)
    - it is not explicit ➔ non linear equation or inverse interpolation



Sun and Moon



$$\zeta^2 + \eta^2 = \rho^2$$

$$f(x, y) = 0$$

$$x = \eta$$

$$y - \mathcal{R}(y) = \zeta$$

non linear and implicit equation

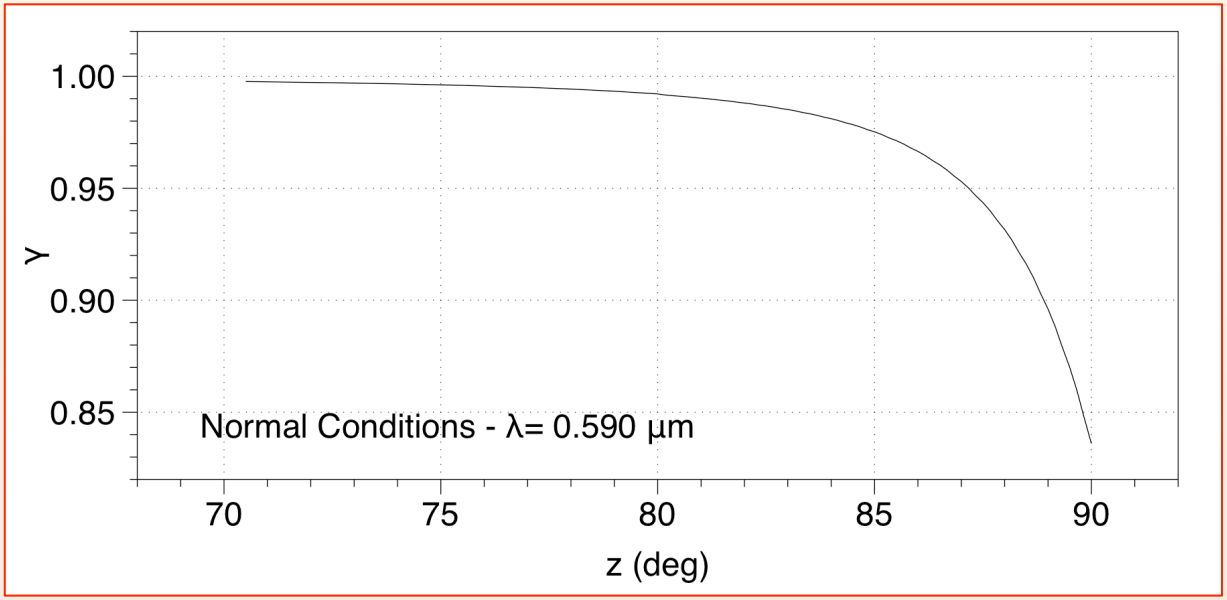
- Can be solved for every point with iterative methods
- Also with numerical refraction one can compute $\bar{\mathcal{R}}(\zeta)$

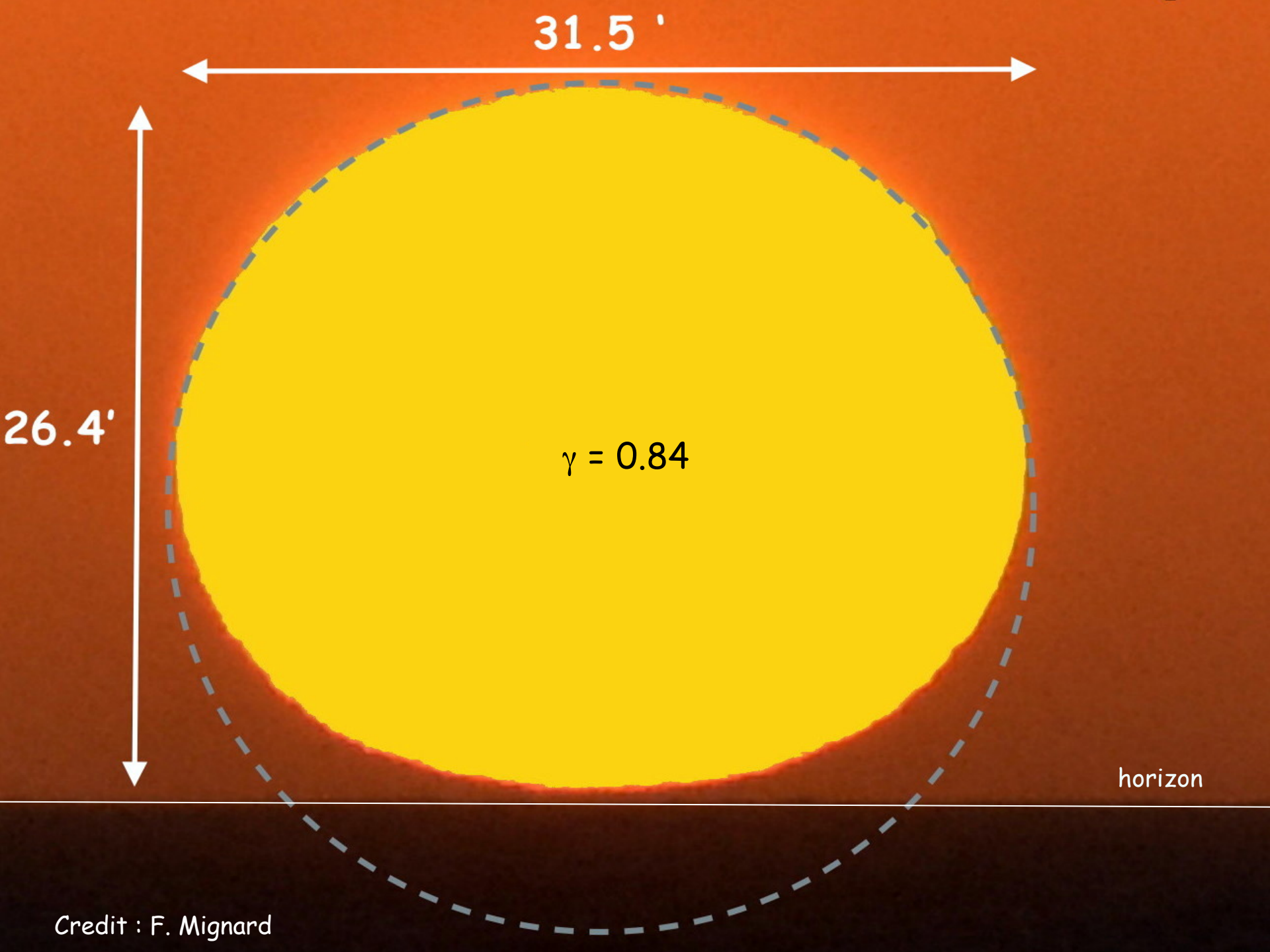
$$x = \eta$$
$$y = \zeta + \bar{\mathcal{R}}(\zeta)$$

Distortion (t : true, a : apparent)

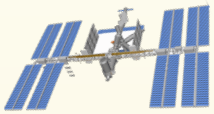
$$z_t = z_a + \mathcal{R}(z_a) \qquad \delta z_a = \frac{\delta z_t}{\left(1 + \frac{d\mathcal{R}(z_a)}{dz_a}\right)}$$

$$\text{Magnification : } \gamma(z_a) = \frac{\delta z_a}{\delta z_t} = \frac{1}{\left(1 + \frac{d\mathcal{R}(z_a)}{dz_a}\right)}$$



