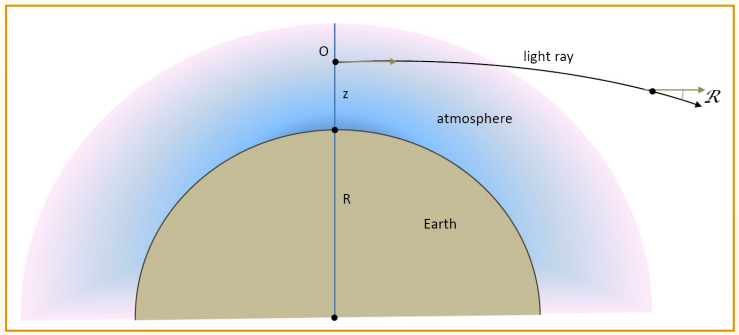
Sunrises and sunsets

Theory and observations

F. Mignard
UCA/CNRS
Observatoire de la Côte d'Azur

Summary



- Sunrises and sunsets
  - opinions

- Theoretical framework
  - astronomical side
  - light-rays in the atmosphere
  - constraints to get a few seconds accuracy in the prediction



- Practical activities
  - when does the sun rise/set ?
  - observations and confrontation to predictions



Quotations …

- In astronomical textbooks or national ephemeris, sunrises/sets are given to the nearest minute
  - horizontal refraction is often taken as a constant (34', 36' … ➔ t = 15°C ?)
  - angular radius of the Sun fixed at 16'
  - observer at sea-level by default, horizon at z = 90°
- Refraction near horizon is the main problem

…giving rising or setting times of a celestial body more accurately than one minute makes no sense. J. Meeus

…the indeterminacy is a geometrical one, and the refraction at any instant may differ by ***several minutes of arc*** *from the most accurate value that can be calculated.* G.M. Clemence

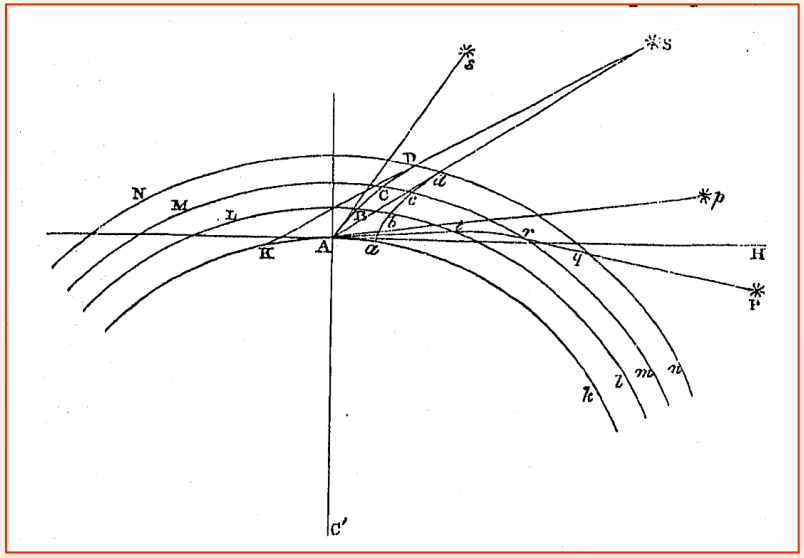
…there is perhaps no branch of practical astronomy on which so much has been written and which is still in so unsatisfactory a state " . S. Newcomb

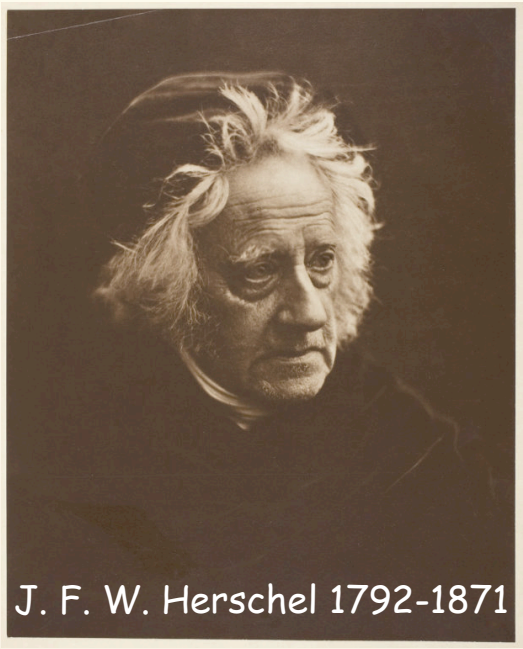
In the polar regions, the observed times of sunrise and sunset can differ from predictions by several days; A.T. Young

Quotations …

…The exact estimation of the amount of atmospheric refraction is a very difficult subject of physical inquiry, one on which geometers are not yet entirely agreed.

J. F. Herschel, Outline of Astronomy, 1849, Sect. 41.




J. F. W. Herschel 1792-1871

Quotations …

From A. T. Young (San Diego Univ.) authoritative website on refraction and green-flash

Why We Can't Predict Sunset Times Exactly

Introduction

So, if we can calculate astronomical refraction to a gnat's eyelash, and we know the position of the Sun to a fraction of a second of arc, we should be able to predict when it will set tonight, with an accuracy of a fraction of a second of time, right?

Wrong. (Although that's a reasonable question.)

SUNSET SCIENCE. IV. LOW-ALTITUDE REFRACTION

ANDREW T. YOUNG
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Received 2004 January 13; accepted 2004 February 25

A.J., 127, 2004

Few Requirements

- My goal is to do better than 10s in prediction and observed time

  Astronomical requirements to get a prediction to 1s of time :

- dz/dt at z ~ 90 °, in mid-latitudes is about 8 "/s
  - a typical sunrise takes between 3 and 4 mn
  - shortest will be 2mn in equatorial regions
- An accuracy of 5 - 10'' in the sun position is sufficient
  - a simple model of solar motion will do it, but not too crude
    - e^2 ~ 60'', Lunar term is 6.4'', Venus or Jupiter ~ 5 ''
  - one hardly needs to make distinction between apparent or geometric directions (maximum error of 2s from annual aberration or nutation)
  - in practice there is no issue with planetary motions
    - all my ephemeris based on VSOP2013 or INPOP

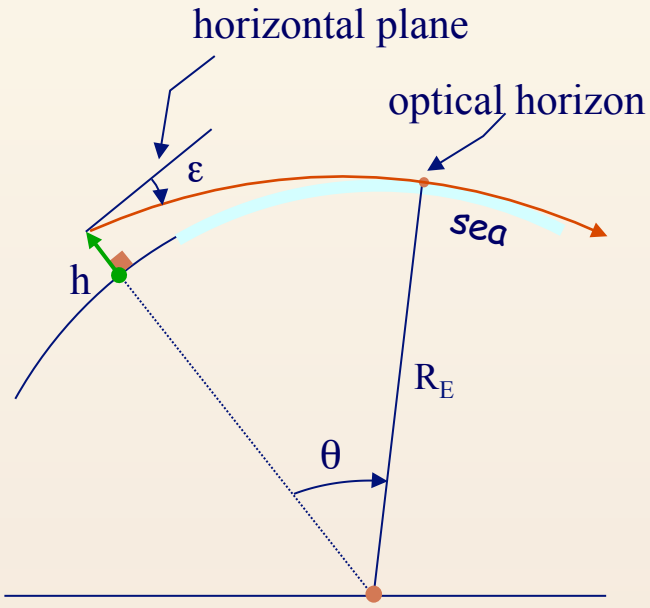
Rise and set : altitude of the Sun

- Observer at height *h*
- Sun rises when seen just touching the optical horizon
- Compute the horizon dip ε and the refraction R on the grazing ray
  - functions of *h*, *T*, *P*, RH, λ
- Add (or remove) solar radius and add equatorial parallax (~ 9'')

$$h_{\odot}^{\mathrm{r,s}} = -\epsilon(h) - R(h) \pm s + \pi_{\odot}$$

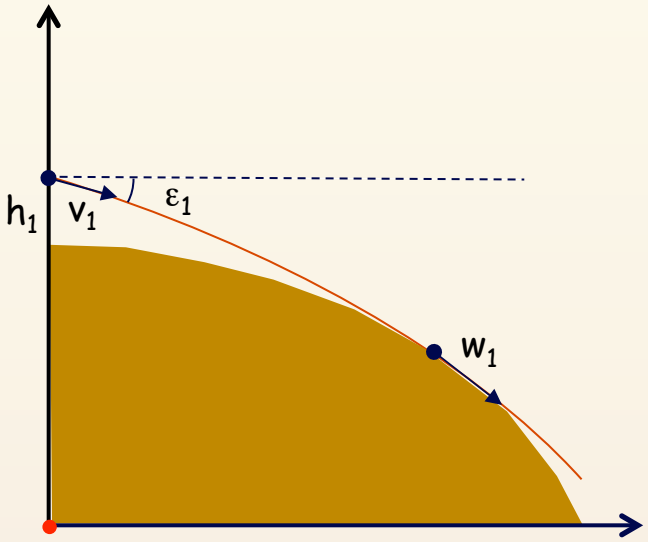
$\epsilon(h)$: horizon dip

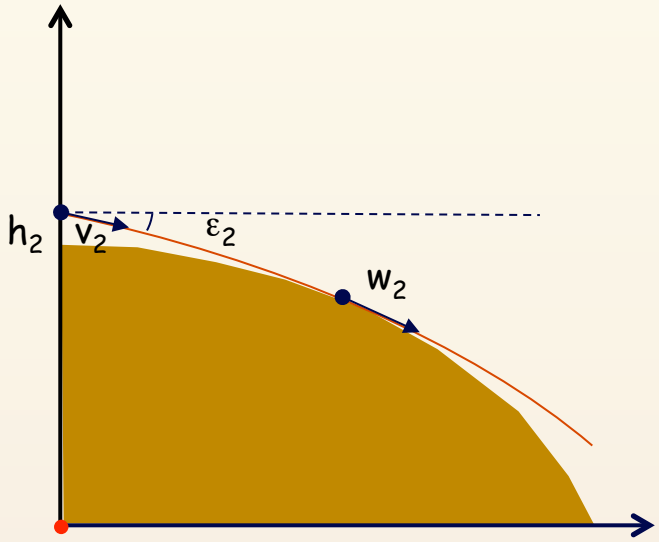
$R(h)$: refraction in horizon



Dip and additional refraction

- Refraction vs dip with varying height





$$R(h_1) = \mathbf{W}_1 - \mathbf{V}_1 + R(90°, 0)$$

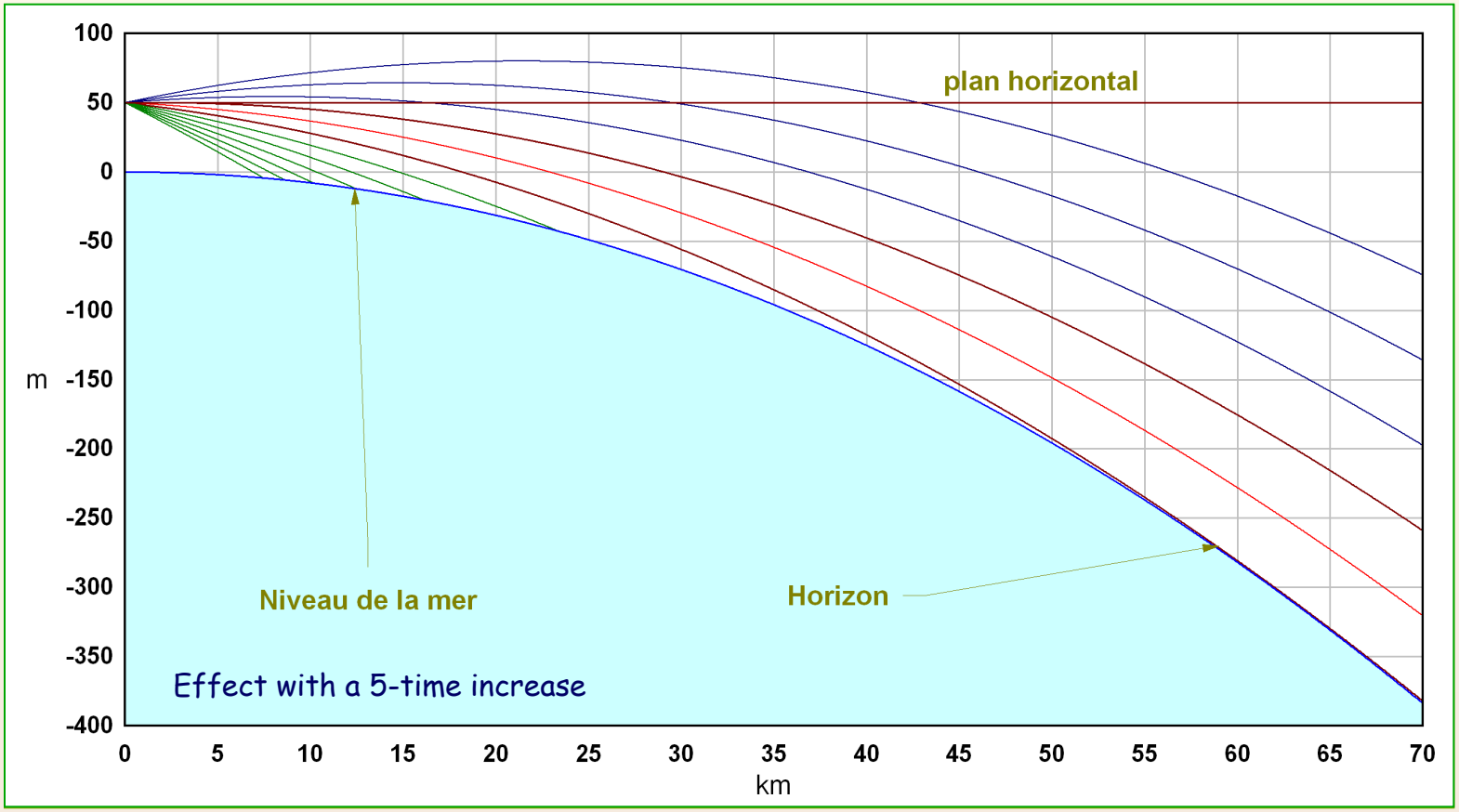
$$\text{dip} = \epsilon_1$$

$$R(h_2) = \mathbf{W}_2 - \mathbf{V}_2 + R(90°, 0)$$

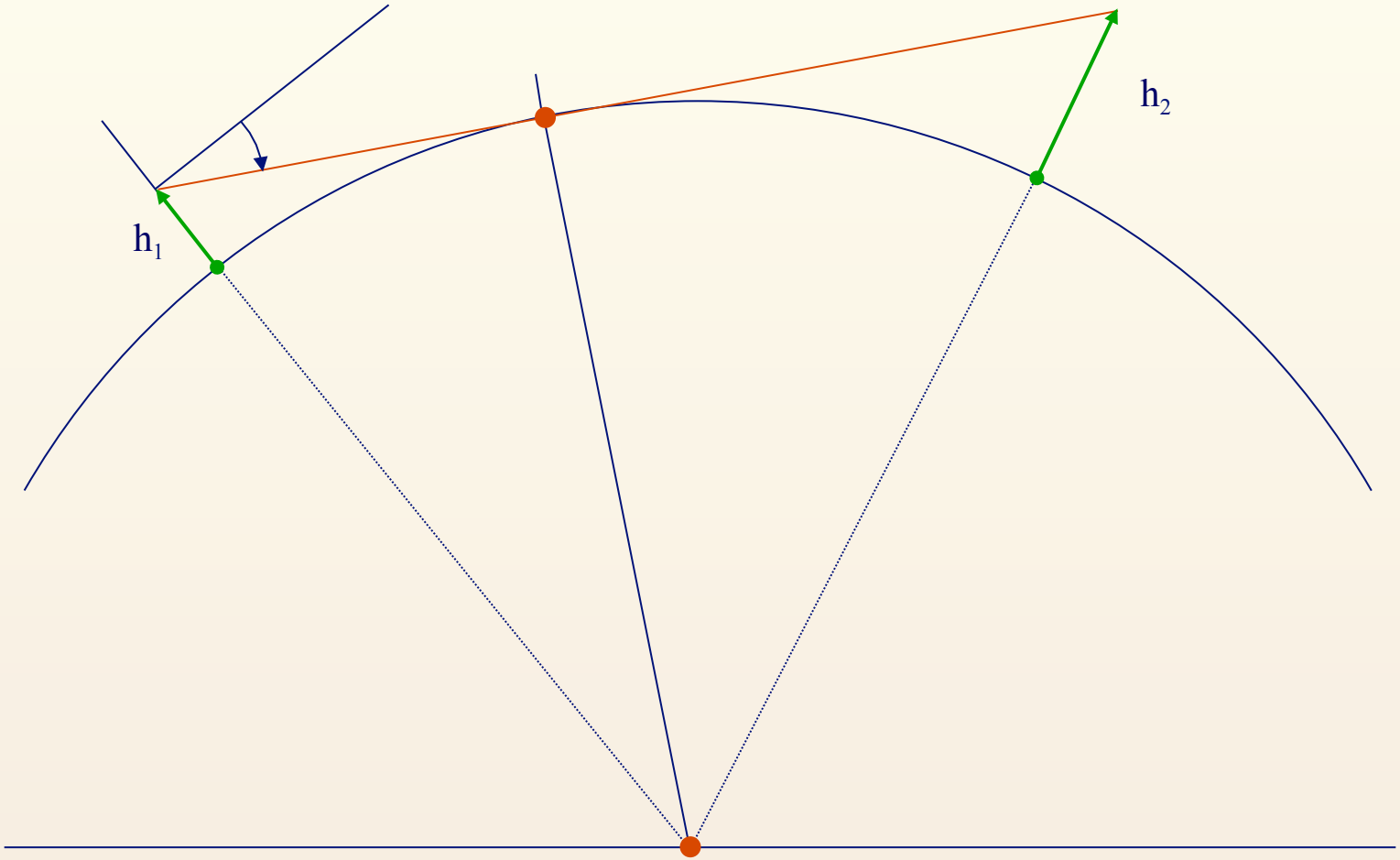
$$\text{dip} = \epsilon_2$$

Light propagation in the atmopshere

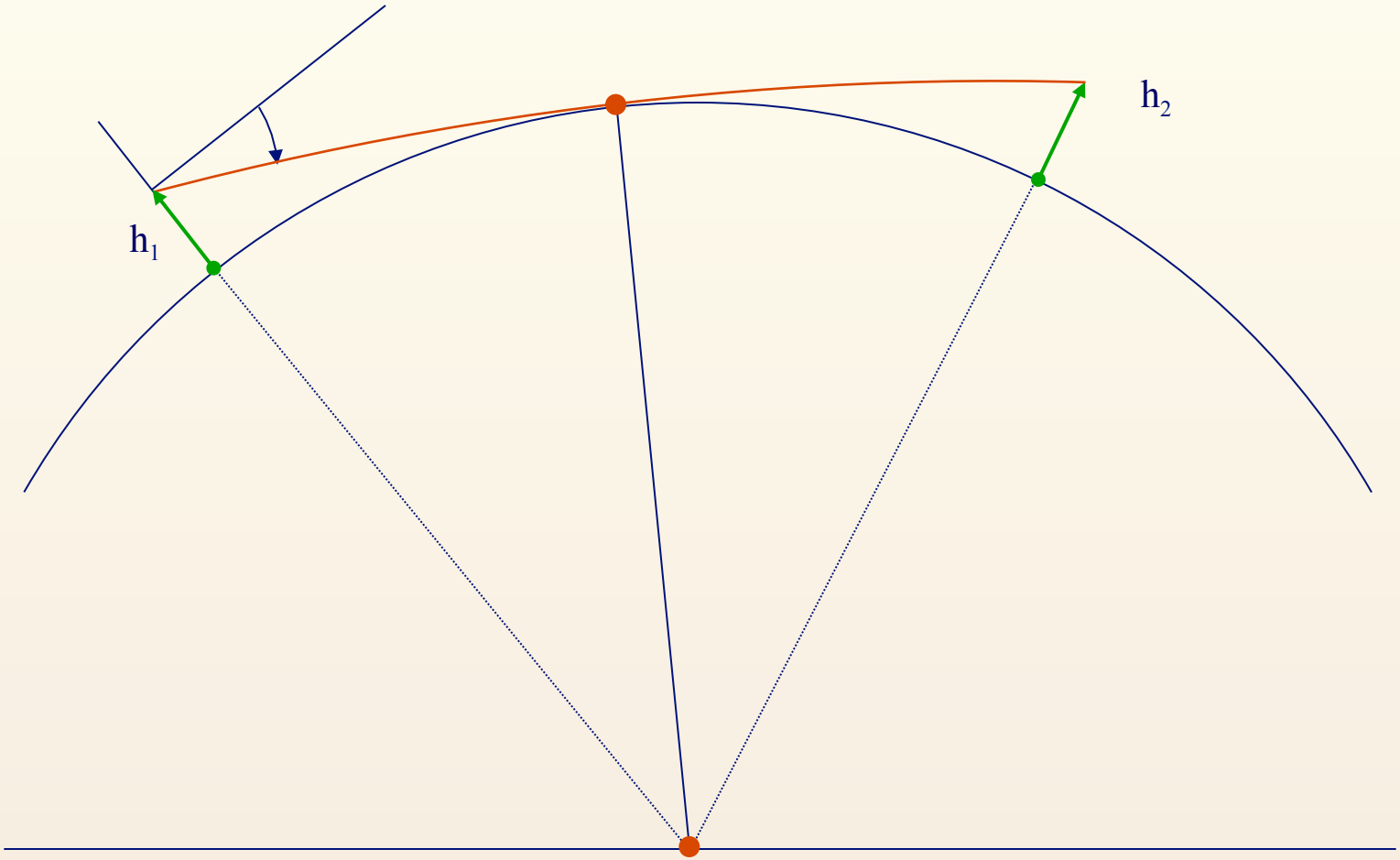
- Light path in the atmosphere is not a straight line
  - because pressure and temperature CHANGE with altitude
  - in normal conditions curved towards the ground
    - but could be in the other way in case of mirages