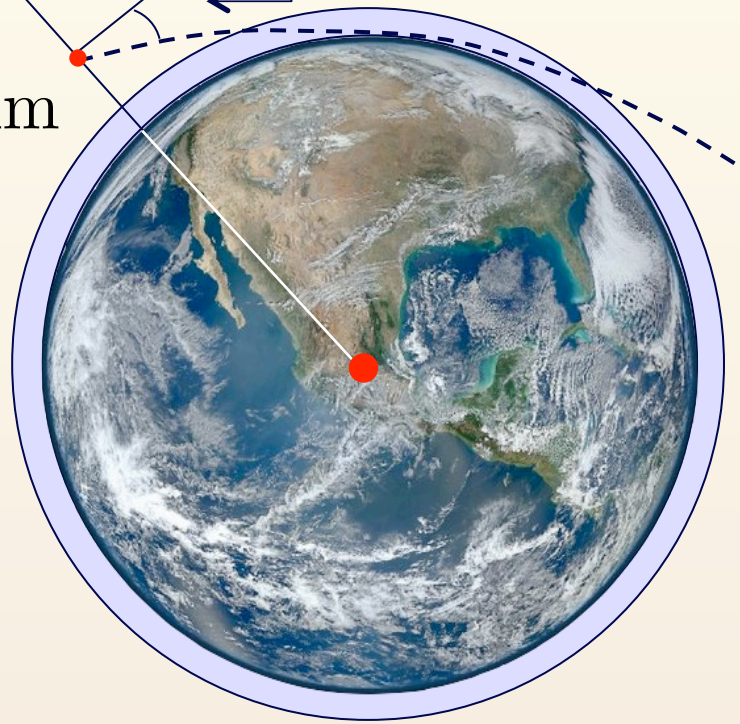

31.5 '
26.4'
γ = 0.84
horizon
Credit : F. Mignard

Imaging from ISS



$\epsilon \approx 19\overset{\circ}{.}74$

ISS $\approx$ 400km



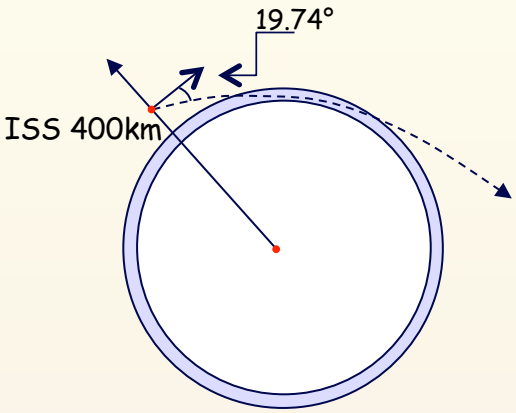
$\mathcal{R} \approx 1\overset{\circ}{.}09$

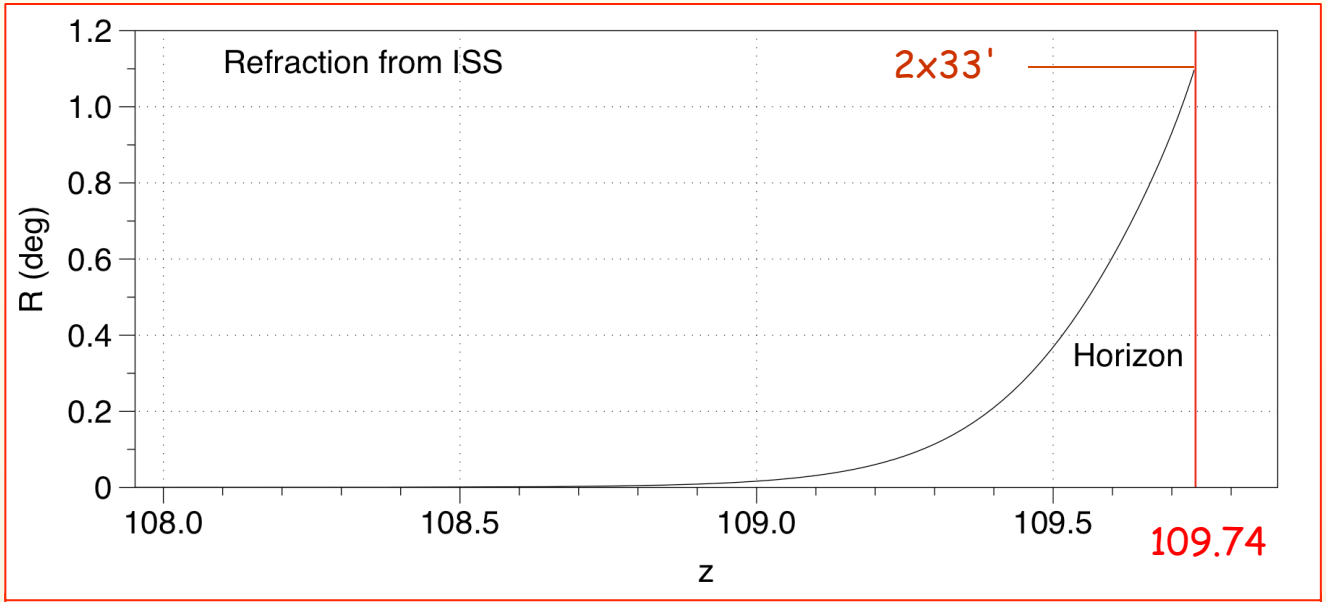


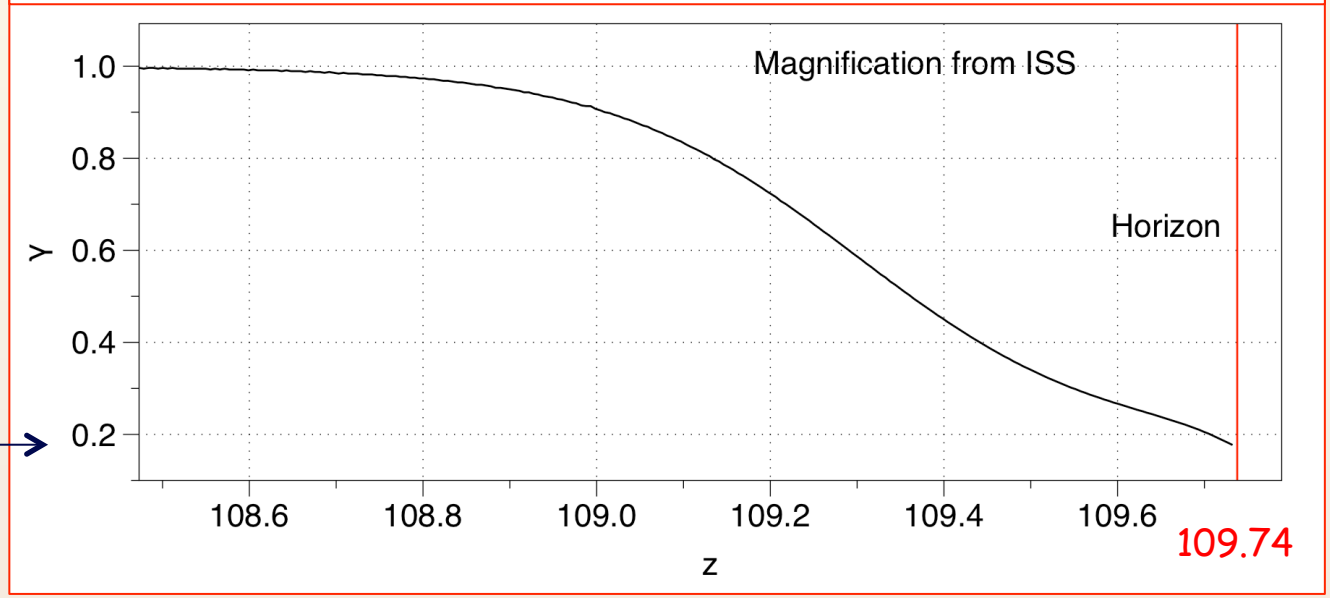
$$\frac{d\mathcal{R}}{d\epsilon} \approx 4.7 \text{ deg/deg}$$

$$\text{Earth}: \frac{d\mathcal{R}}{d\epsilon} \approx 0.18 \text{ deg/deg at } z = 90°$$

Refraction from ISS (h = 400 km)







huge flattening →

Moonset from ISS



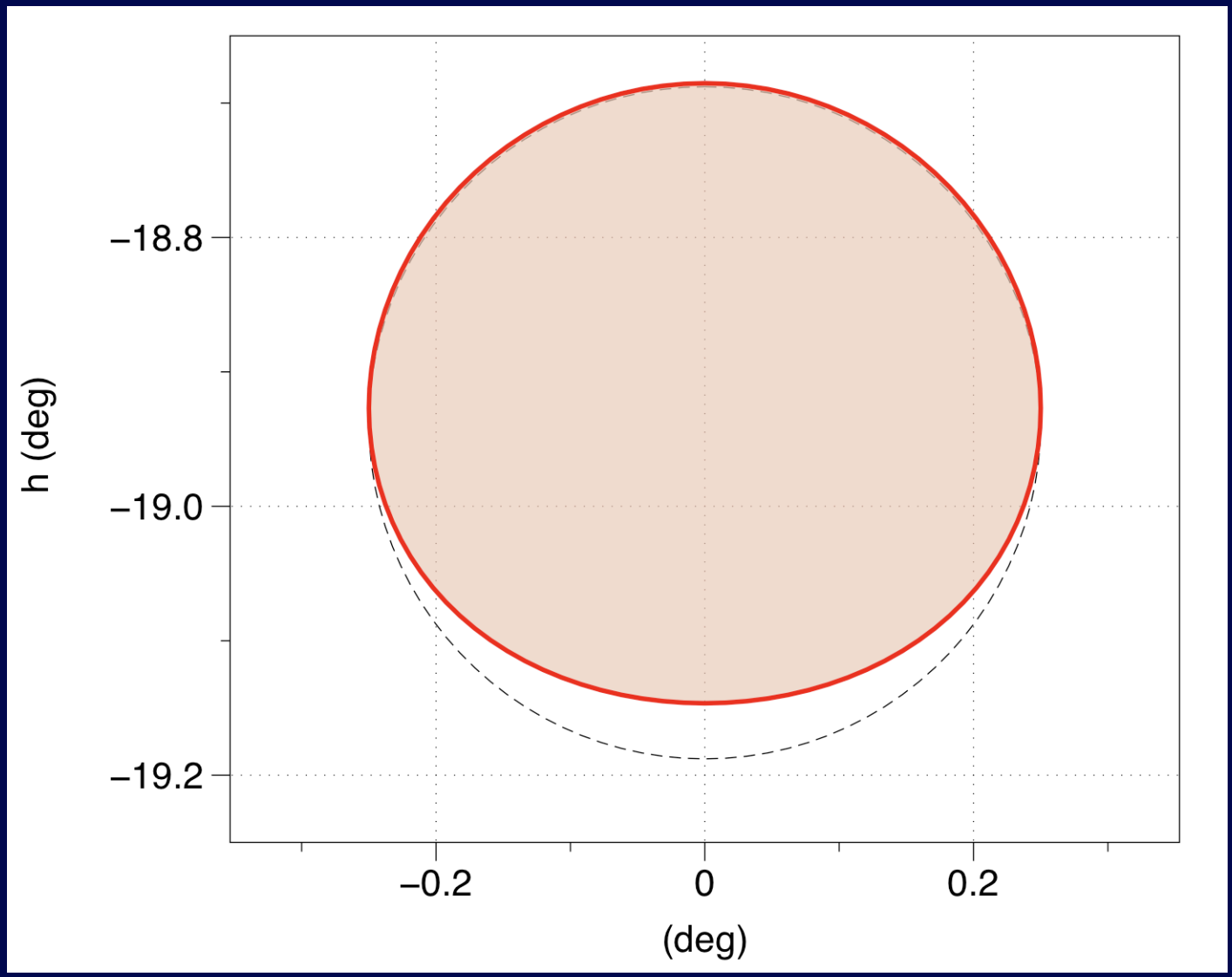
ISS, 16 April 2003

Credit : Don Pettit, Les Cowley, ISS, NASA

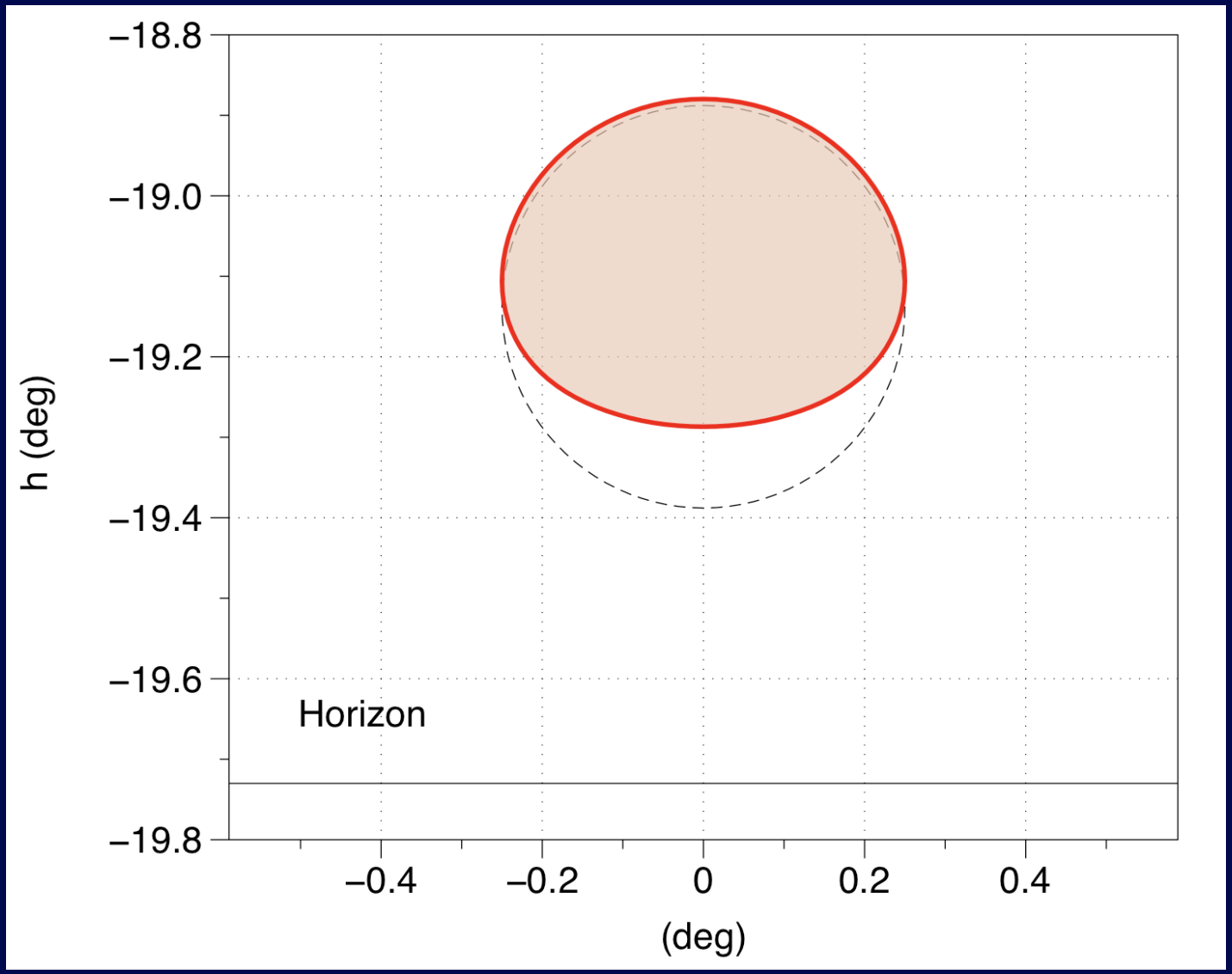