Straight line propagation



Curved light-ray



Optical horizon on the sea



Optical horizon on the sea



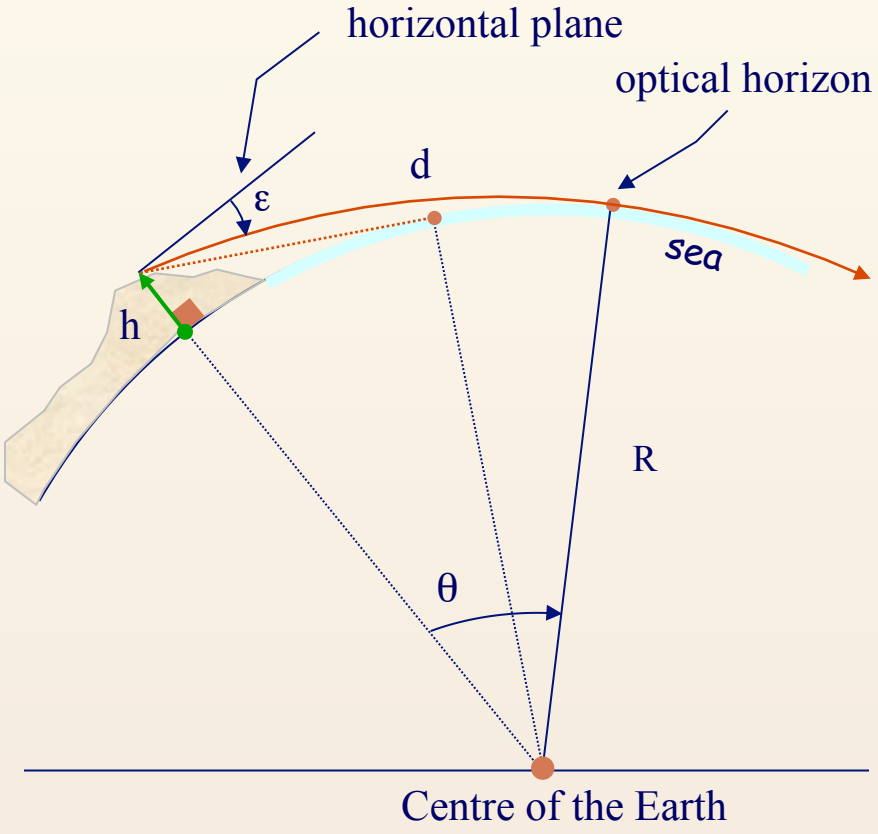
Horizon dip

- From an altitude h, one follows the grazing lightray
- 2nd assumption : Refraction and constant curvature
  - geometry of circles

$$\beta = 1 - R_\oplus \frac{dn}{dh} \simeq 0.83$$

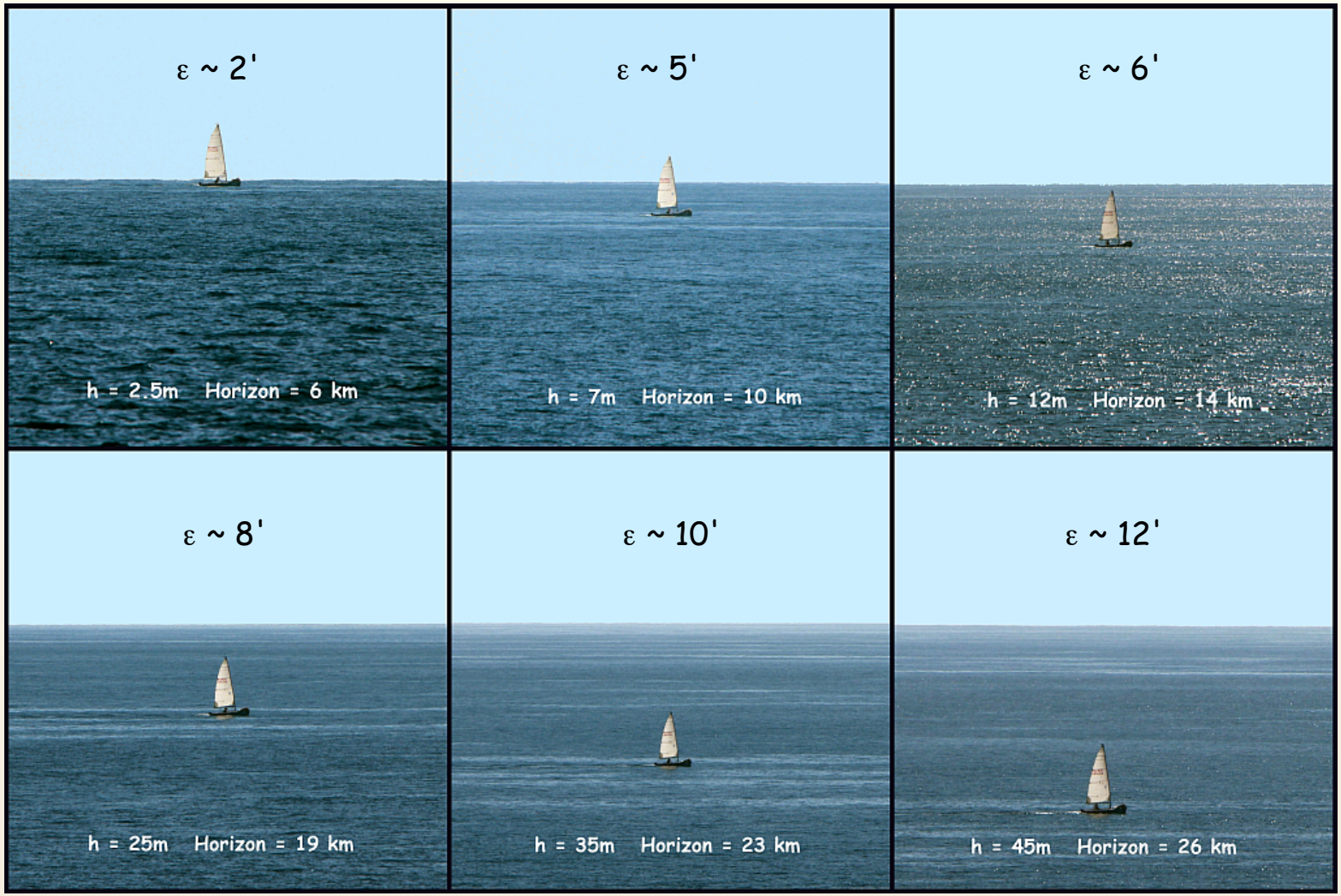
$$d = \sqrt{\frac{2Rh}{\beta}} \qquad \varepsilon = \sqrt{\frac{2\beta h}{R}}$$

h (m)	d (km)	ε (')	dε/dh ('' /m)
0	0	0	∞
2	5.5	2.5	36
5	9	3.9	23
10	12	5.5	16
50	27	13	7.3
100	40	17 [19]	5
250	61	27 [30]	3.3
500	88	39 [43]	2.3



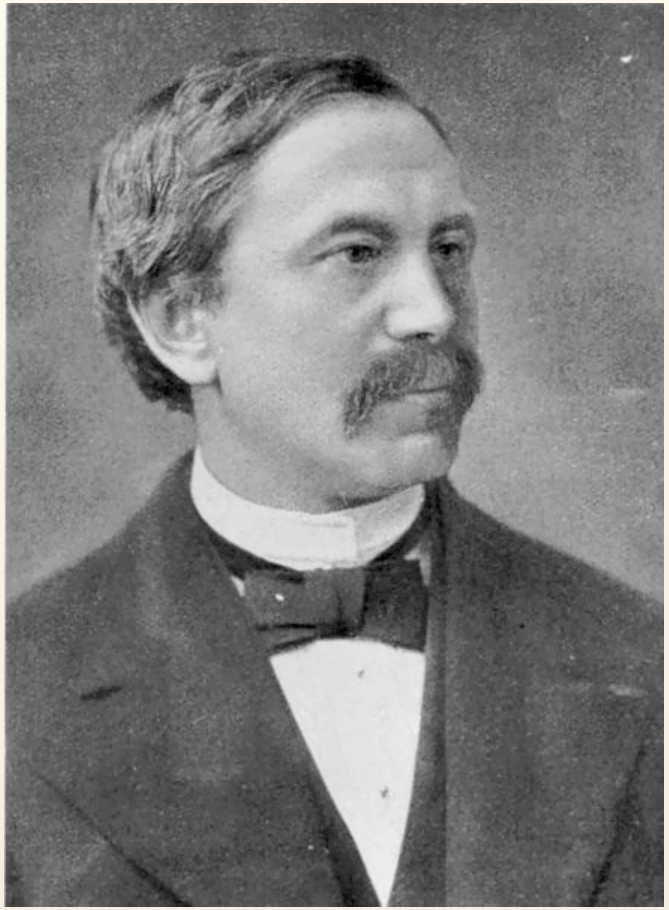
A visual demonstration

- Pictures taken at different altitudes
  - total time range of ~ 2 mn



Refraction: History

- Age-old problem of practical astronomy



K. C. Bruhns, 1830-1881

DIE

ASTRONOMISCHE

STRAHLENBRECHUNG

IN

IHRER HISTORISCHEN ENTWICKELUNG

DARGESTELLT

VON

DR. C. BRUHNS,

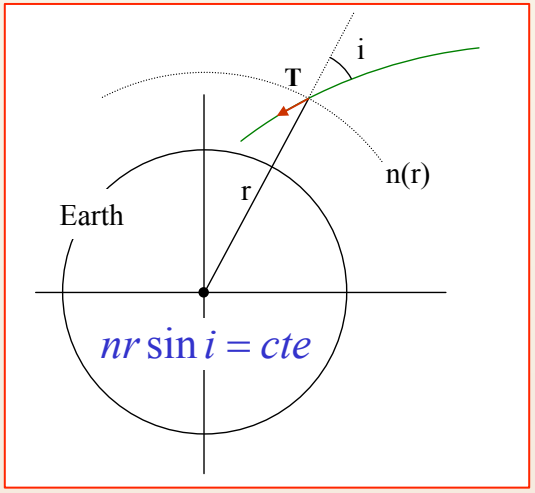
ASTRONOM DER NEUEN STERNWARTE UND PROFESSOR AN DER UNIVERSITÄT ZU LEIPZIG.

EINE GEKRÖNTE PREISSCHRIFT.

LEIPZIG,
VOIGT & GÜNTHER.
1861.

Refraction

- Almost countless papers, monographs, models throughout history
  - started very early in modern astronomy
    - Tycho Brahe computed tables from observations ➔ polar distances of stars
    - Tables from Cassini, Flamsteed, Halley, Bradley and empirical rules
    - models from B. Taylor, P. Bouguer, J. Cassini, L. Euler, J.H. Lambert, Bessel …
  - elaborate mathematical modelling to achieve the arcsec accuracy
  - see Laplace (100 p), Biot (100p), Delambre (30 p), Chauvenet (70p), Garfinkel (1944, 1967) for extensive analytical theories
  - very complex and extensive tables (eg Pulkovo)
  - see Biot(1836), Auer & Standish (2000), Hohenkerk & Sinclair (1985) for numerical methods
- Well defined problem for a spherically symmetric atmosphere



Could be very intricate …

F.W. Bessel

und

$$\left(\frac{d^2Q}{d\beta^2}\right) - \frac{\cot g^2 z}{2}\left(\frac{dQ'}{d\beta}\right) + \frac{\cot g\, z}{2(2\beta)^{\frac{3}{2}}}\frac{x}{1-x}$$
$$-\frac{\cot g^4 z}{4}Q' - \sqrt{\frac{\beta}{2}}\frac{\cot g^2 z}{4\beta}\frac{x}{(1-x)^2} + \sqrt{\frac{\beta}{2}}\frac{\cot g\, z}{4\beta^2}\frac{x}{(1-x)} \cdots\cdots (k)$$

Aus (g) ist

$$\left(\frac{dQ}{dx}\right) = \frac{1-x}{x}Q'$$

mithin

$$\left(\frac{d^2Q}{dx\,.\,d\beta}\right) - \frac{1-x}{x}\frac{\cot g^2 z}{2}Q'' - \sqrt{\frac{\beta}{2}}\frac{\cot g\, z}{2\beta}\frac{1}{(1-x)^2} \cdots\cdots (l)$$

Aus

$$\beta = \frac{g-1}{gl}a,\ l = l_0(1+mr),\ \alpha = \alpha_0\frac{b}{B}\frac{1}{1+mr}^{1}),\ x = \frac{\alpha\beta}{\sin^2 z}$$

folgt:

$$\left(\frac{d\beta}{dl}\right) = -\frac{a}{l^2},\ \left(\frac{dl}{dr}\right) = lm,\ \left(\frac{d\alpha}{dr}\right) = -\alpha m,$$
$$\left(\frac{dx}{d\beta}\right) = \frac{x}{\beta},\ \left(\frac{dx}{d\alpha}\right) = \frac{x}{\alpha},\ \left(\frac{d^2\beta}{dl^2}\right) = \frac{2a}{l^3},\ \left(\frac{d^2l}{dr^2}\right) = 0.$$

also:

$$\left.\begin{aligned}&\left(\frac{d\beta}{dr}\right) = \left(\frac{d\beta}{dl}\right)\left(\frac{dl}{dr}\right) = -\frac{am}{l} = -m\beta\frac{g}{g-1}\\ &\left(\frac{d^2\beta}{dr^2}\right) = m^2 2\beta\frac{g}{g-1},\ \left(\frac{dx}{dr}\right) = \left(\frac{dx}{d\beta}\right)\left(\frac{d\beta}{dr}\right) + \left(\frac{dx}{d\alpha}\right)\left(\frac{d\alpha}{dr}\right) = -mx\left(\frac{2g-1}{g-1}\right)\\ &\left(\frac{d^2x}{dr^2}\right) = 2m^2x\left(\frac{3g-1}{g-1}\right)\end{aligned}\right\} (m)$$

Nach den Formeln (f), (g), (i), (l), (m) erhält man

$$\left(\frac{dQ}{dr}\right) = -mQ'\left\{\frac{2g-1}{g-1}(1-x) + \frac{g}{g-1}\frac{\beta}{2}\cot g^2 z\right\} + m\sqrt{\frac{\beta}{2}}\frac{\cot g\, z}{2}\frac{g}{g-1}\frac{x}{1-x} \cdots (n)$$

$$\left.\begin{aligned}&\left(\frac{d^2Q}{dr^2}\right) = m^2Q''\left\{\frac{2g-1}{g-1}(1-x) + \frac{g}{g-1}\frac{\beta}{2}\cot g^2 z\right\}^2\\ &+ m^2Q'\left\{\frac{6g-2}{g-1}(1-x) - \left(\frac{2g-1}{g-1}\right)^2 + \frac{\beta}{2}\cot g^2 z\,\frac{2g}{g-1}\right\}\\ &- m^2\frac{x}{1-x}\sqrt{\frac{\beta}{2}}\cot g\, z\,\frac{g}{g-1}\left\{\frac{g}{g-1}\cdot\frac{\beta}{4}\cdot\frac{\cot g^2 z}{(1-x)^2} + \frac{2g-1}{g-1}\frac{1}{1-x}\right.\\ &\left.+ \frac{3g-4}{4g-4}\right\}\end{aligned}\right\} (o)$$

und

$$\left(\frac{dQ}{db}\right) = \left(\frac{dQ}{dx}\right)\left(\frac{dx}{db}\right) = \frac{1-x}{x}Q'\left(\frac{dx}{db}\right) = \frac{1-x}{x}Q'\frac{x}{\alpha}\left(\frac{d\alpha}{db}\right) = \frac{1-x}{b}Q' \cdots\cdots (p)$$

¹) Den Factor $1 + mr'$ kann man sich mit in b enthalten denken.

Ch. Kramp

$+\frac{63\,r^3}{5^6} - \frac{17.28\,r^4}{5^8} + \frac{154.408\,r^5}{5^{11}}$. Telle est l'expression générale de toute faculté, ayant $\frac{1}{2}$ pour exposant.

141. Enfin, voici les développemens des facultés à exposant fractionnaire, dans lesquelles les dénominateurs de ces exposants se trouvent entre les limites *un* et *six*.

$$1^{\frac{1}{2}}Ir = 1 - \frac{5r}{2^3 3^2} - \frac{55r^2}{2^7 3^4} + \frac{4675}{2^{10}3^7}r^3 + \frac{37697}{2^{11}3^9}r^4 - \frac{11.29.35.37}{2^{14}3^{10}}r^5 - \text{etc.}$$
$$1^{\frac{1}{3}}Ir = 1 - \frac{2r}{5^2} - \frac{3r^2}{5^4} + \frac{42}{5^6}r^3 + \frac{19.105}{5^8}r^4 - \frac{19.41.42}{5^{11}}r^5 - \text{etc.}$$
$$1^{\frac{1}{4}}Ir = 1 - \frac{3r}{2^5} - \frac{7r^2}{2^{11}} + \frac{231}{2^{16}}r^3 + \frac{77.103}{2^{23}}r^4 - \frac{11.63.361}{2^{28}}r^5 - \text{etc.}$$
$$1^{\frac{1}{5}}Ir = 1 - \frac{r}{3^2} \ldots\ldots + \frac{10}{3^7}r^3 + \frac{11}{3^9}r^4 - \frac{77}{3^{12}}r^5 - \text{etc.}$$
$$1^{\frac{2}{3}}Ir = 1 - \frac{3r}{5^2} + \frac{2r^2}{5^4} + \frac{78}{5^6}r^3 + \frac{39}{5^8}r^4 - \frac{13.23.243}{5^{11}}r^5 - \text{etc.}$$
$$1^{\frac{1}{2}}Ir = 1 - \frac{r}{2^3} + \frac{r^2}{2^7} + \frac{5}{2^{10}}r^3 - \frac{21}{2^{15}}r^4 - \frac{399}{2^{18}}r^5 + \text{etc.}$$
$$1^{\frac{3}{4}}Ir = 1 - \frac{3r}{5^2} + \frac{7r^2}{5^4} + \frac{63}{5^6}r^3 - \frac{17.28}{5^8}r^4 - \frac{17.24.154}{5^{11}}r^5 + \text{etc.}$$
$$1^{\frac{2}{3}}Ir = 1 - \frac{r}{3^2} + \frac{r^2}{3^4} + \frac{7}{3^7}r^3 - \frac{28}{3^9}r^4 - \frac{182}{3^{11}}r^5 + \text{etc.}$$
$$1^{\frac{3}{4}}Ir = 1 - \frac{3r}{2^5} + \frac{25r^2}{2^{11}} + \frac{135}{2^{16}}r^3 - \frac{39.307}{2^{23}}r^4 - \frac{17.89.117}{2^{28}}r^5 + \text{etc.}$$
$$1^{\frac{4}{5}}Ir = 1 - \frac{2r}{5^2} + \frac{7r^2}{5^4} + \frac{22}{5^6}r^3 - \frac{22.23}{5^8}r^4 - \frac{11\,28.71}{5^{11}}r^5 + \text{etc.}$$
$$1^{\frac{5}{6}}Ir = 1 - \frac{5r}{2^3 3^2} + \frac{35r^2}{2^7 3^3} + \frac{13.25}{2^{10}3^7}r^3 - \frac{19.91.131}{2^{15}3^9}r^4 - \frac{19.23.91.125}{2^{18}3^{10}}r^5 + \text{etc.}$$

Ainsi donc, pour achever entièrement l'ouvrage que nous nous étions proposé, il ne reste plus qu'à résoudre le problème suivant.