Moonset from ISS



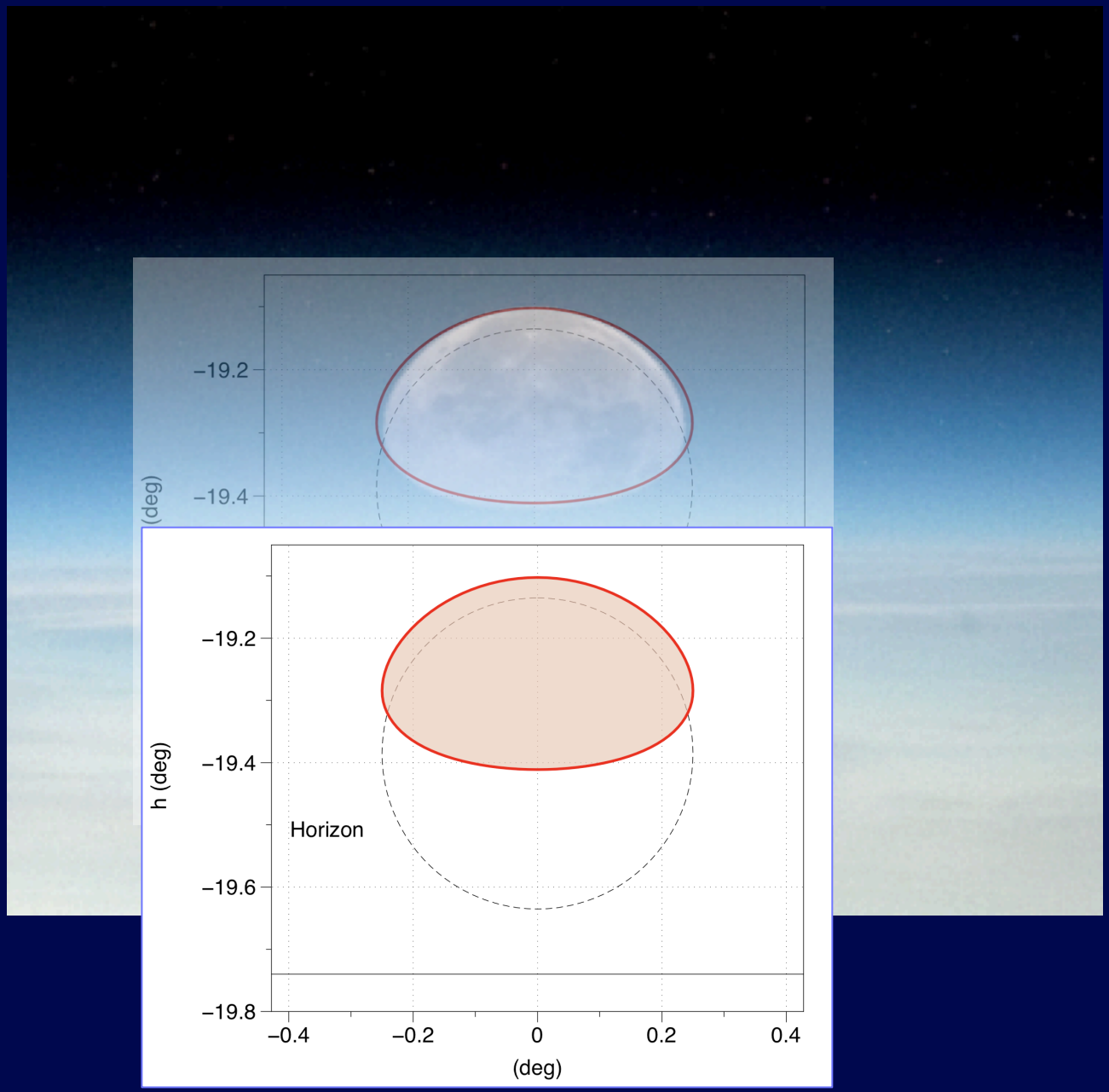
Computed shapes at setting



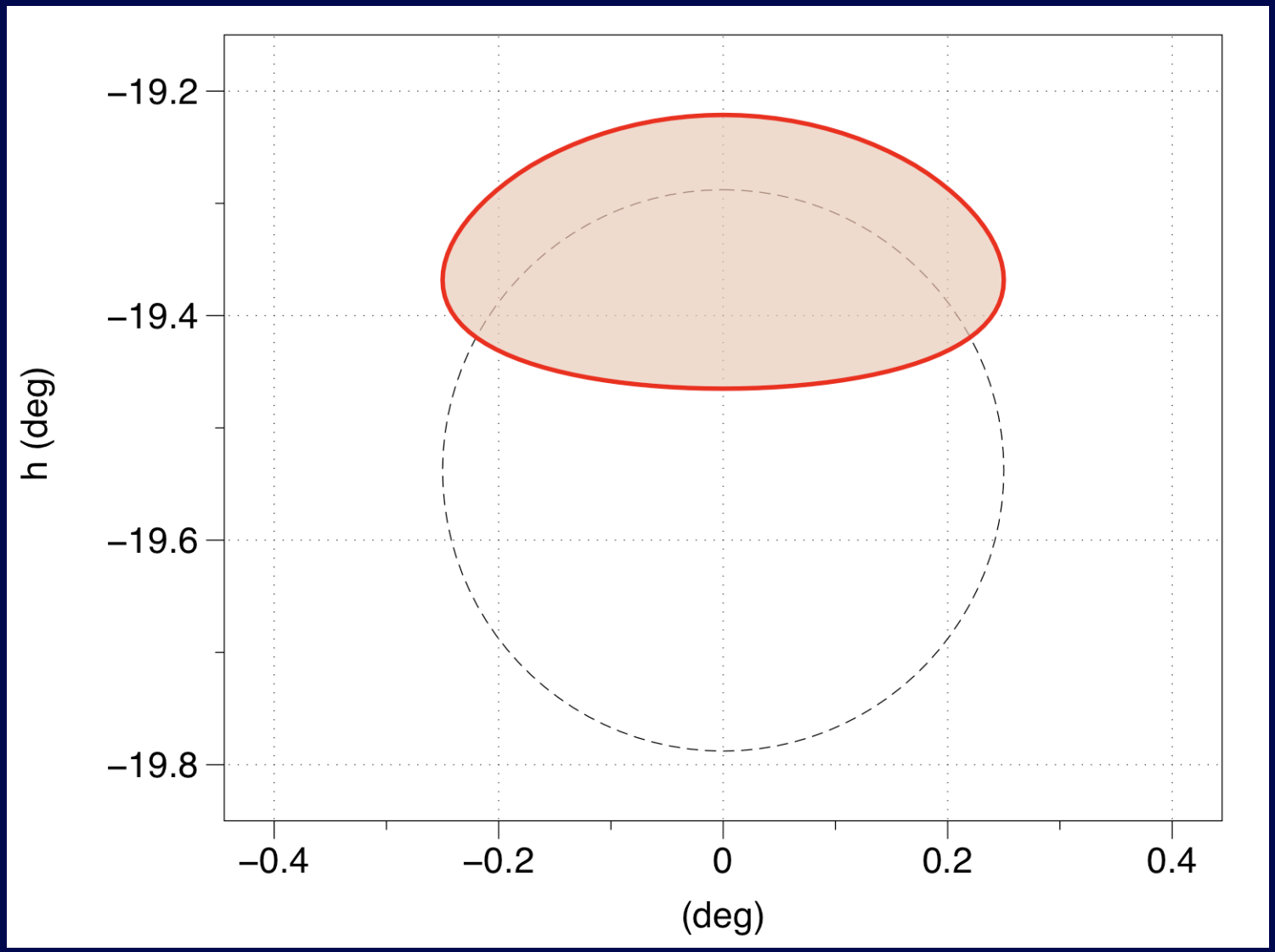
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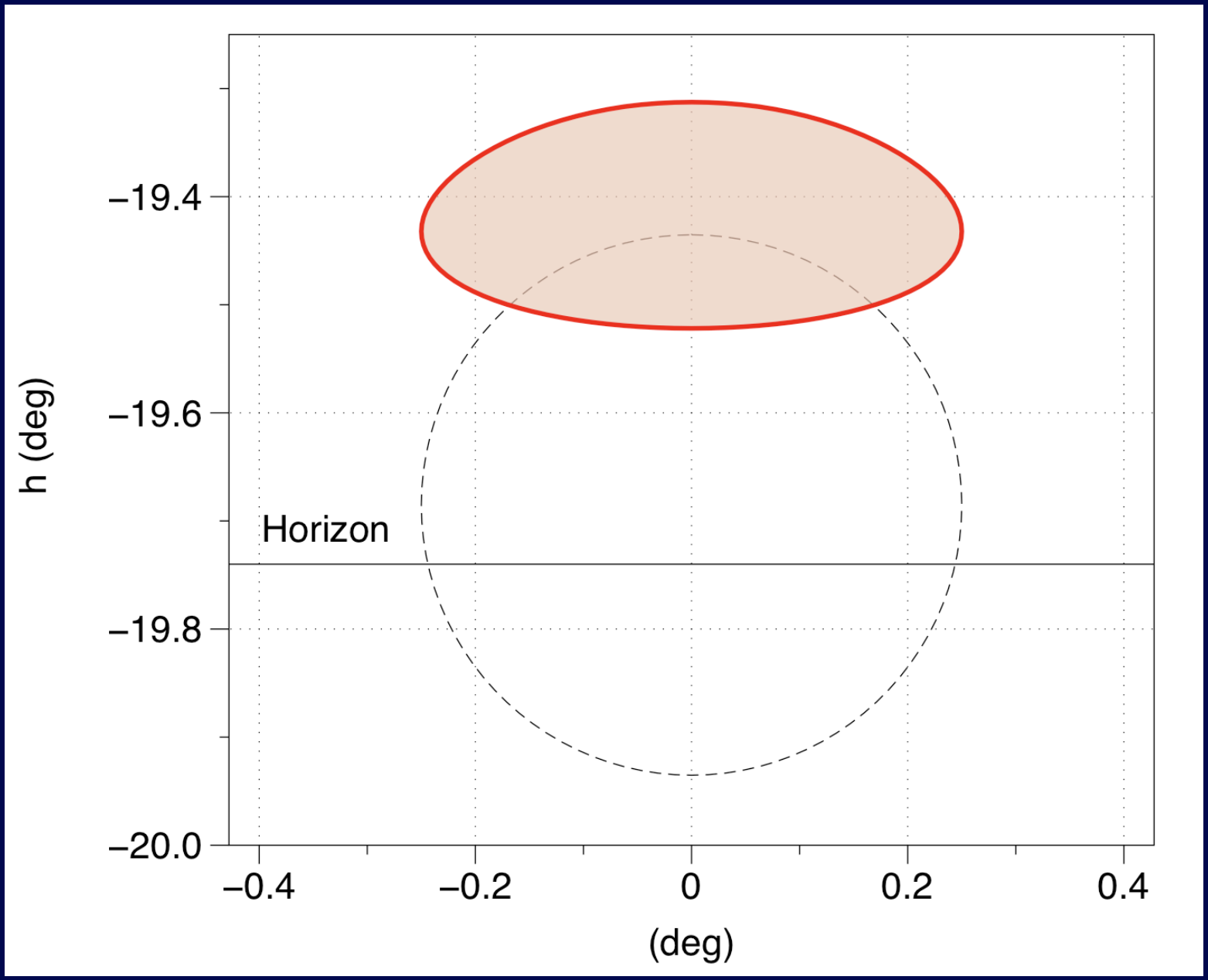
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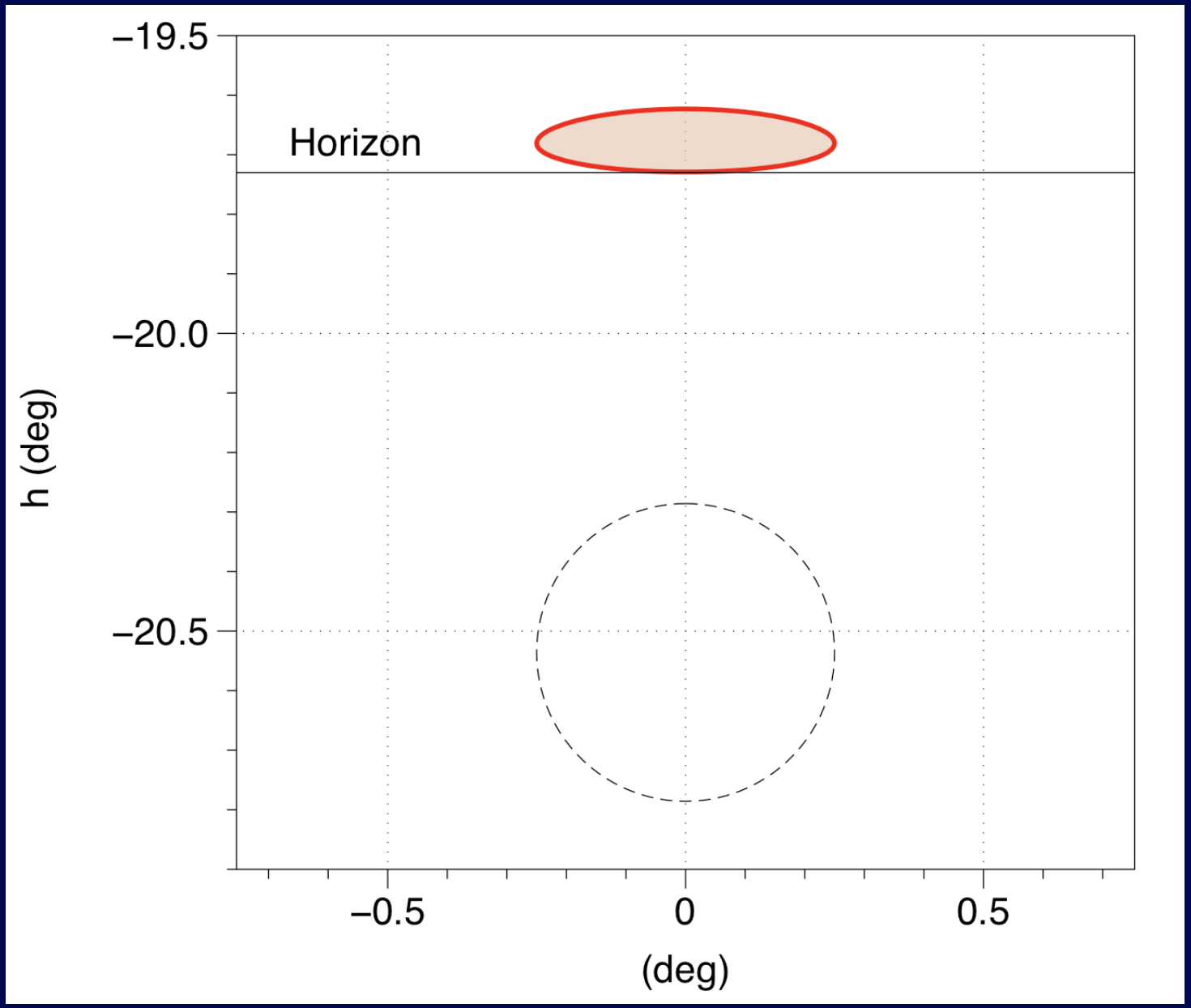
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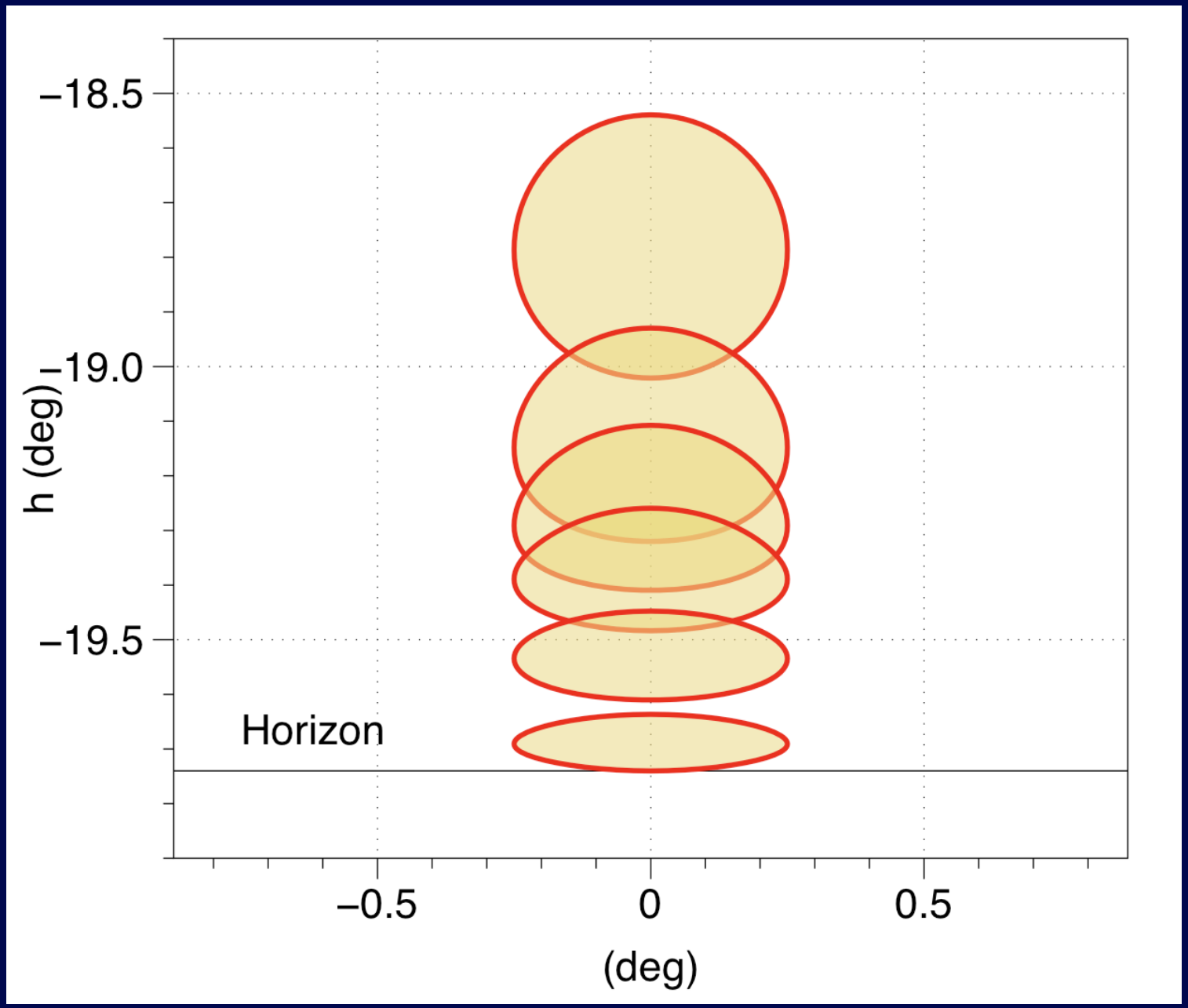