Semi-empirical rules

- From Bradley and quoted by Lalande (Astronomie, art. 2202)

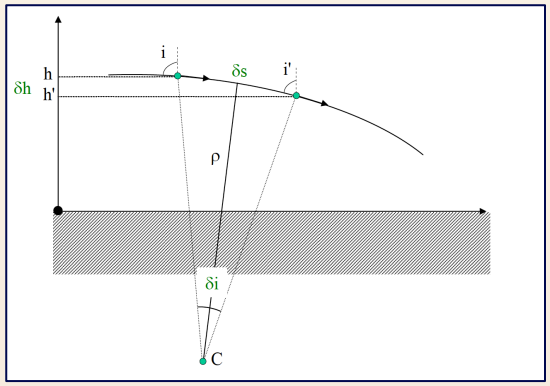
"la réfraction est comme la tangente de la distance apparente au zénith diminuée de trois fois la réfraction"

$$R \sim \tan(z - 3R)$$

➔ very similar to Laplace : $R = \alpha \tan z - 3\alpha^2 \tan^3 z$

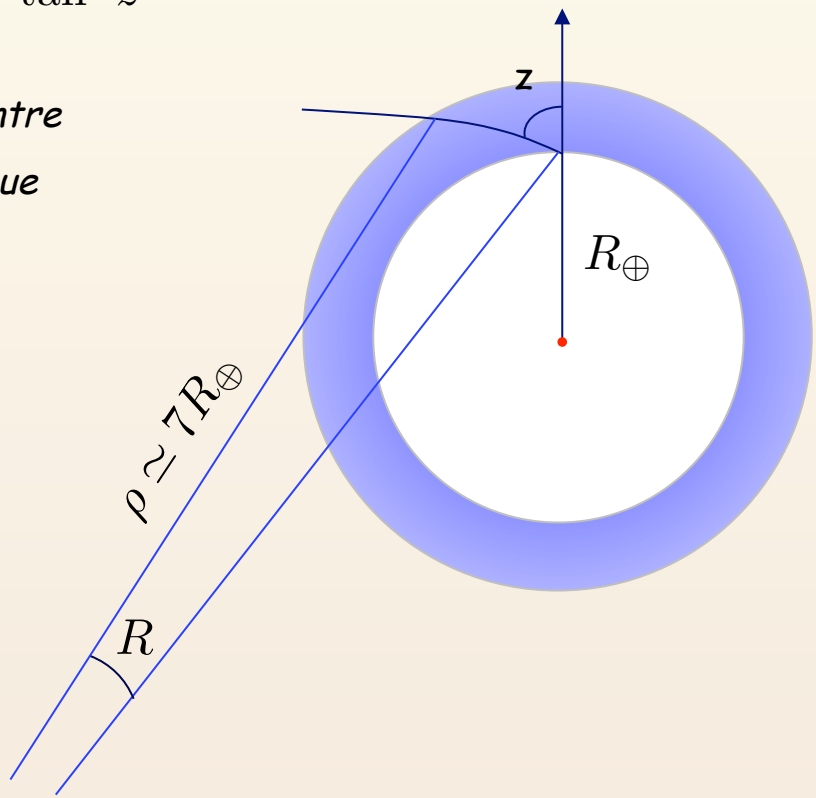
"la réfraction est la septième partie de l'angle au centre de la Terre, dans lequel est renfermé tout l'espace que le rayon a parcouru dans l'atmosphère"

Quite good approximation since



$$\frac{1}{\rho} \sim -\frac{dn}{dh}$$

$$\rightarrow \rho \sim 40,000\,\mathrm{km}$$



First theoretical modelling of the astronomical refraction

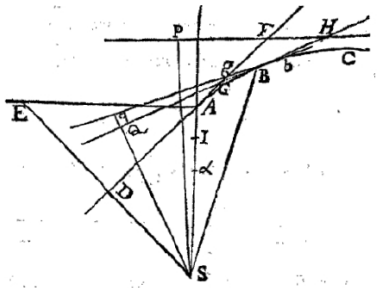
- J.D. Cassini in 1650, with uniform density spherical slab
  - basically wrong, numerically rather accurate to z ~ 75°
- Brook Taylor in 1715 (yes, the one of the Taylor series)

PROP. XXVII. PROB. XXII.

To find the refraction of the light rays passing through the atmosphere of the earth.

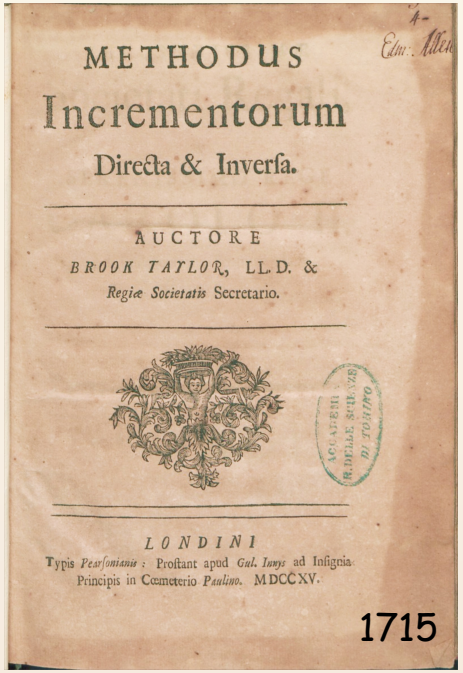


B. Taylor 1685-1731



Let S be the centre of the earth, ABC the radius of a curved light ray, that touches the lines AG; BG in A and B, and these tangents mutually cross each other in G, and the perpendiculars *s*D and SQ are sent to the tangents, and SA and SB are drawn; and at A the normal of SA, AE is drawn crossing SD in E, and A is a given fixed point on the ray, and B is a variable point. Let SA = *a*, SD = *b*, SE = *t* (= $\frac{aa}{b}$,) SB = *x*, *d* is the density at A, *y* the density at B. [p. 109.]

$$\frac{a^2 b y \dot{x}}{2cx^2 \times \overline{1+y} \times \sqrt{\frac{1+d}{1+y}x^2 - b^2}}$$

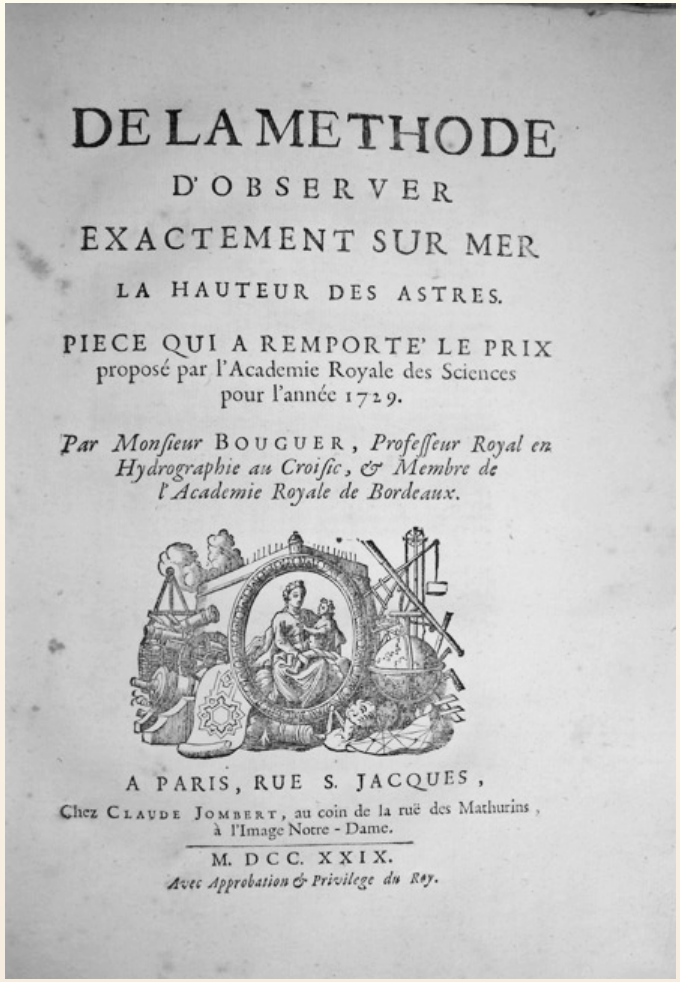


Refraction

- P. Bouguer treatise awarded by the Académie in 1729



P. Bouguer 1698-1758

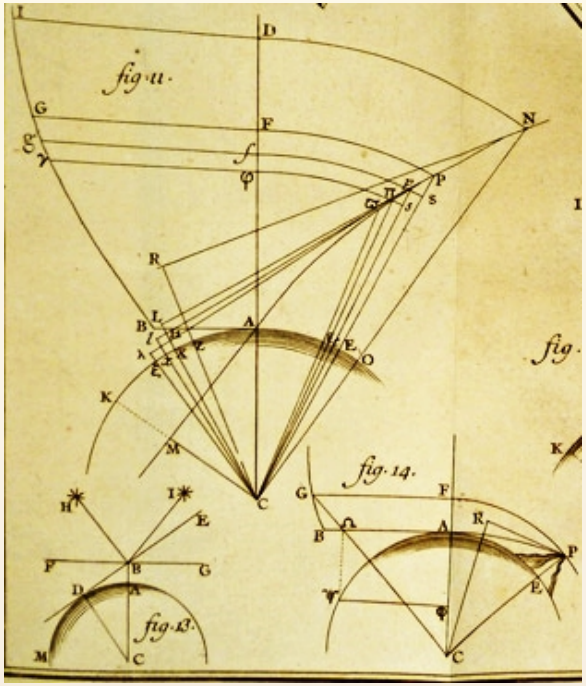


DE LA METHODE
D'OBSERVER
EXACTEMENT SUR MER
LA HAUTEUR DES ASTRES.

PIECE QUI A REMPORTE' LE PRIX
proposé par l'Academie Royale des Sciences
pour l'année 1729.

Par Monſieur BOUGUER, *Profeſſeur Royal en Hydrographie au Croiſic, & Membre de l'Academie Royale de Bordeaux.*

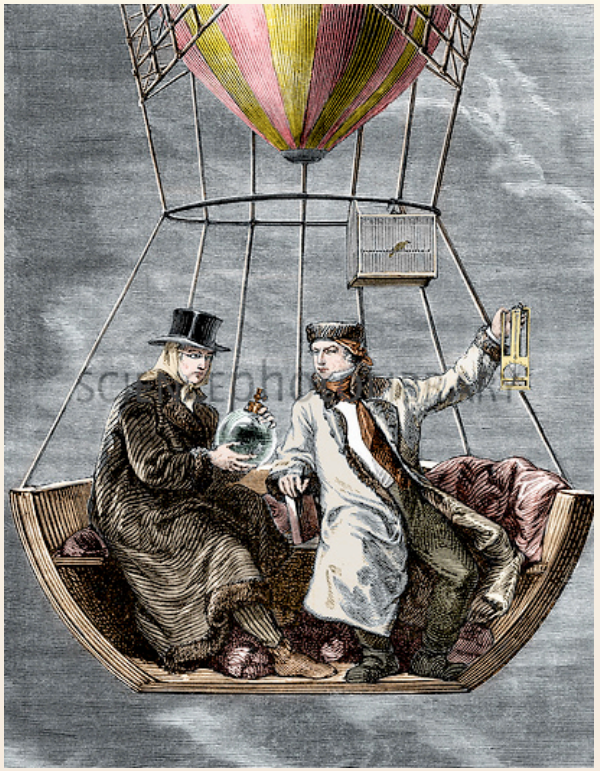
A PARIS, RUE S. JACQUES,
Chez CLAUDE JOMBERT, au coin de la ruë des Mathurins, à l'Image Notre-Dame.
M. DCC. XXIX.
Avec Approbation & Privilege du Roy.



Gay-Lussac and Biot

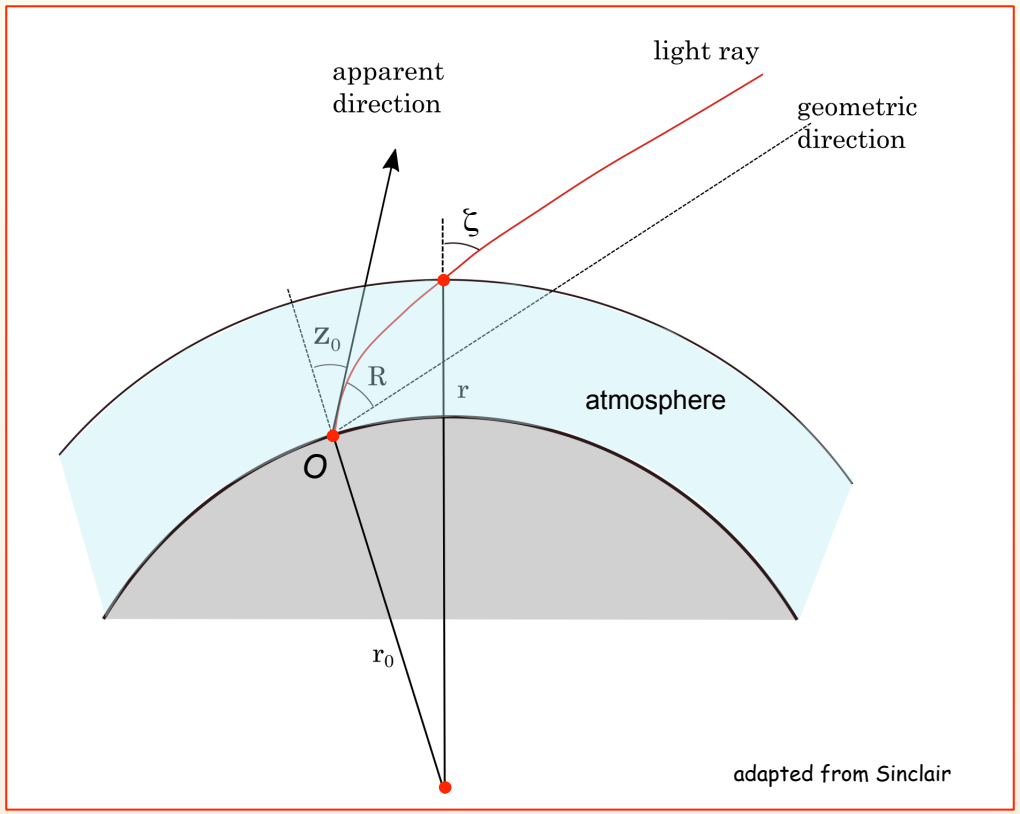
- First balloon ascent specifically for scientific purposes
  - main gaol : variation of magnetism with altitude
  - but measurements of pressure, humidity, temperature in upper atmosphere
- Gay-Lussac reached 7016m on 16 Sep 1804 (temperature = -9°C)
  - he noticed the regular decrease of temperature with altitude


J. Gay-Lussac 1778-1850



Refraction

- Mathematically well defined problem



$$R = \int_1^{n_0} \frac{\tan\zeta}{n} dn = \int_\infty^{r_0} \frac{\tan\zeta}{n} \frac{dn}{dr} dr$$

$$R = -\int_0^{z_0} \frac{r\frac{dn}{dr}}{n + r\frac{dn}{dr}} d\zeta$$

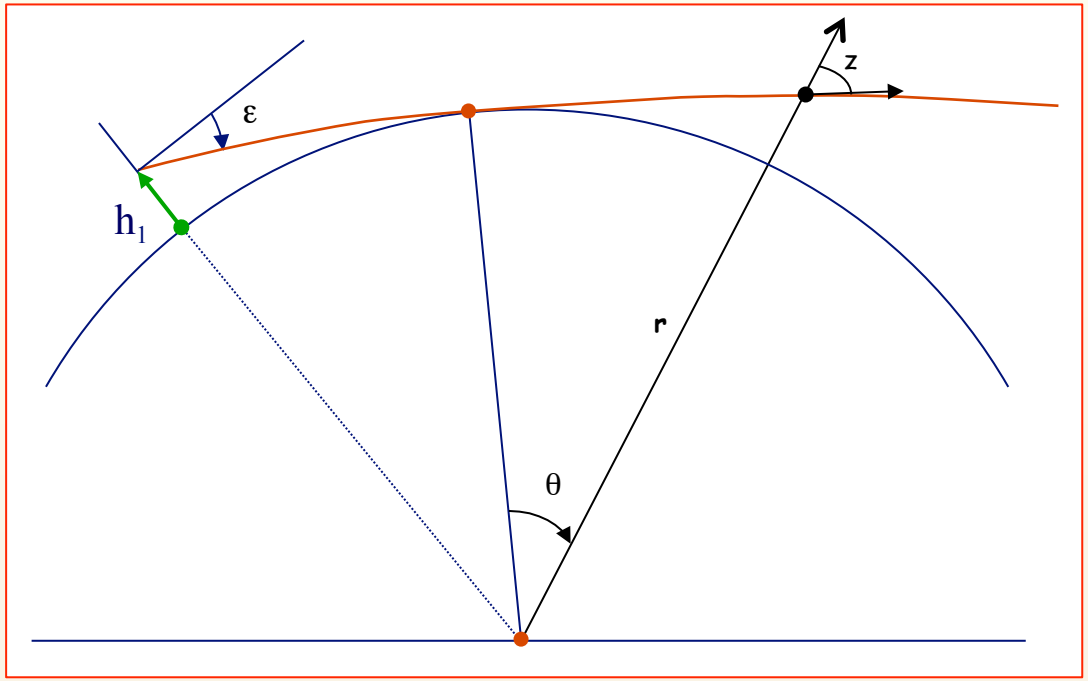
Auer & Standish, 2000

$$nr\sin\zeta = cte$$

$$R(z_0),\ z_0 \in 0, \frac{\pi}{2}$$

- Physically one needs the law $n(r)$
  - atmospheric model
  - several free parameters

Not exactly my problem



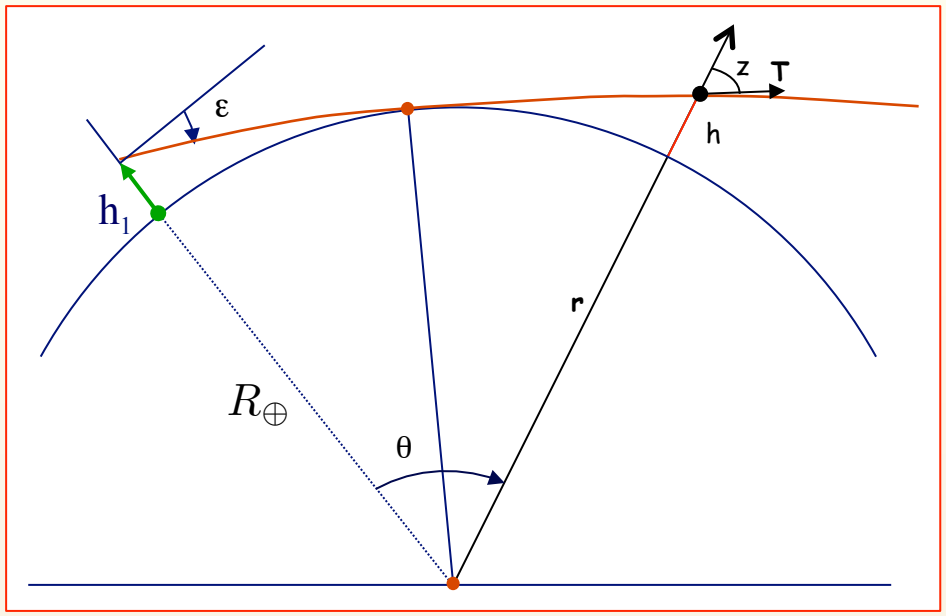
$$z_0 > \pi/2$$

$$nr\sin z = cte = C$$

$$R(\theta) = \eta \int_{r_0}^{r} \frac{dr}{r\left(\frac{n^2 r^2}{C^2} - 1\right)^{1/2}}$$

- One needs to integrate on two branches (η = +/- 1)
  - but one can use the symmetry with respect to perigee to go round
- The initial direction and the contact point are not given
  - the solution for the grazing ray is closer to a boundary conditions problem
- More generally I am interested also by the trajectory

Back to basics



- General equation

$$\frac{d(n\mathbf{T})}{ds} = \nabla n(\mathbf{r})$$

- Spherical symmetry

$$\frac{d(n\mathbf{r} \times \mathbf{T})}{ds} = 0$$

$$nr \sin z = cte = C$$

- Regular ODE for all θ

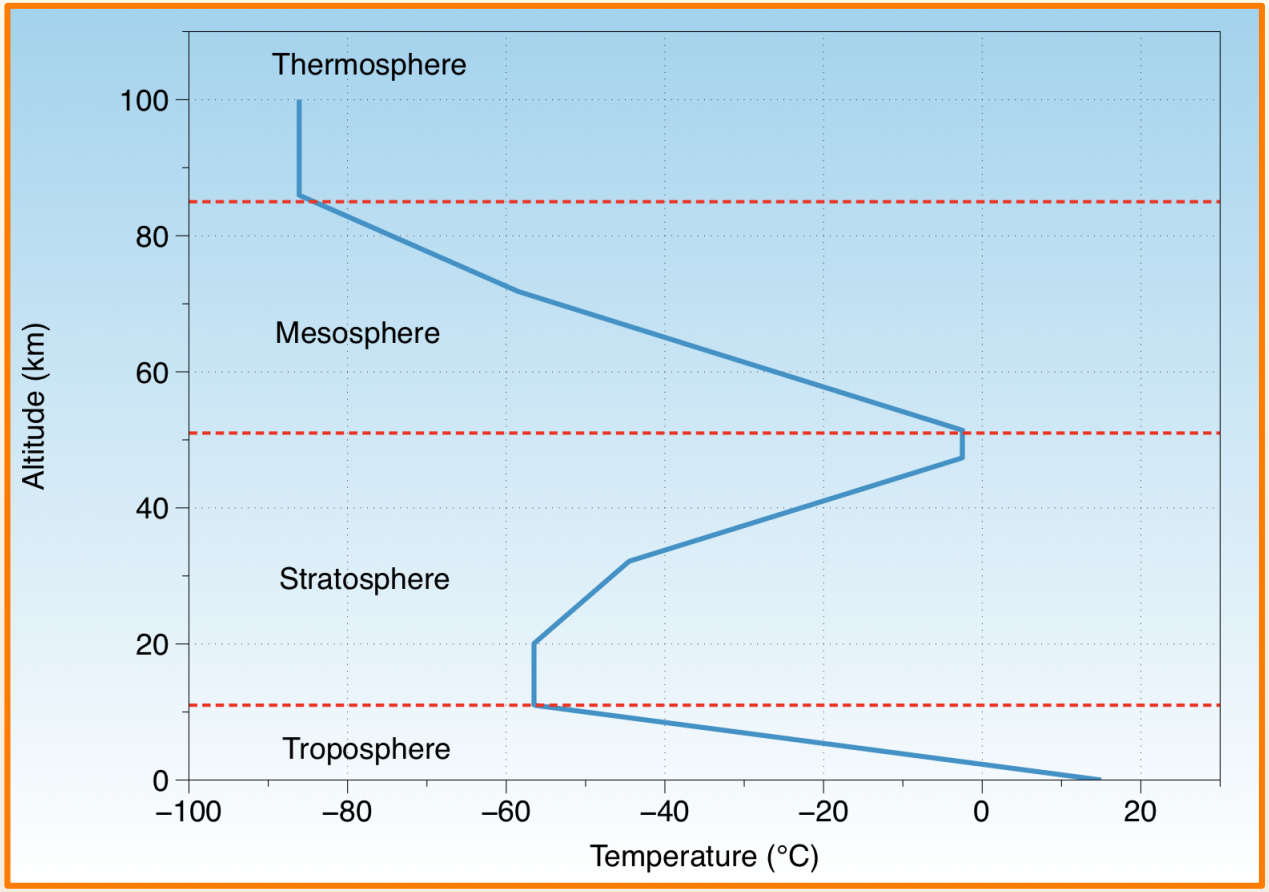
$$\frac{d^2 r}{d\theta^2} = r\left[\frac{2n^2\, r^2}{C^2} + \frac{n\, r^3}{C^2}\frac{dn}{dr} - 1\right]$$

$$r = R_\oplus + h$$

$$\boxed{\frac{d^2 h}{d\theta^2} = (R_\oplus + h)\left[\frac{2n^2\,(R_\oplus + h)^2}{C^2} + \frac{n\,(R_\oplus + h)^3}{C^2}\frac{dn}{dh} - 1\right]}$$

Atmospheric model

- Baseline is the International Standard Atmosphere
  - a multi-layers atmosphere with dT/dh = 0 or cte
  - but P(h=0), T(h=0) can be different from standard conditions
  - includes a dry and a humid component



Atmospheric model: humidity

- Serious complications for little at the end
  - only significant in the troposphere
  - vapour content decreases quickly with altitude
- The model includes a dry and a humid component in the troposphere
  - mixture of perfect gas
  - water vapour pressure determined by temperature

$$P = P_\mathrm{d} + P_\mathrm{w} \qquad \rho = \rho_\mathrm{d} + \rho_\mathrm{w} \qquad T = T_0 - \gamma h \qquad \gamma \simeq 6.5\mathrm{K/km}$$

$$P_\mathrm{d} = \frac{\rho_\mathrm{d} RT}{M_\mathrm{d}} \qquad P_\mathrm{w} = \frac{\rho_\mathrm{w} RT}{M_\mathrm{w}} \qquad M_\mathrm{d} = 28.963\ \mathrm{g/mol} \quad M_\mathrm{w} = 18.015\ \mathrm{g/mol}$$

$$\frac{dP}{dh} = -\rho g = -(\rho_\mathrm{d} + \rho_\mathrm{w}) g$$

$$\boxed{\frac{dP}{dT} = \beta \frac{P}{T} - \beta \left[1 - \frac{M_\mathrm{w}}{M_\mathrm{d}}\right] \frac{P_\mathrm{w}}{T}}$$

$$\beta = \frac{M_\mathrm{d} g}{\gamma R} \simeq 5.3$$

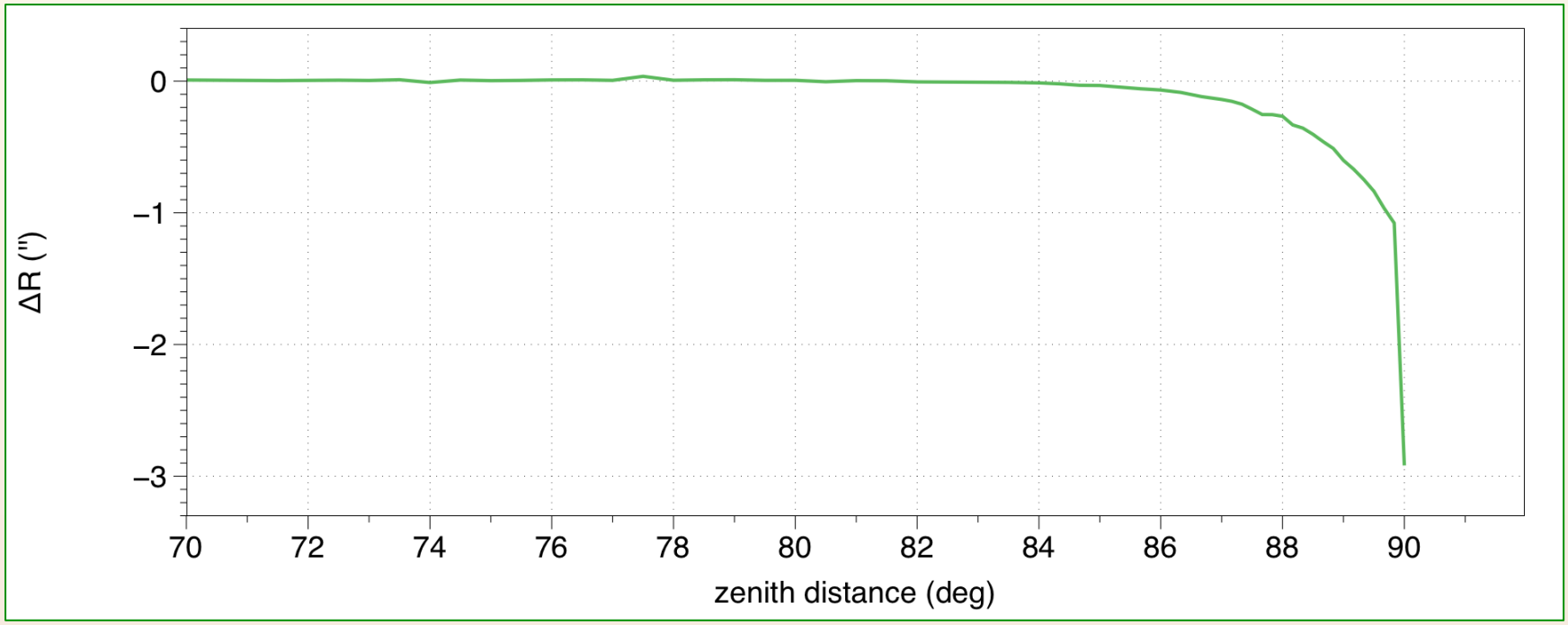
$$P_\mathrm{w} = f P_\mathrm{ws}(T)$$

Air refractive index

- Model from Ciddor (Applied Optics, 35, 9, 1996)
  - Designed for metrological purposes in laboratory conditions
    - much too accurate given the uncertainty in the atmospheric conditions
  - Determined by temperature, pressure and water vapour content
    - computed from humidity and saturation vapour pressure at T
    - humidity is kept constant in the troposphere
  - Depends also on the wavelength
    - smaller λ, larger n
- Discontinuities
  - n(h) is a continuous function
  - dn/dh has discontinuities at atmospheric layer boundaries
    - care needed in the numerical integration

Validation of refraction model

- Comparison to the table in "Introduction aux Ephémérides"
  - P = 1013.25 hPa , t = 15 °C
  - humidity = 0
  - altitude = 0
- The plot gives ΔR(z) = My_value - Ref_value in arcsec
  - R(85) ~ 10' ~ 600 " R(90) ~ 33' ~ 2000 "



Model sensitivity

- Change of refraction (R) in horizon for a change Δ in physical parameter

Parameter	Δ	δR1	δR2	δt (rise/set)
		h = 5m	h= 500m	
T	1°C	-11 "	-15"	1.5 - 2s
P	10 hPa	+20''	+27''	3s
h	10m	50''(*)	6'' (*)	6s – 1s
λ	0.1 μm	-9''	-12''	1s
humidity	50%	-5 ''	-8''	< 1s
dT/dh(**)	0.5 °C/km	15''	24''	3 s

* Refraction to optical horizon, but the change of dip is much larger

** Nominal value is -6.5 °C/km

Refraction and Dip

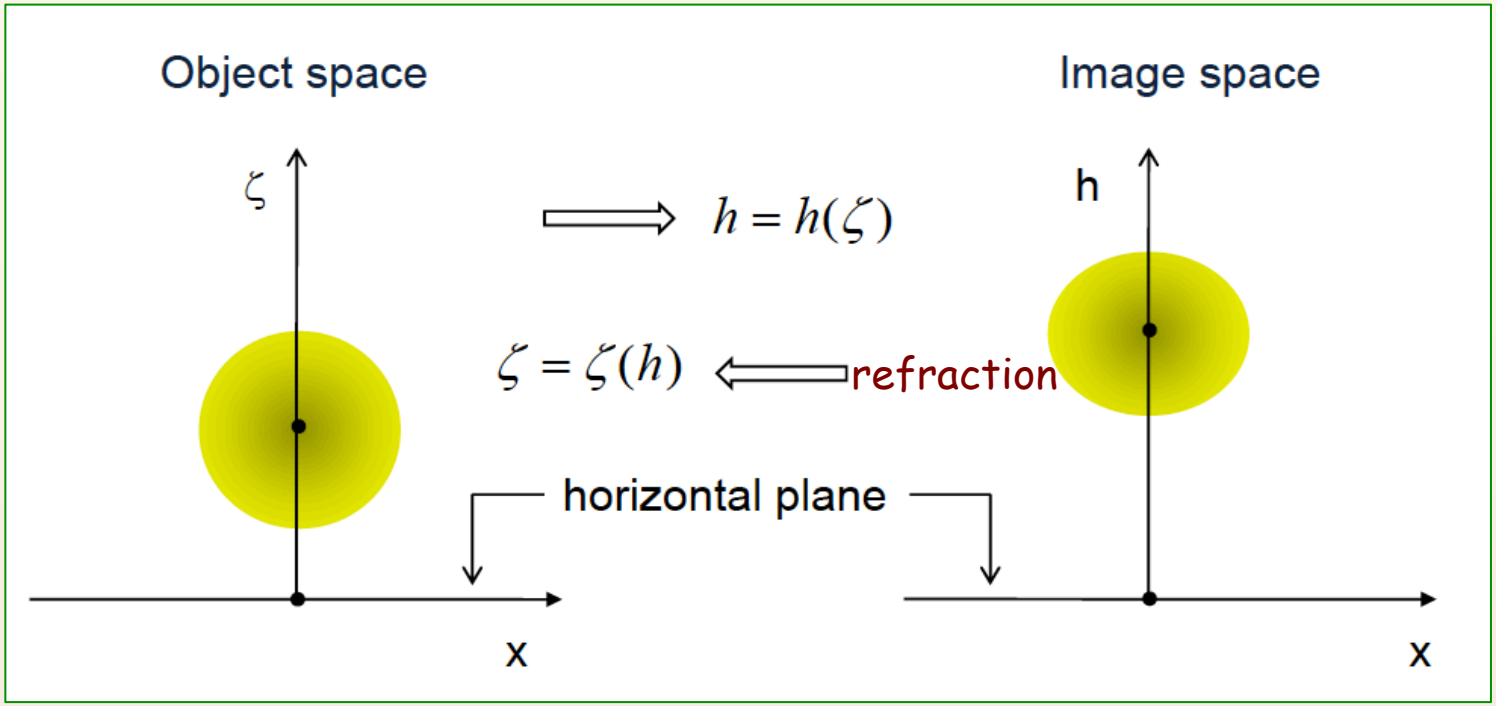
- Horizontal refraction, refraction in horizon, dip

Altitude	R(90°)	R(horizon)	Dip
m	arcmin	arcmin	arcmin
0	32.9	32.9	0
50	32.8	35.4	12.4
100	32.6	36.5	17.5
200	32.3	37.9	24.8
500	31.4	40.8	39.3

Standard temperature and pressure, no humidity, λ = 0.590 μm

Image of extended sources

- The atmosphere seen as an optical system
  - the refraction is a single value function
  - it gives explicitly $\zeta(h)$
  - the inverse function $h(\zeta)$ is needed to map sources in the image space
    - it is not necessarily single valued (multiple images)
    - it is not explicit ➔ non linear equation or inverse interpolation

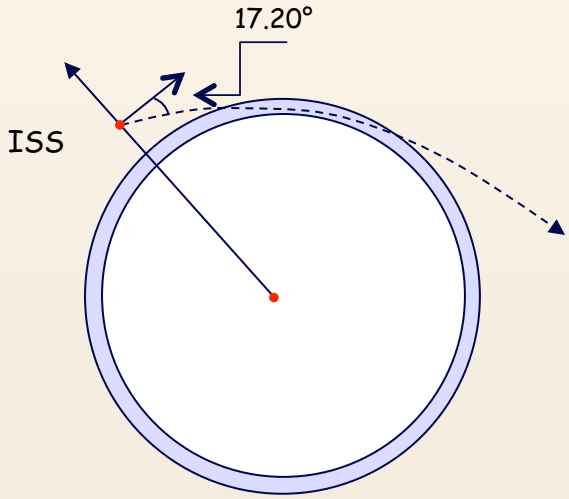


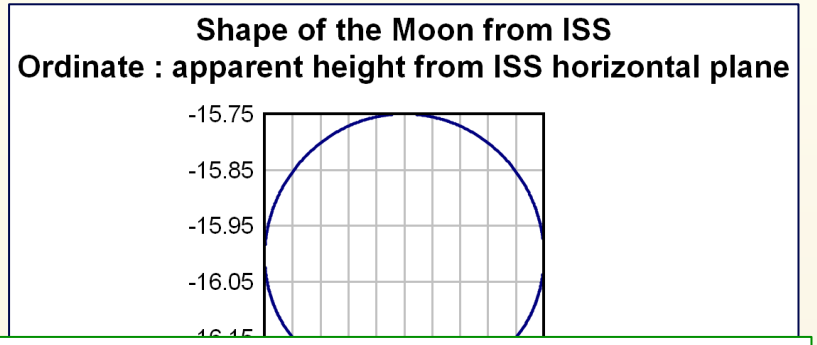
A side application of the model

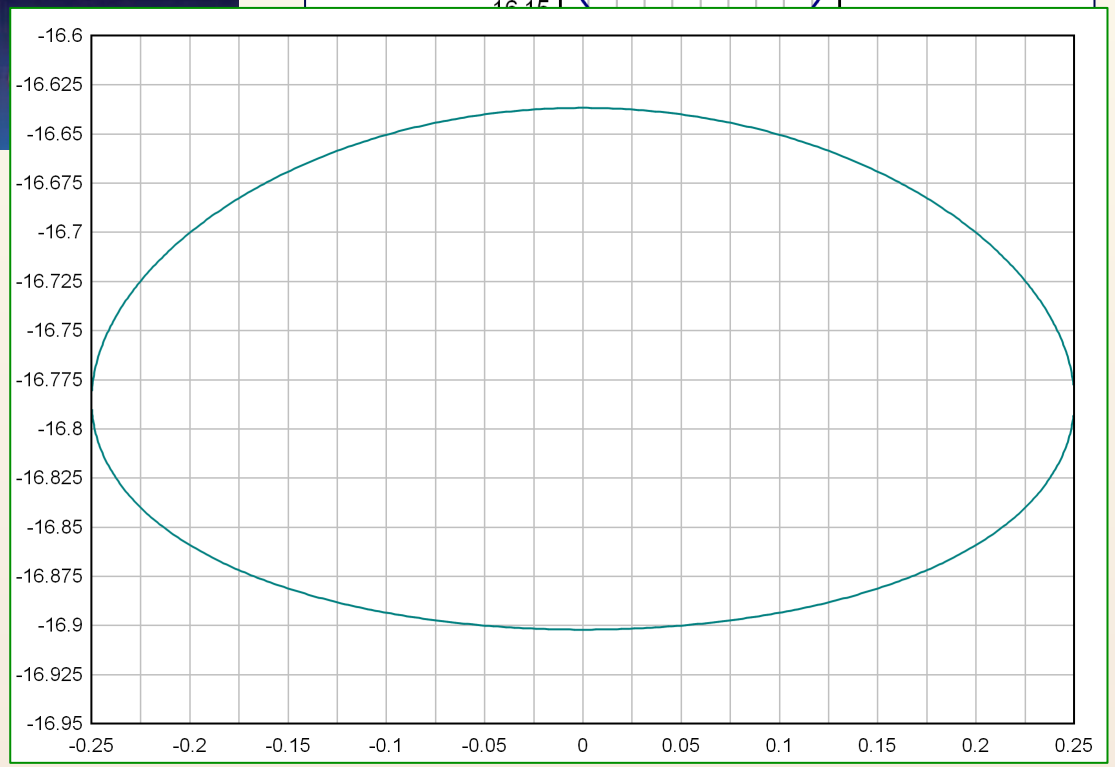
- Moon vanishing behind the Earth from ISS