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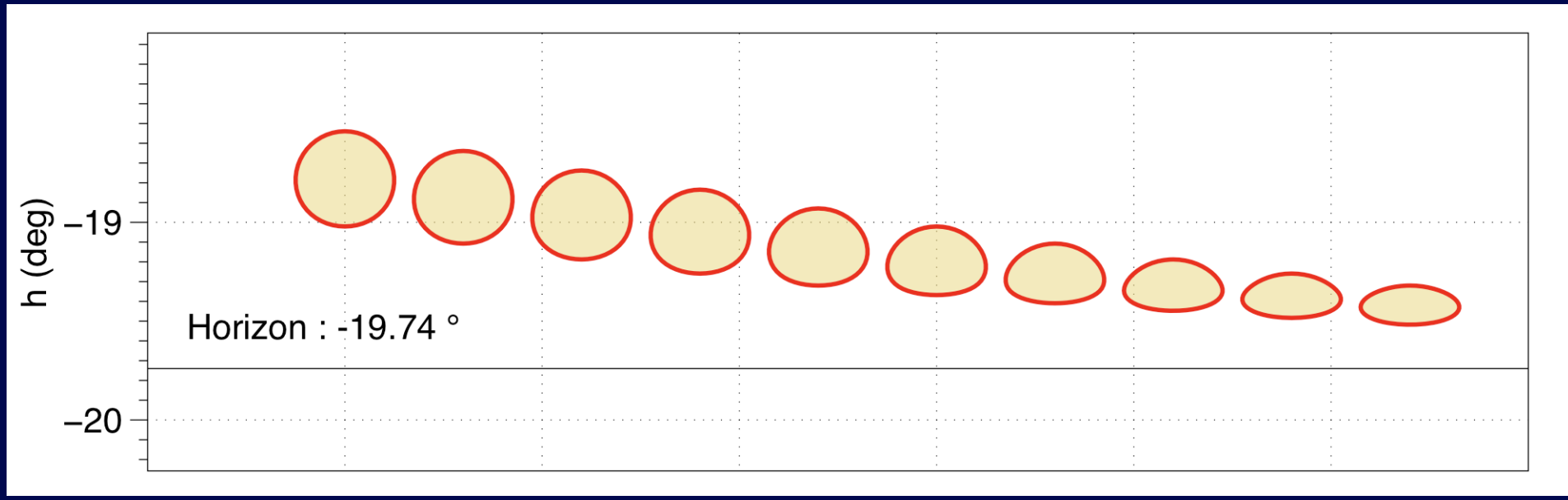
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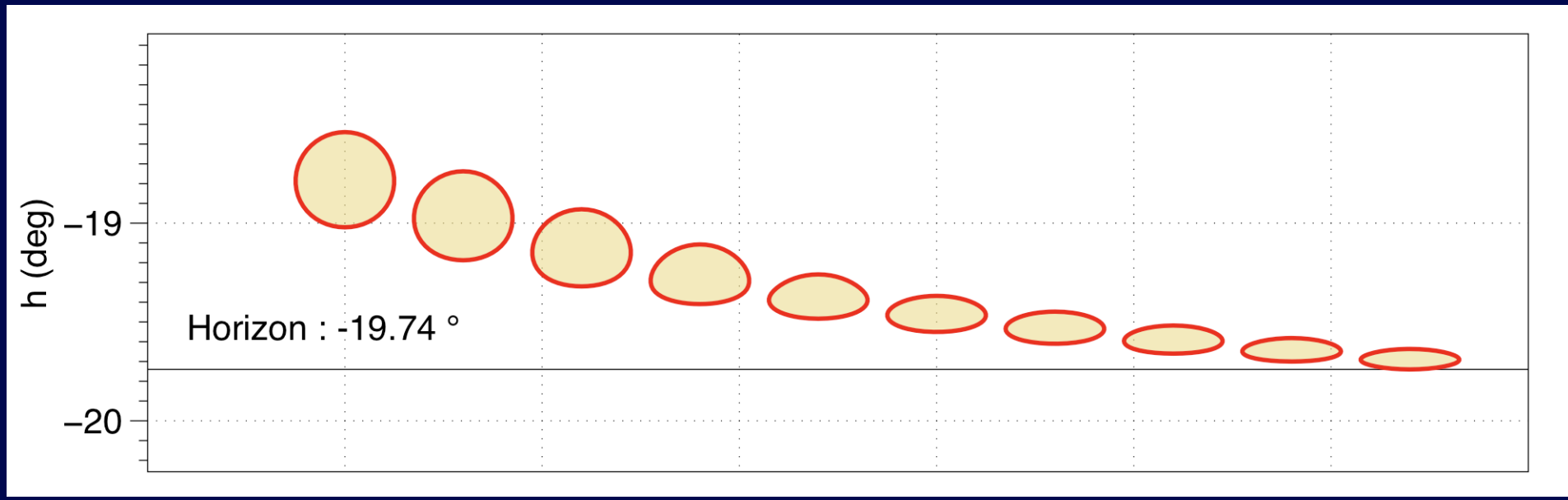