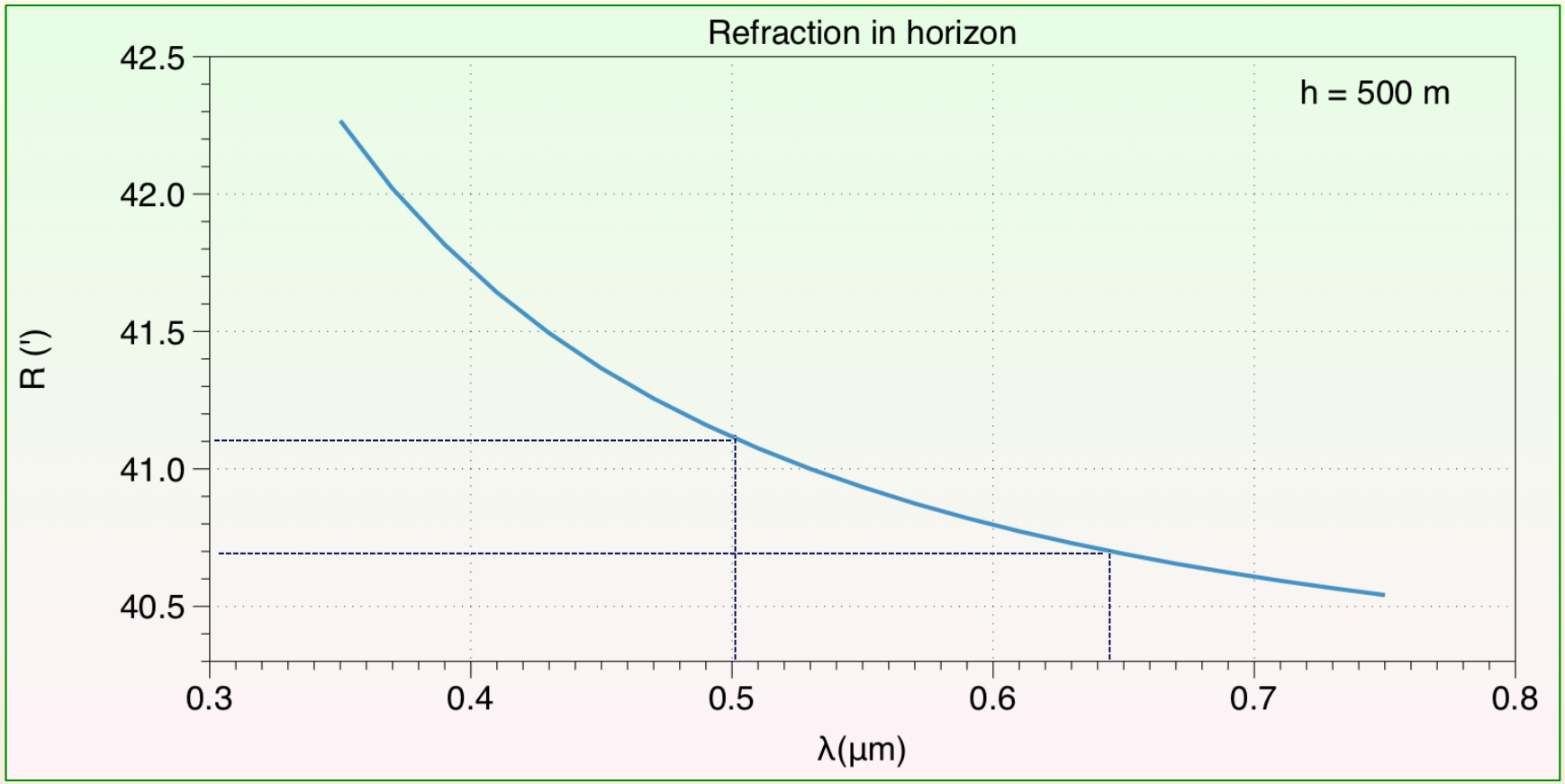
Credit : D. Petit, April 2003







Effect of wavelength



blue red

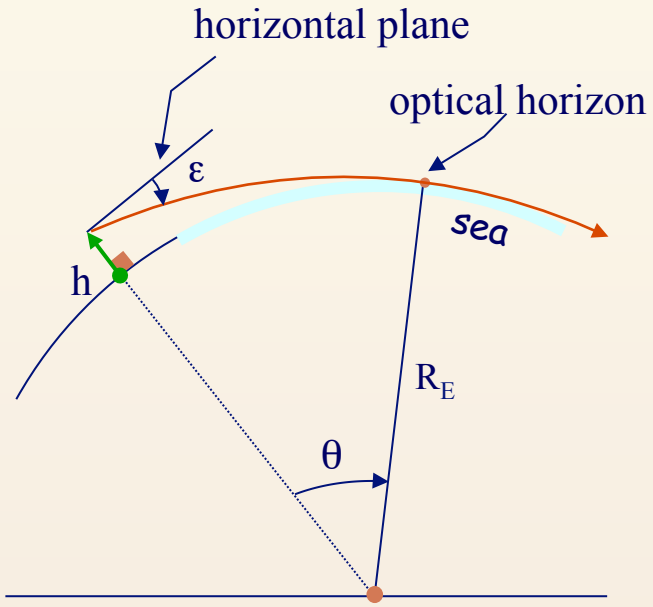
Green Flash



Time of Rise and Set

- Observer at height *h*
- Compute the horizon dip ε and the refraction R on the grazing ray
  - functions of *h*, *T*, *P*, RH, λ
- Add (or remove) solar radius and add equatorial parallax (~ 9'')

$$h_{\odot}^{\mathrm{r,s}} = -\epsilon(h) - R(h) \pm s + \pi_{\odot}$$



- Then root(s) in time of the equation:

$$\cos H = \frac{\sin h_{\odot} - \sin\phi \sin\delta}{\cos\phi \cos\delta}$$

- Solve with Newton-Raphson iterations
  - manage the 'no root' cases or more than 2 roots in 24h

Horizon dip in practice

- From altitude h, one follows the grazing light ray
- General refraction model
  - solve an ODE with a boundary condition: lowest ray must graze the sea level

$\epsilon \sim \sqrt{\frac{2\beta h}{R_\oplus}}$ is a very good starting value

- But an exact solution is obtained with the first integral

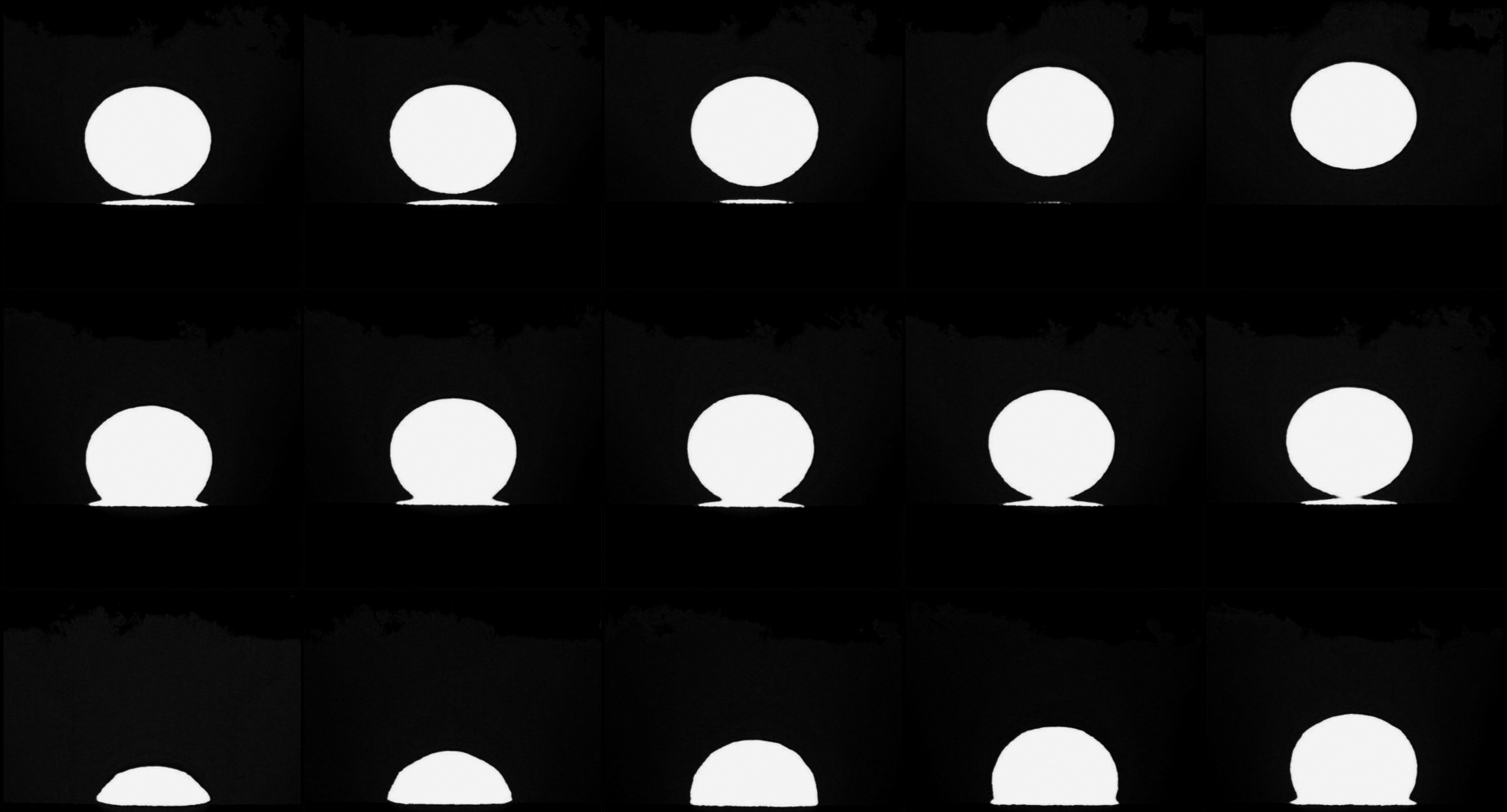
$$(R_\oplus + h)n(h)\cos(\epsilon) = R_\oplus n_0$$

  - with $n(h) \sim n_0 + \frac{dn}{dh}h$ and $\beta = \beta = 1 - \mathrm{R}_\oplus \frac{\mathrm{dn}}{\mathrm{dh}}$

    the two methods are identical for small h

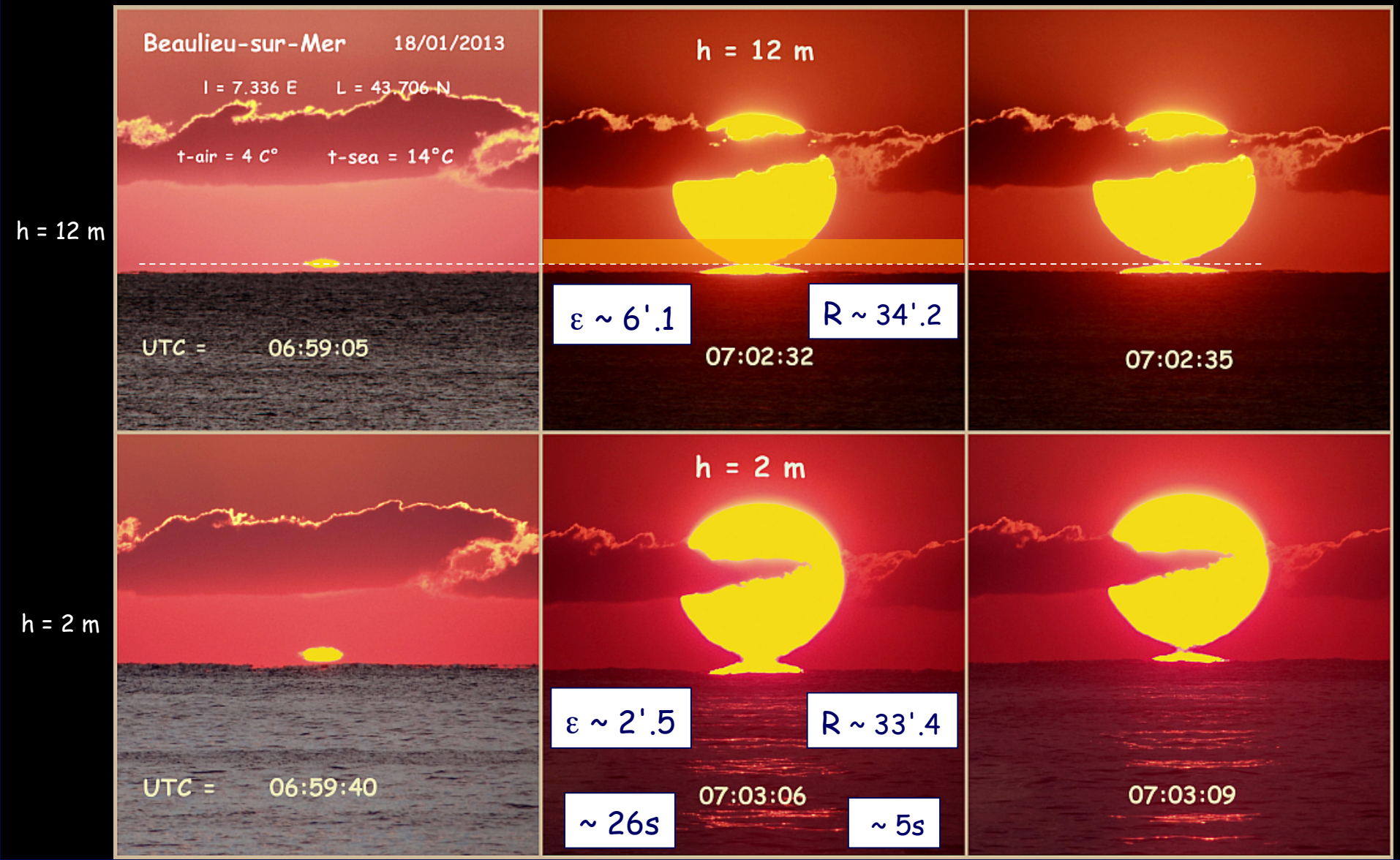
  - now an ODE with initial conditions

How does it fit in ?



Dip : a spectacular consequence

- Sunrise from two points at altitude ~ 2m and ~ 12 m
  - same place, same day, two cameras



Time of sunrise

- In principle take the picture at the right moment
- Match your camera clock to UTC





Time of sunrise/sunset

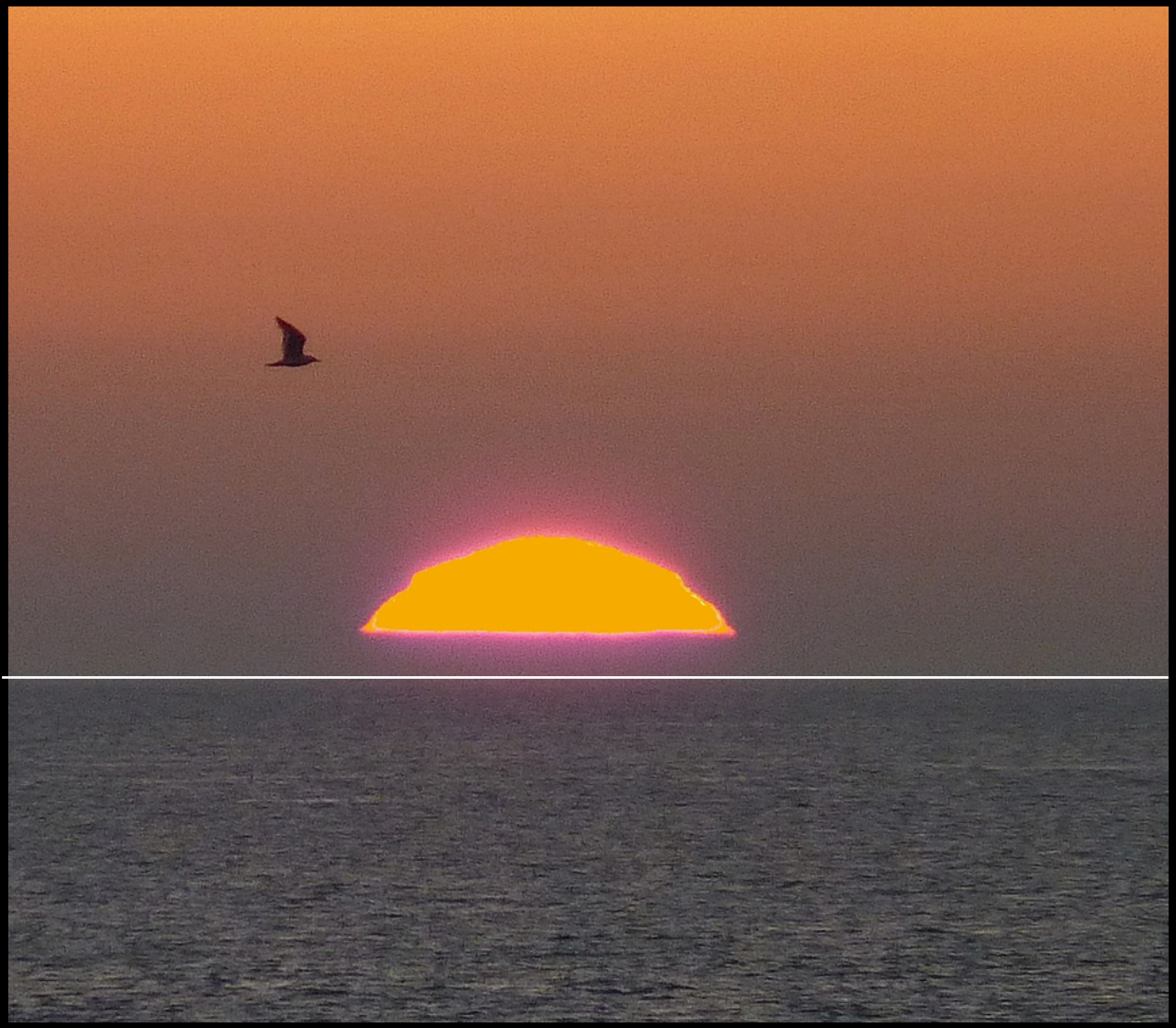
But this is rarely like that…



still 2-3s before sunset…

Time of sunrise

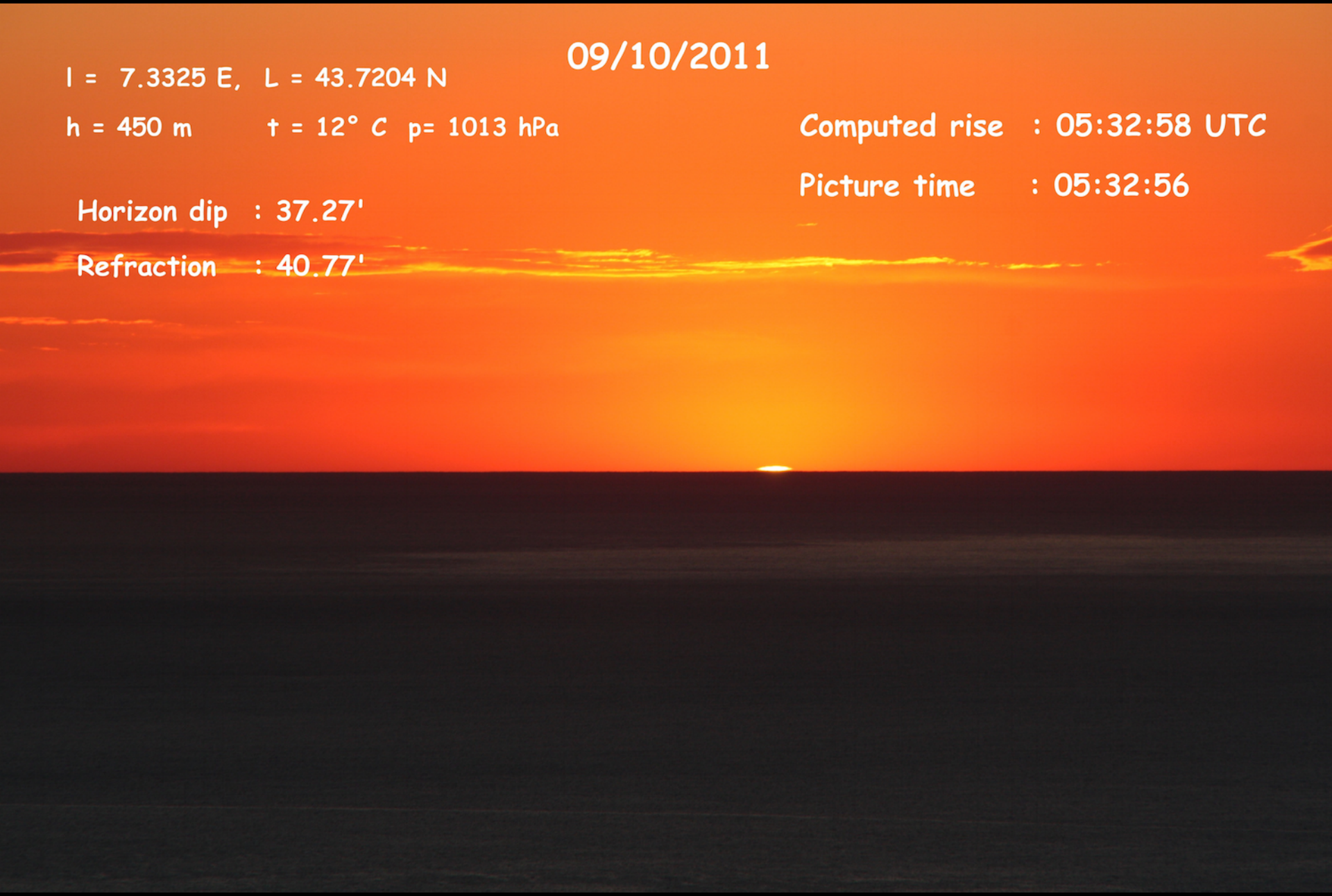
…. and it could be very bad as well



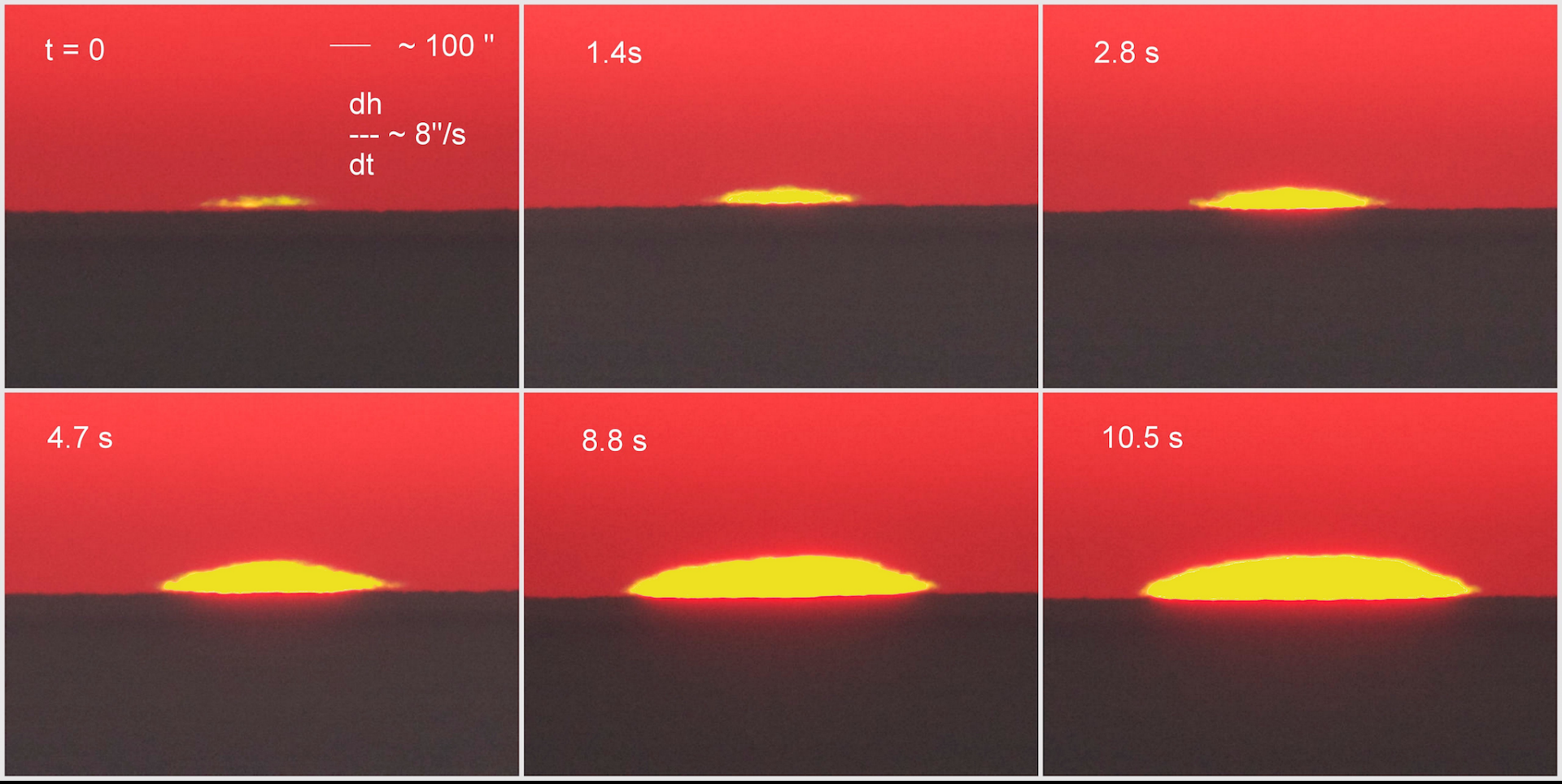
Time of sunrise



Time of sunrise



Time of sunrise

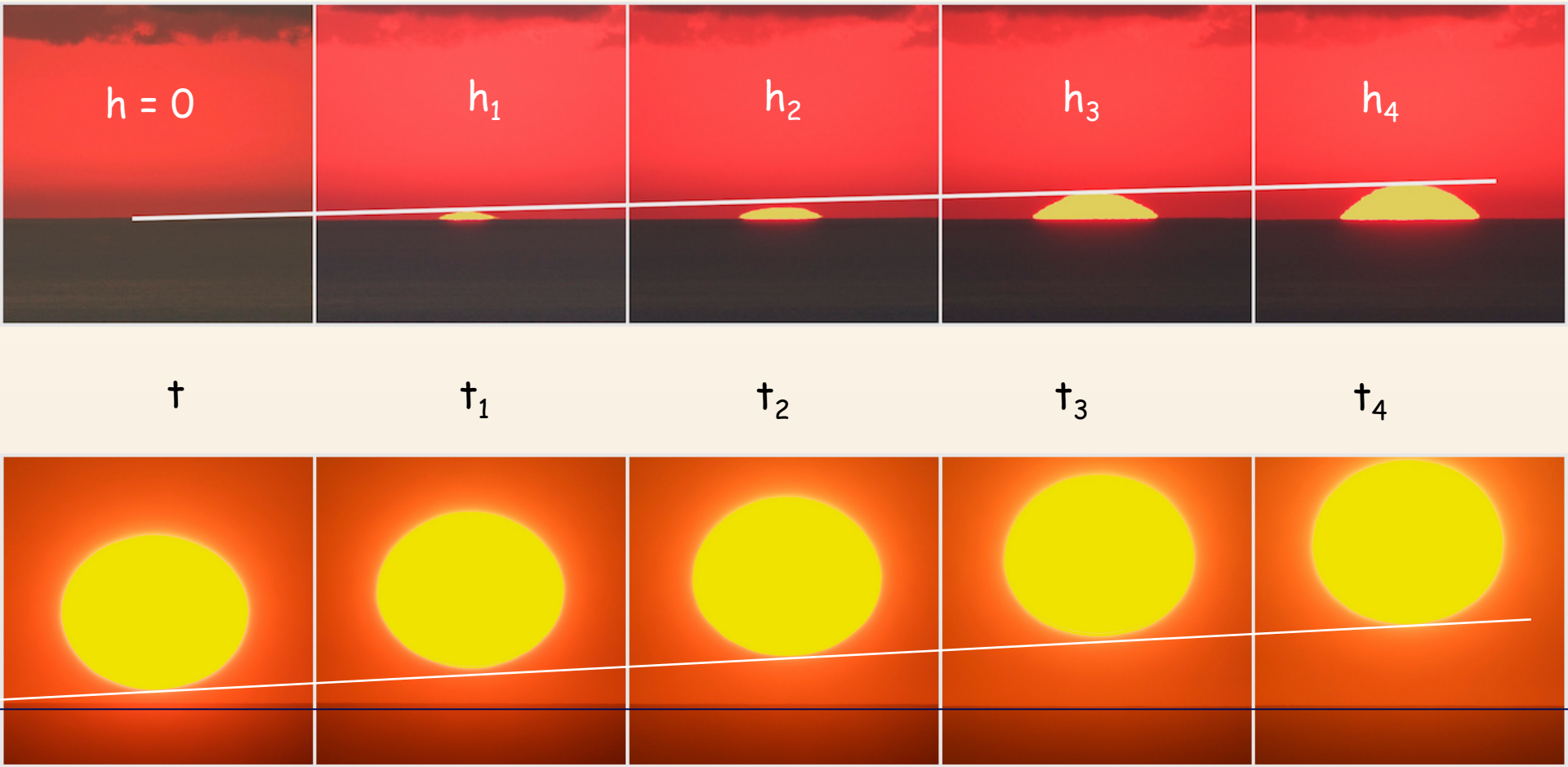


Which is the correct event ?

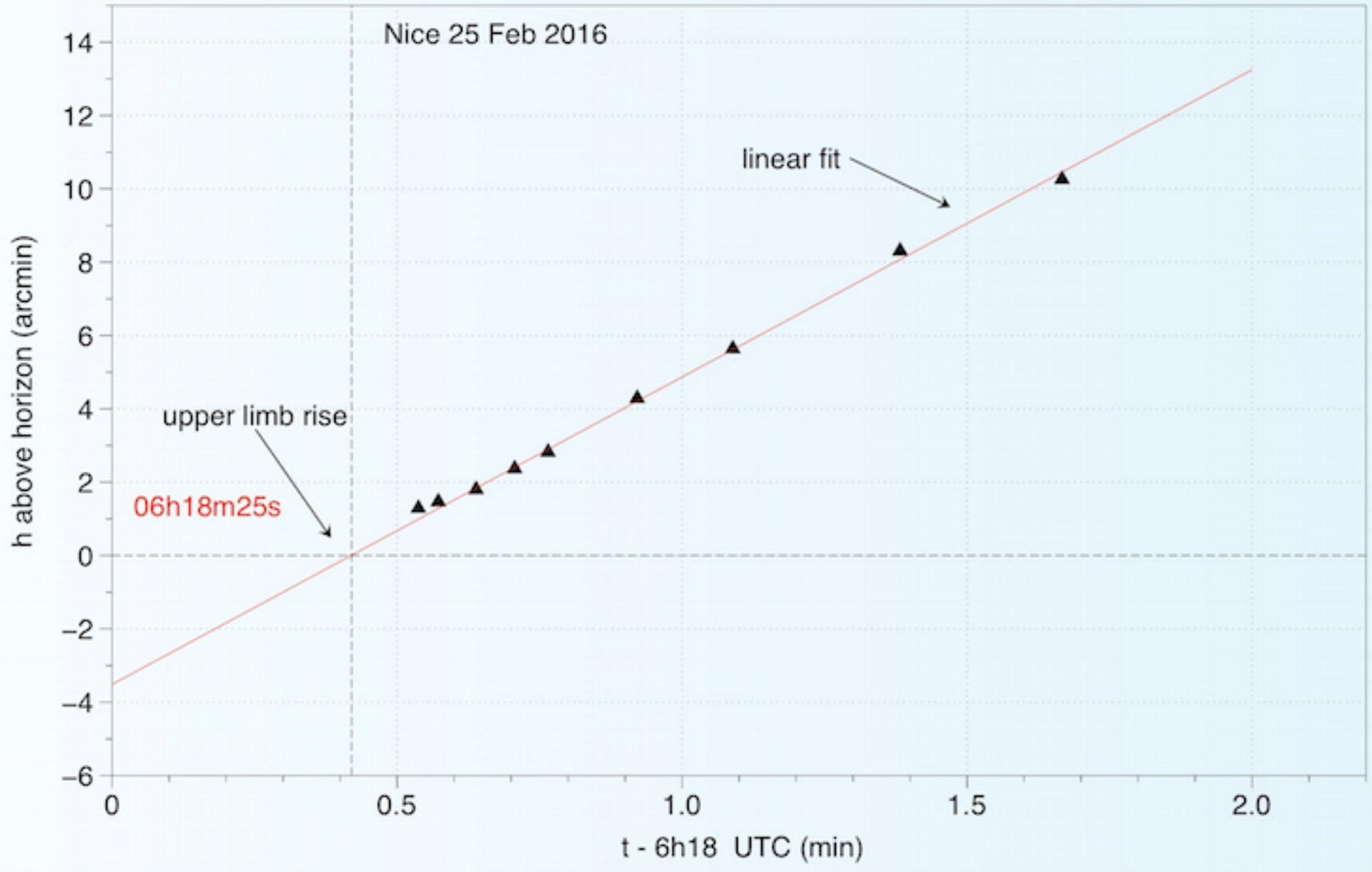


Time of sunrise

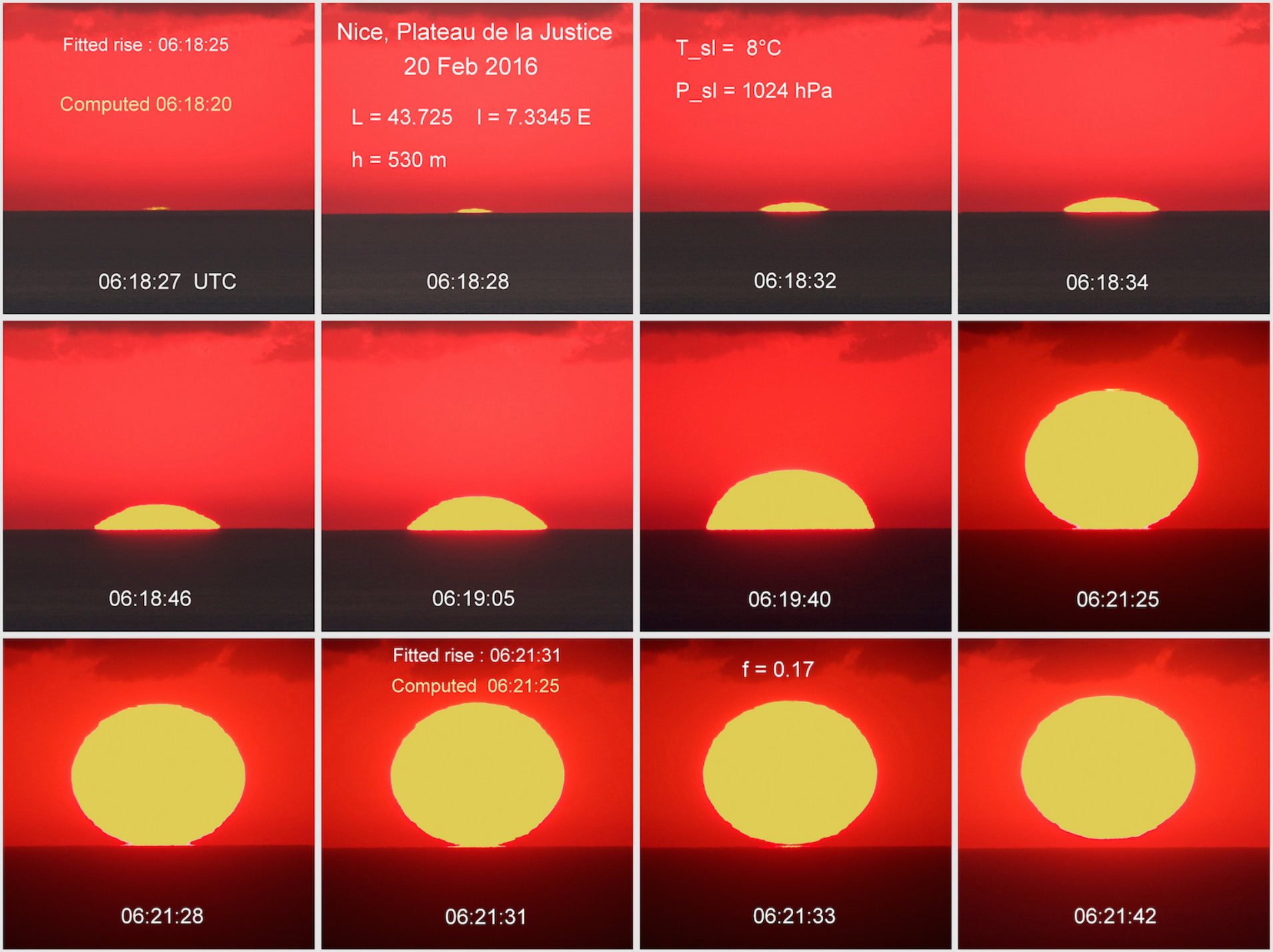
- Much better to take a sequence
- Linear fit between sun altitude and time in UTC
- Extrapolate for h = 0



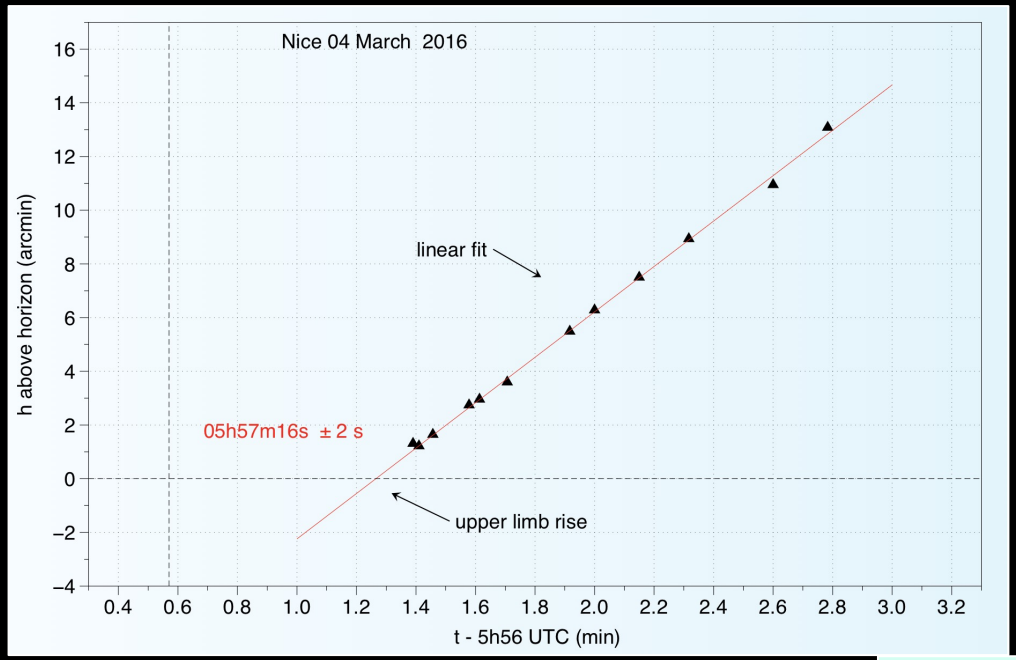
Time of sunrise

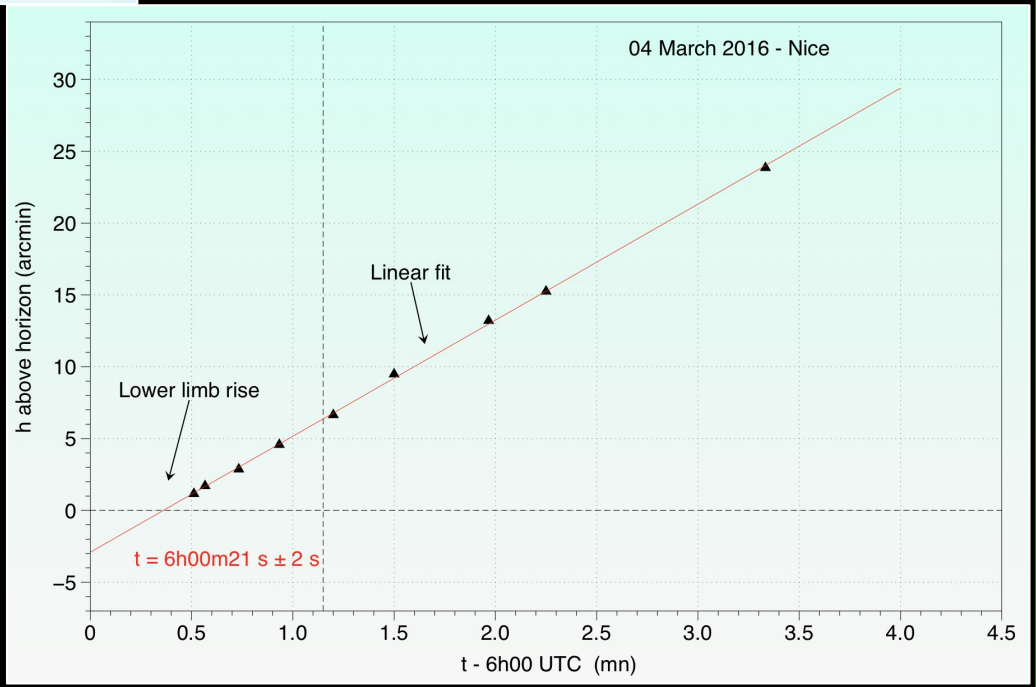



Upper limb rise
Estimated from images : 05:35:09.4
Computed 05:35:09
Shot time : 05:35:18.5 UTC
Nice, Plateau de la Justice
L = 43.7257
l = 7.3345 E
h = 530 m
t_sl = 16 °C
P_sl = 1006 hPa
11 October 2015
05:35:27.0
05:35:52.0
05:36:16.0
05:36:49.0
05:37:54.2
Lower limb rise
Estimated from images : 05:38:10.1
Computed : 05:38:08
05:38:07.0
05:38:10.0
05:38:13.6
05:38:15.5
05:38:17.7
05:38:22.5