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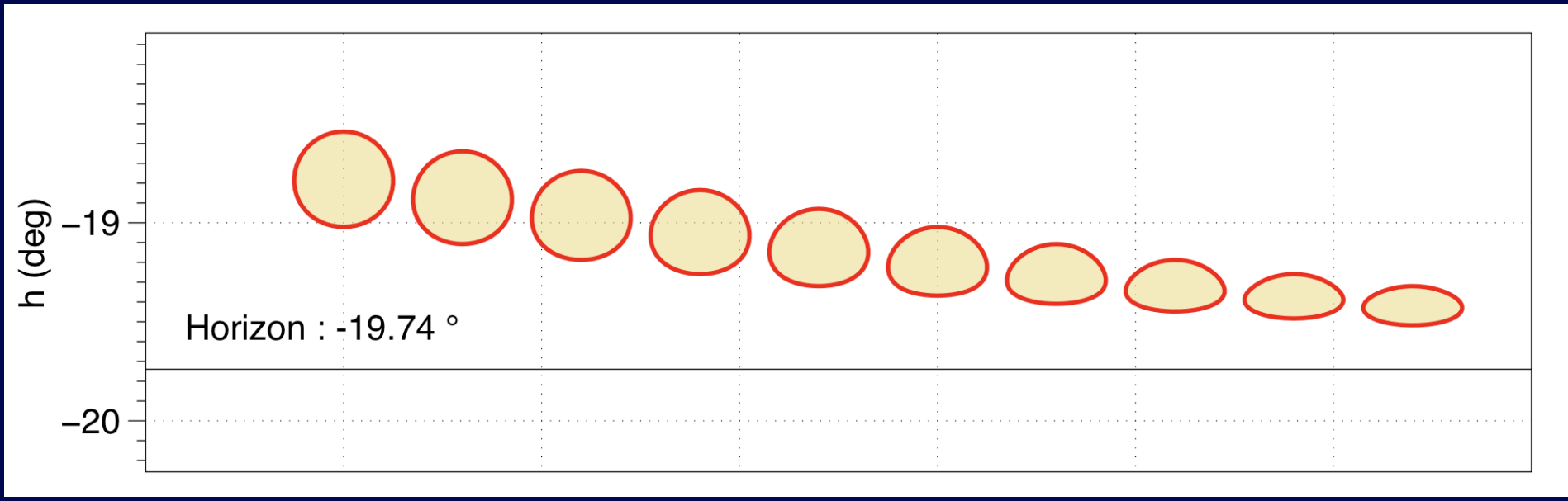
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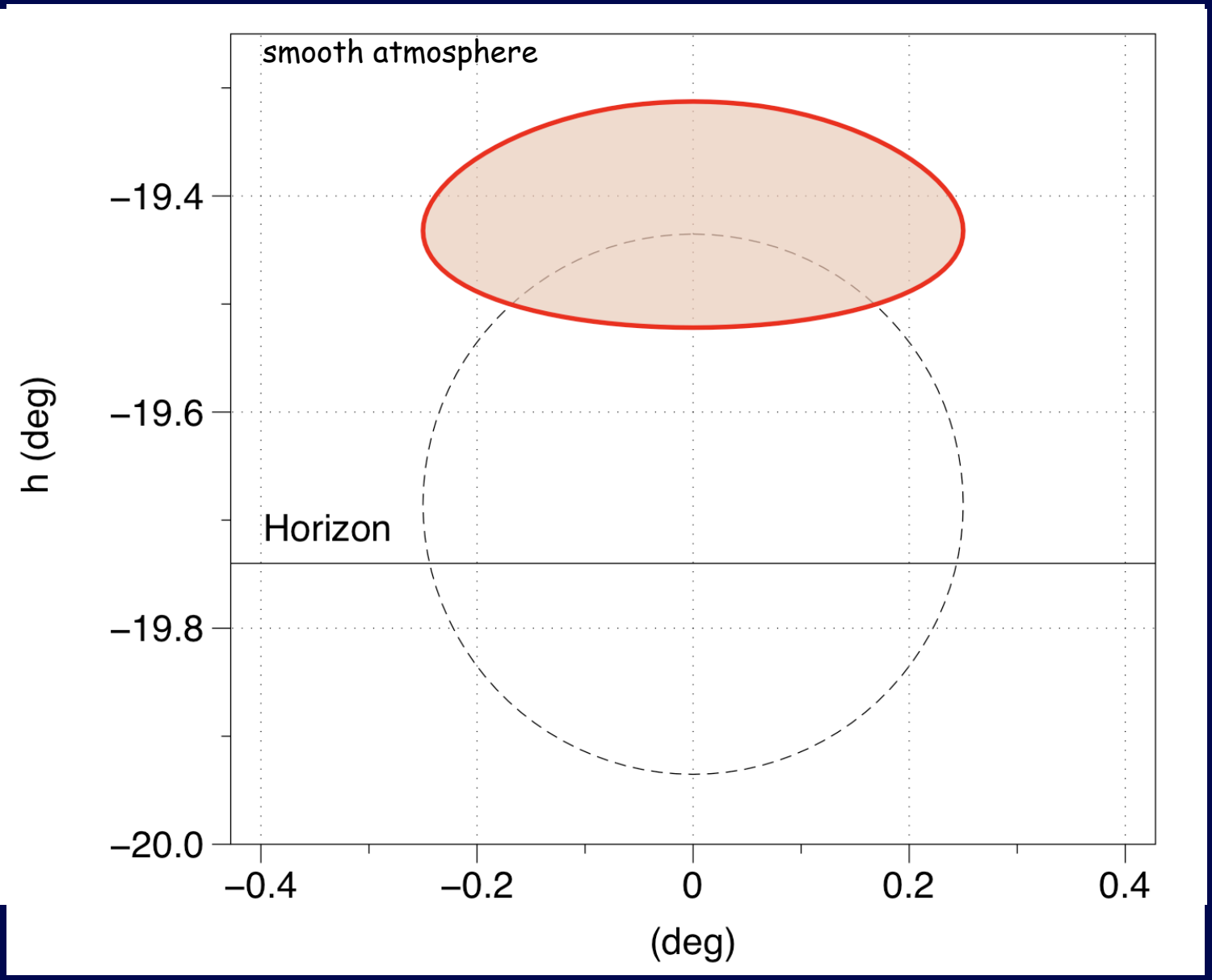