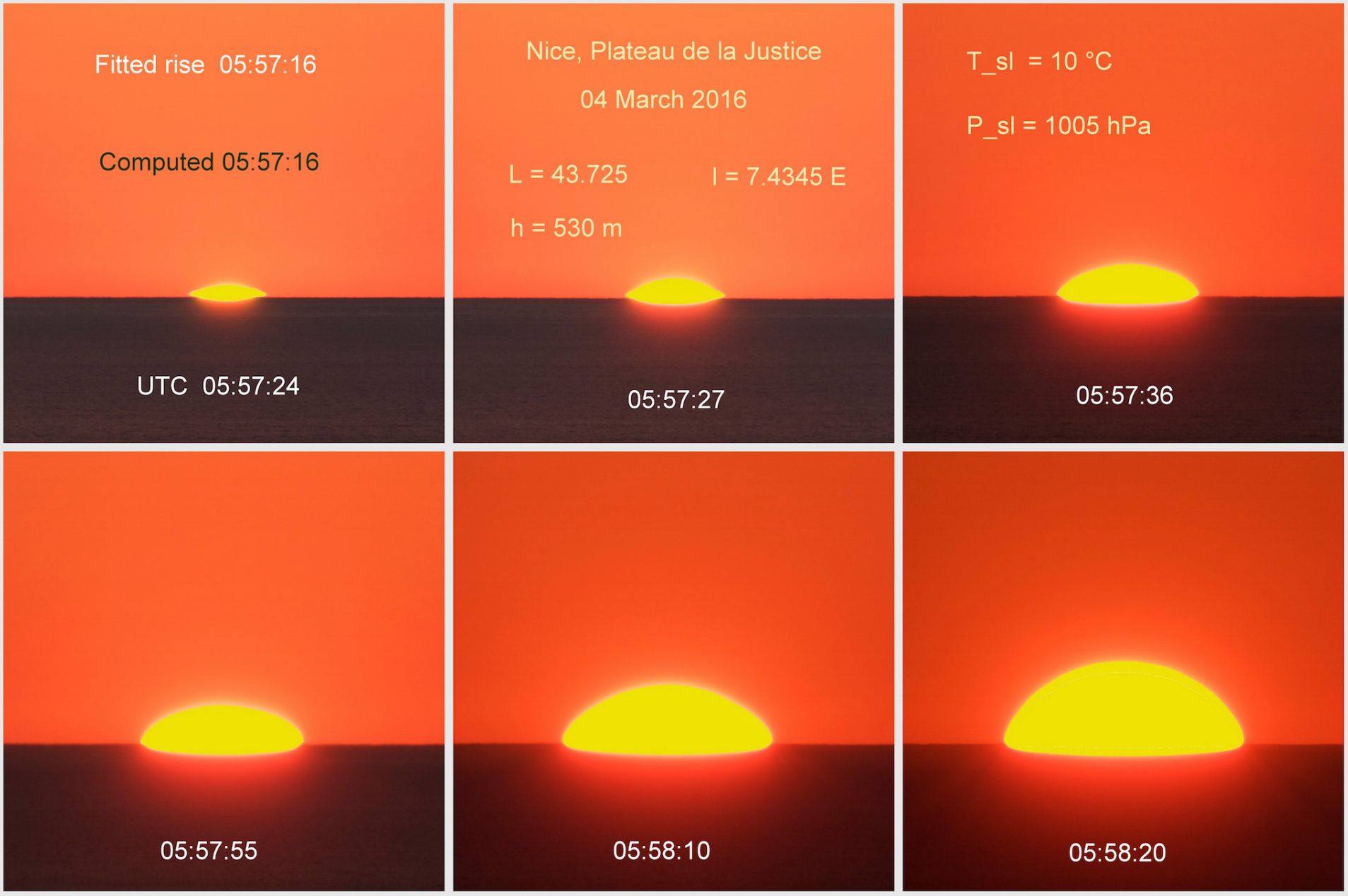

Fitted rise : 06:18:25
Computed 06:18:20
06:18:27 UTC
Nice, Plateau de la Justice
20 Feb 2016
L = 43.725 l = 7.3345 E
h = 530 m
06:18:28
T_sl = 8°C
P_sl = 1024 hPa
06:18:32
06:18:34
06:18:46
06:19:05
06:19:40
06:21:25
06:21:28
Fitted rise : 06:21:31
Computed 06:21:25
06:21:31
f = 0.17
06:21:33
06:21:42

Time of sunrise

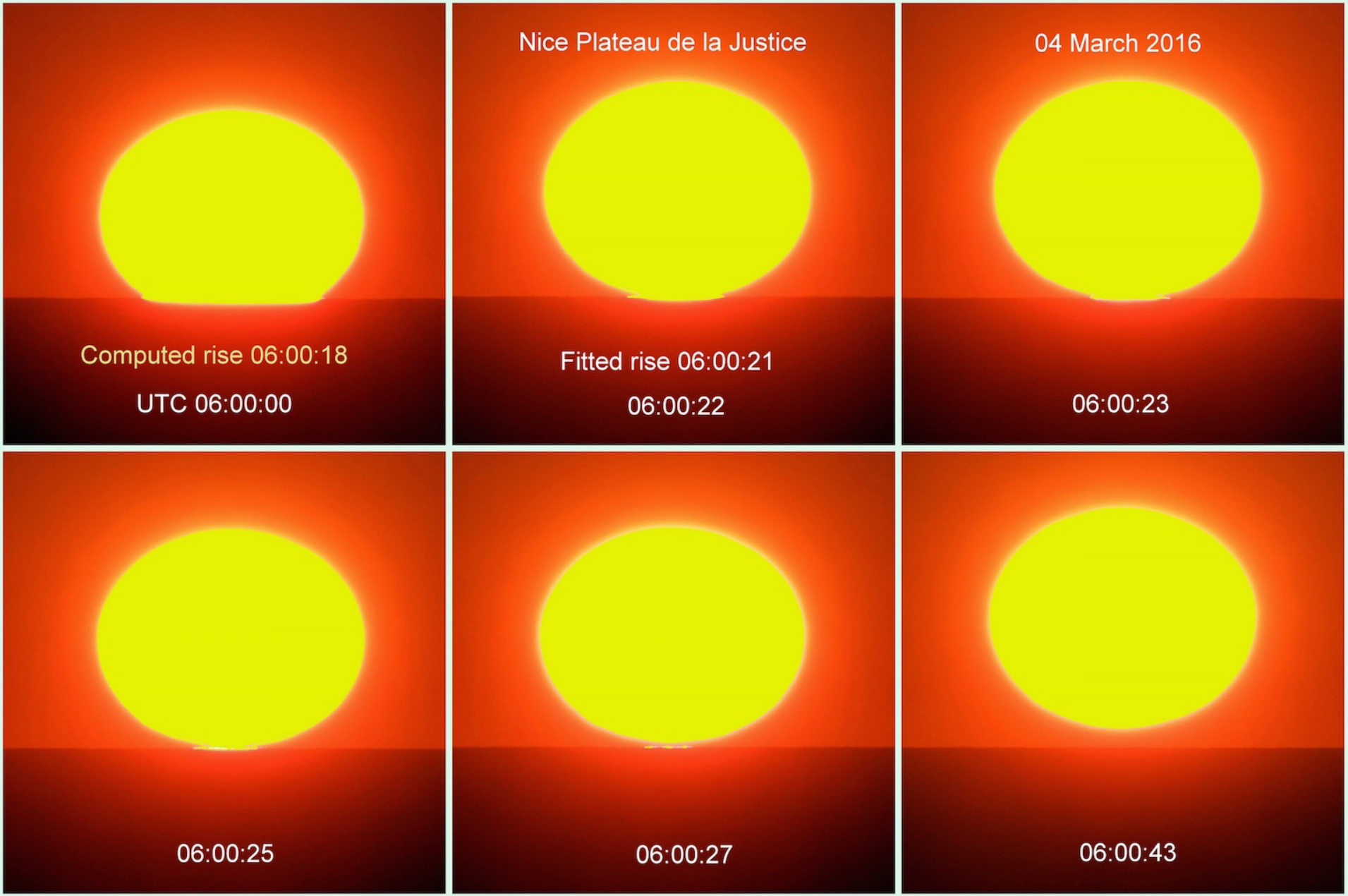




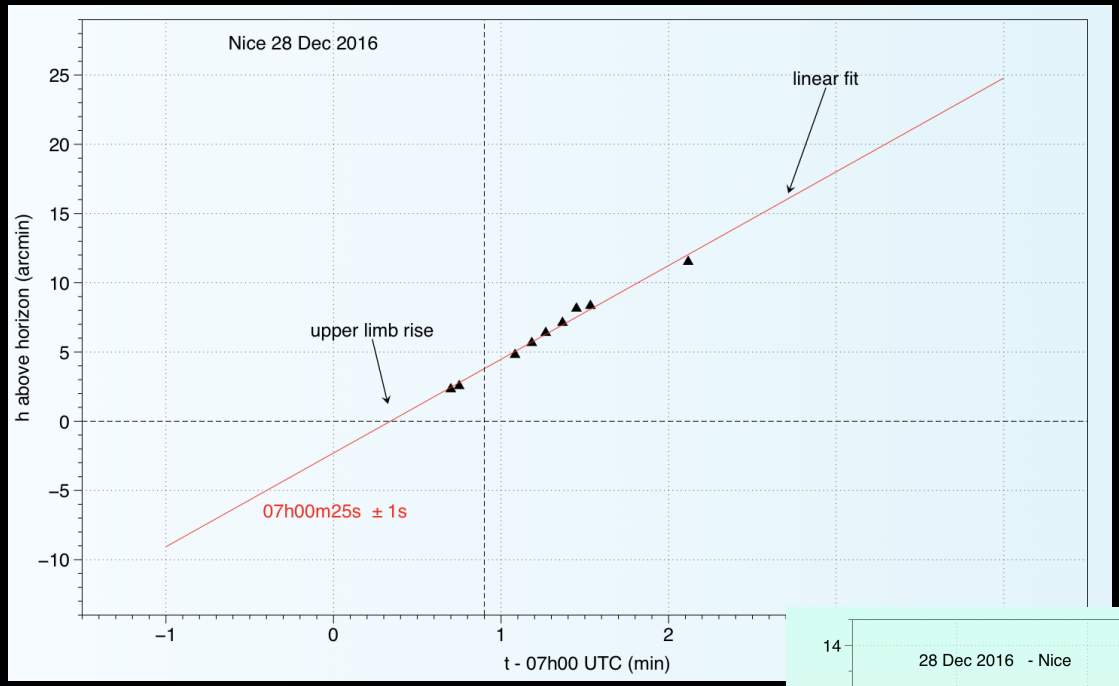
Time of sunrise

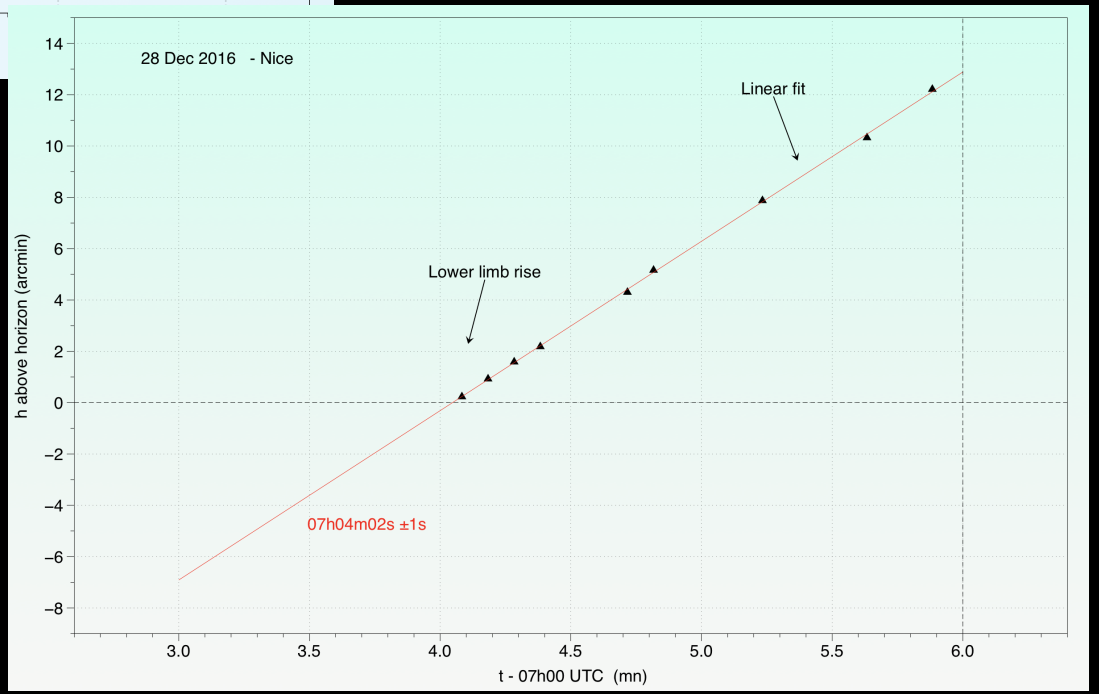


Time of sunrise



Time of sunrise

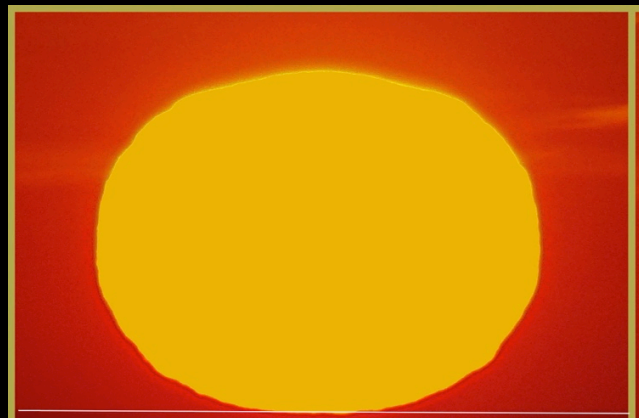




Time of sunrise



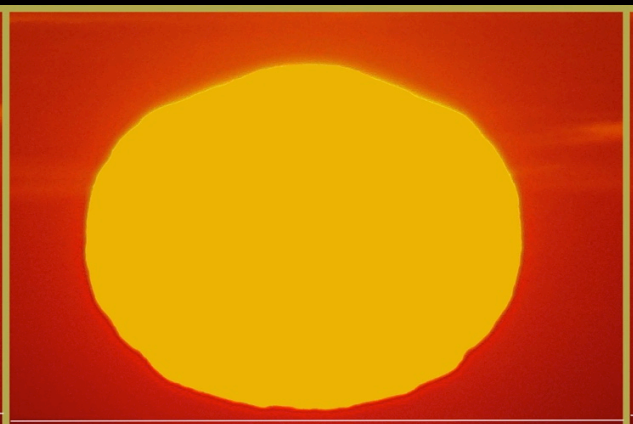
Time of sunrise



fitted rise : 07:04:02 ± 1 s

comp. rise : 07:03:59

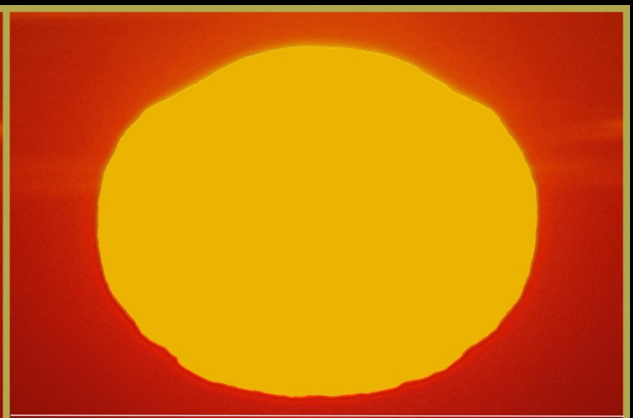
UTC 07:04:00



Nice Mont Alban 28/12/2016

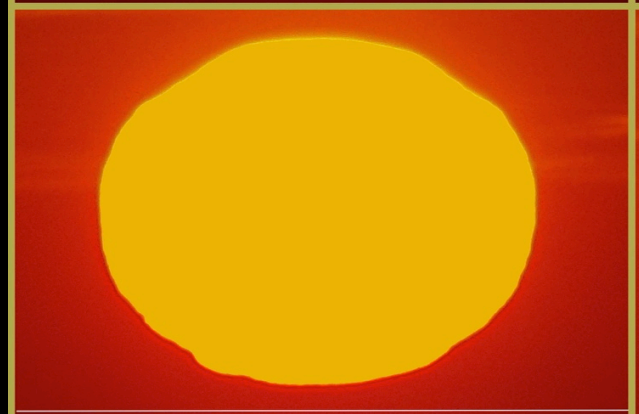
L : 43.701 l : 7.301E h : 218 m

07:04:05

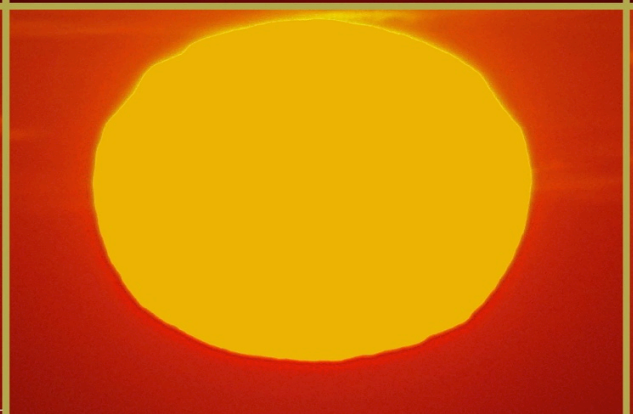


t_sl : 8.5 C P_sl : 1035 hPa RH : 0.5

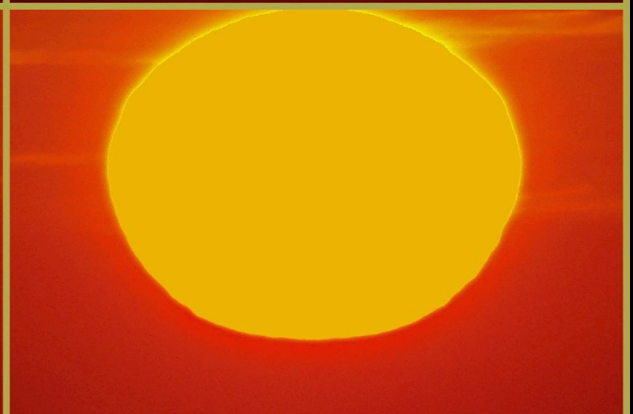
07:04:11



07:04:23



07:04:49



07:05:14

18 July 2015

lat : 42.224 N long : 8.554 E t_{air} = 26 °C h = 1m

Lower contact

time from pictures 18:55:12.2 UTC
computed time 18:55:08

Upper contact

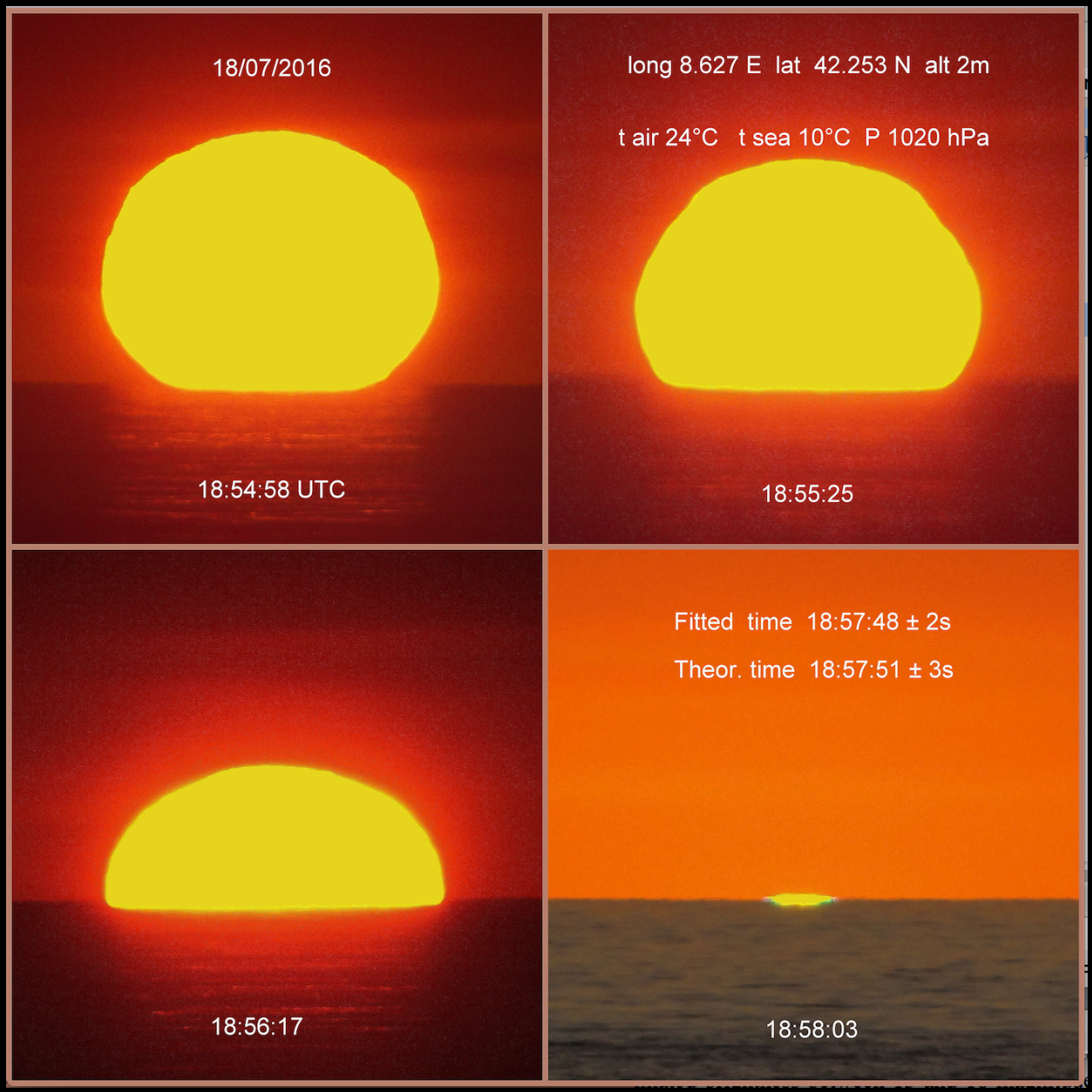
time from pictures 18:58:25.5 UTC
computed time 18:58:24