Moonset from ISS

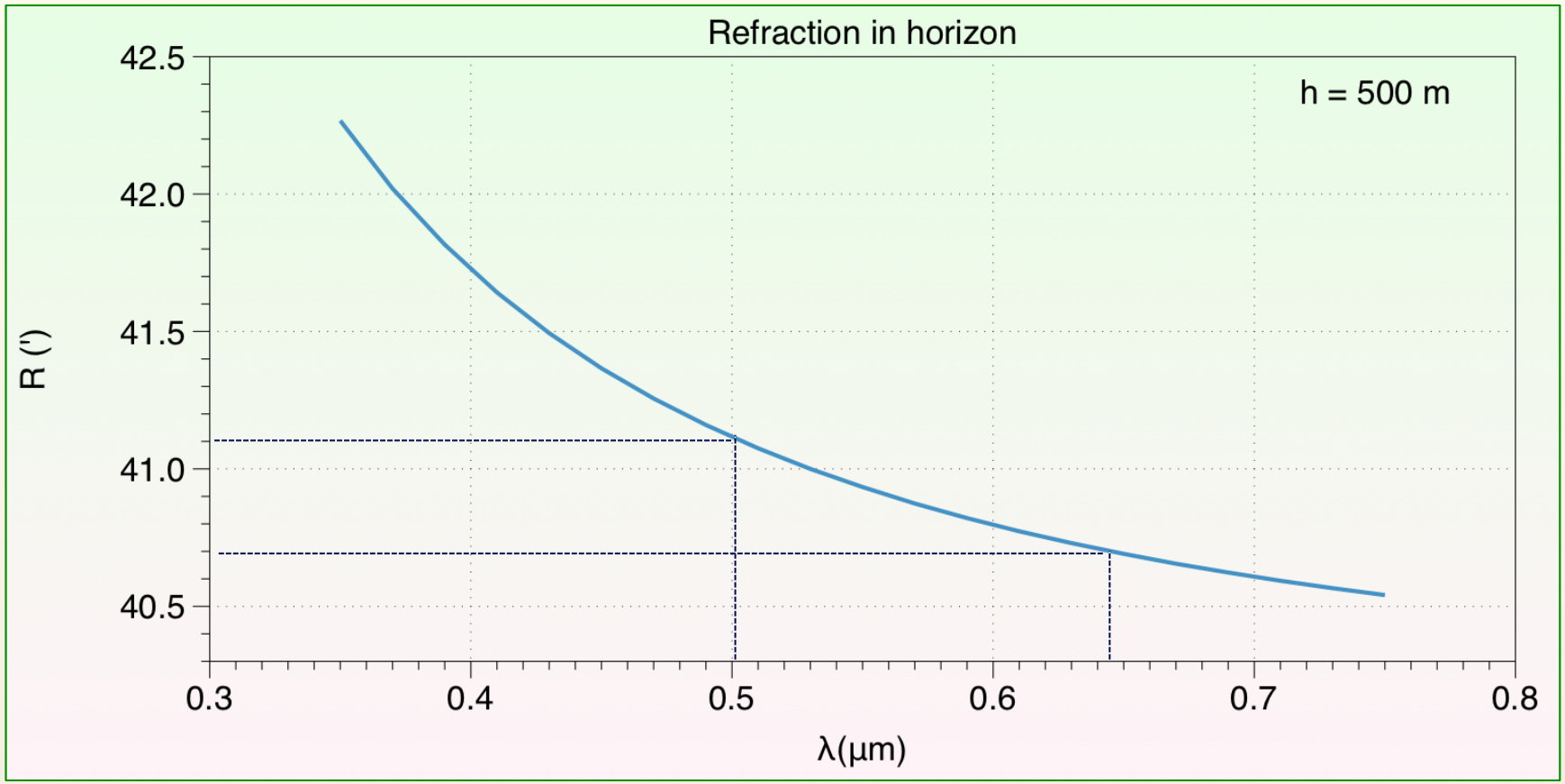




Layer discontinuity issue



Effect of wavelength



blue red

Green Flash



Thanks for your attention