Time of sunset

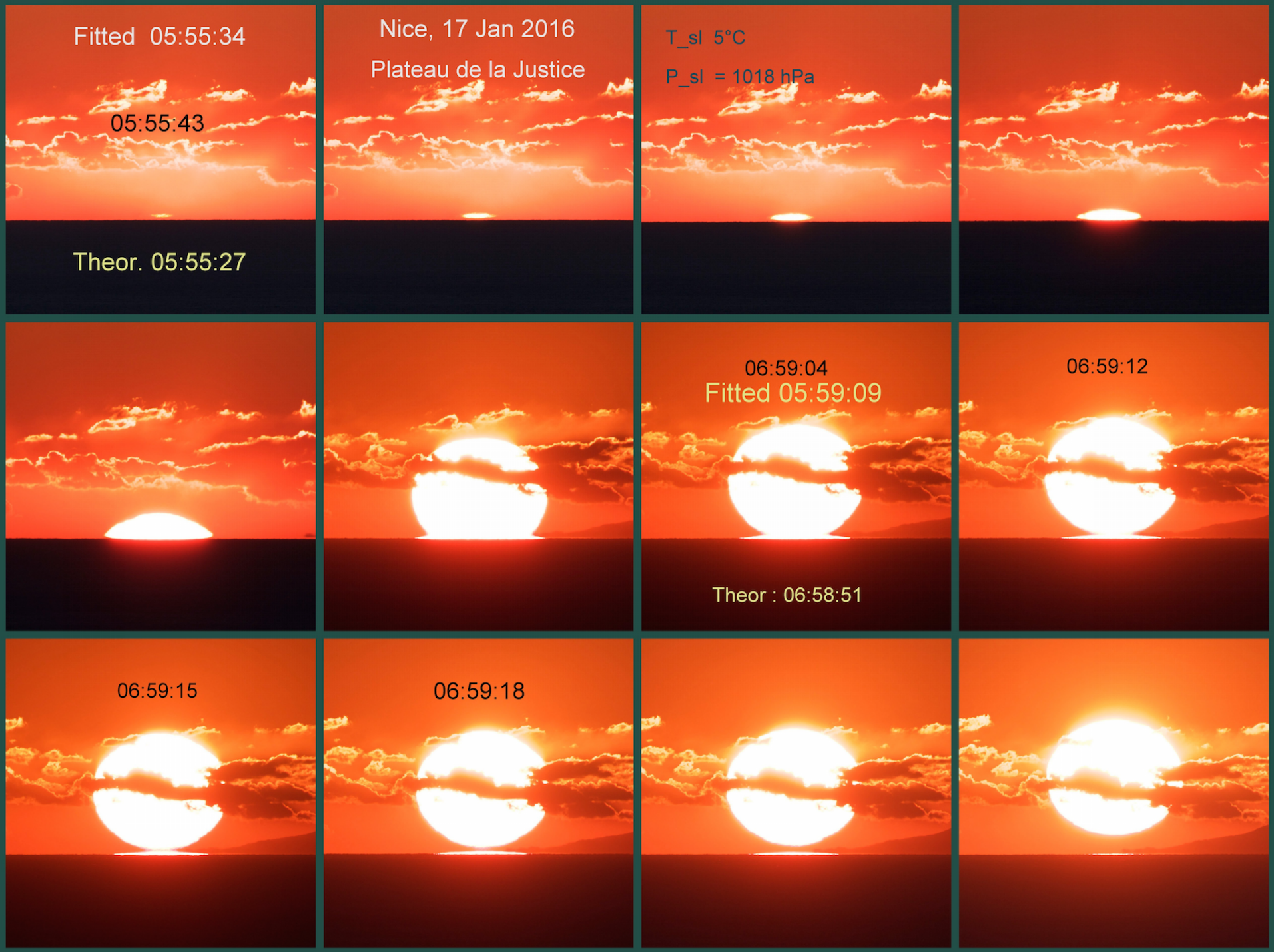


Time of sunset



When it does not work

- A generic case
  - lower layers of atmosphere do not follow the mean rule
- Typical winter situation
  - air is cold , < 5°C
  - sea temperature typically 13 °C
  - a layer of 20 to 100 m to reach the equilibrium temperature
  - Very large temperature gradient
    - same sign as usual, but just very large 100°/km instead of 6.5°/km
    - dn/dh changes of sign with dn/dh > 0 in this layer
      - air density increases with altitude
    - light-rays curvature is inversed
  - This produces an inferior mirage
    - the visible horizon is not the same as used in the computation


Fitted 05:55:34
05:55:43
Theor. 05:55:27
Nice, 17 Jan 2016
Plateau de la Justice
T_sl 5°C
P_sl = 1018 hPa
06:59:04
Fitted 05:59:09
Theor : 06:58:51
06:59:12
06:59:15
06:59:18

First light with mirage

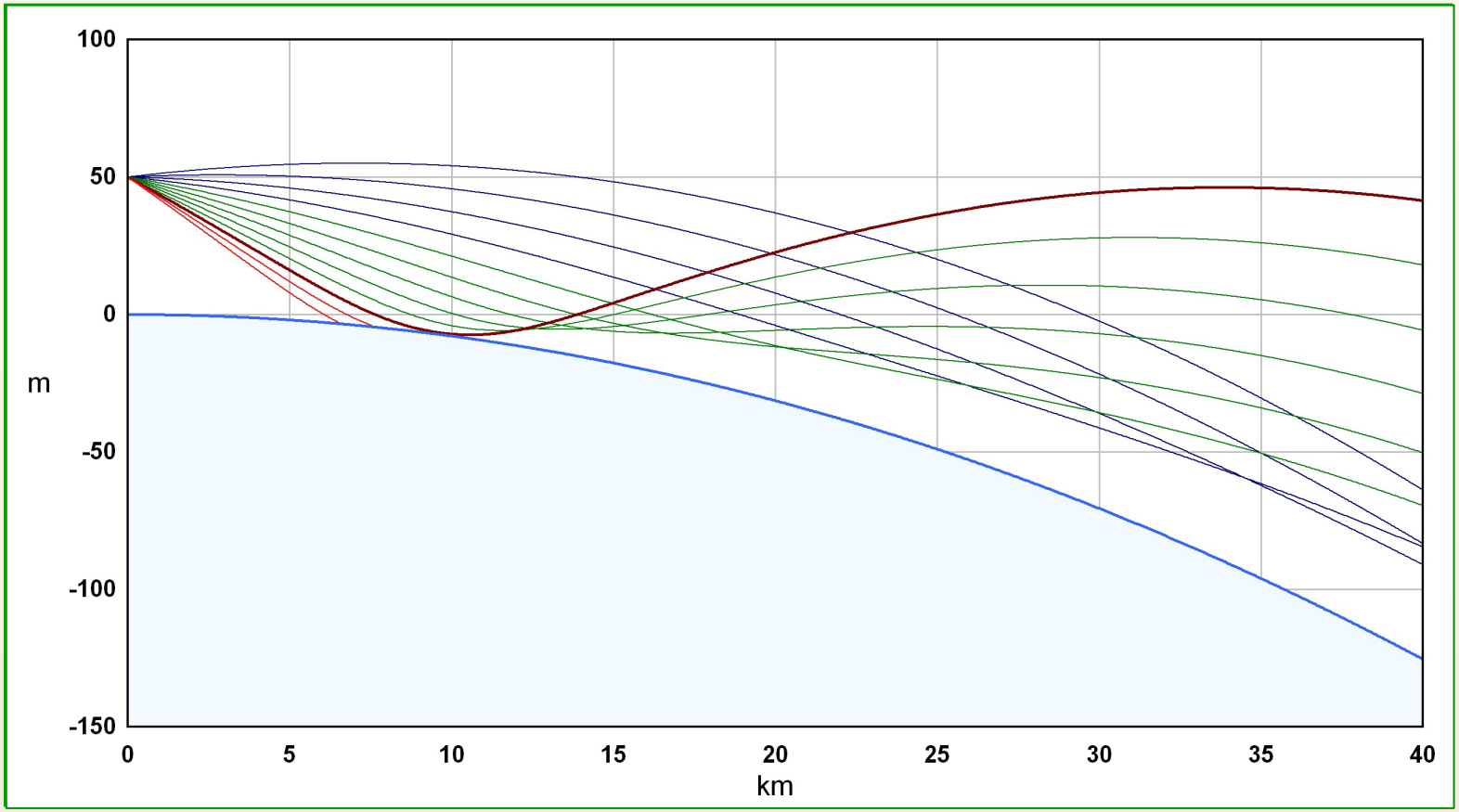




First light is not exactly on the optical horizon

Light propagation in the atmopshere

- Light path in the atmosphere is not a straight line
  - because pressure and temperature CHANGE with altitude
  - in normal conditions curved towards the ground
    - but could be in the other way in case of mirages



The effect with inferior mirage



… but it may work !



Conclusions

- I have essentially achieved my objective
  - sunrises or sunsets can be reasonably predicted within +/- 5s
  - this can be checked with observations
- Even near horizon, the atmosphere at mid-latitude can be modelled with simple physics
  - numerical refraction should be preferred to cumbersome tables
- Very near horizon, within 2-3 arcmin, thermal layers can produce serious deviations from the nominal conditions
  - but even in these case, the 15s accuracy seems achievable
  - probably not true in polar regions
  - these situations are not the most common

Merci de votre attention

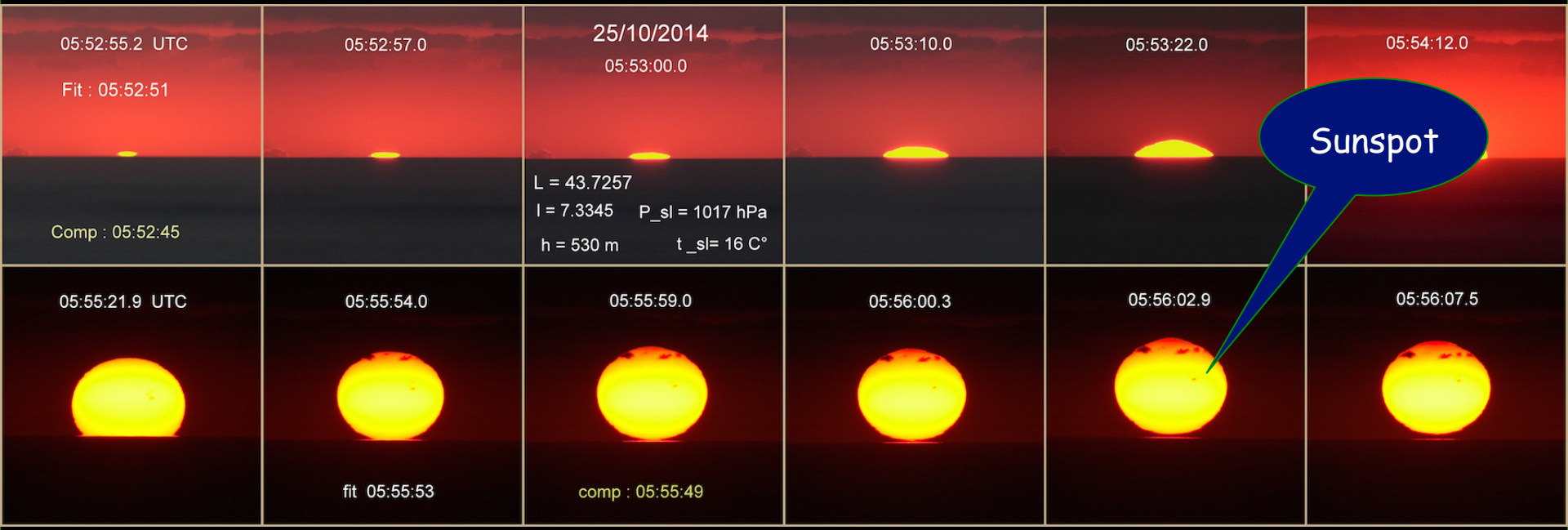


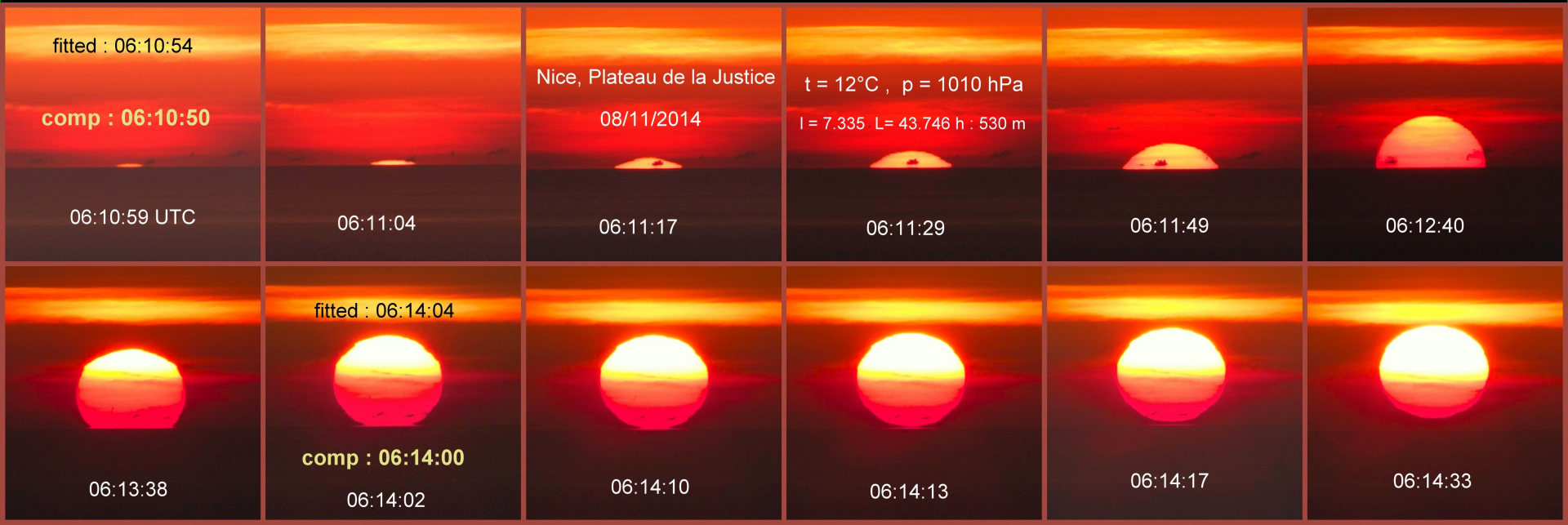

SFR 3G 07:18 60 %
samedi 8 novembre 2014
24 UTC +3,238
06:19 00,4
06:19:00.406
06:19:00.278
06:19:00.426
06:19:00.408
EMERALD ❖ SEQUOIA

The initial goal …



Time of sunrise



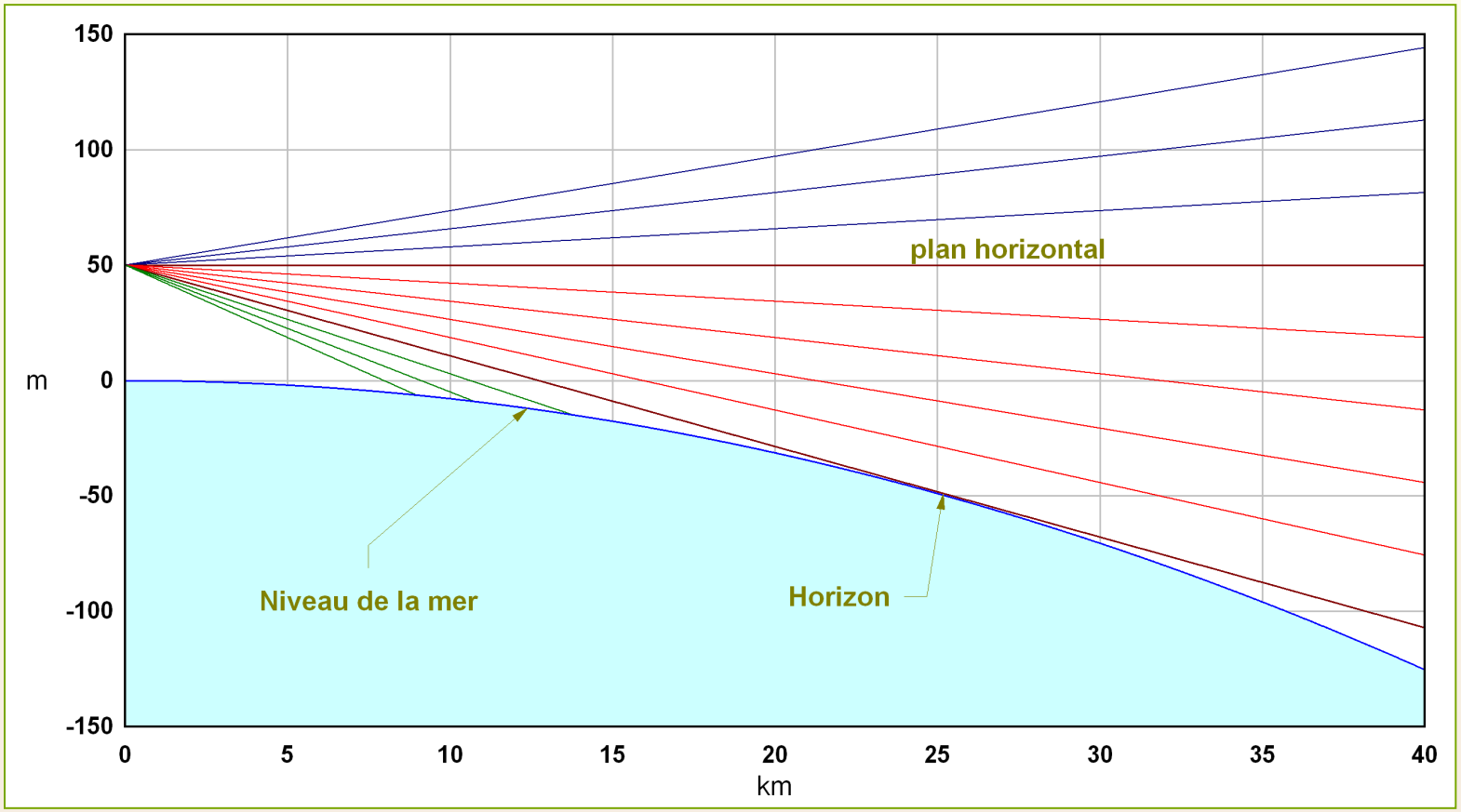


The same day (25/10/2014)



Light propagation in homogeneous medium

- Light path in straight line
  - true for vacuum or homogenous dielectric medium



Horizon dip

- From an altitude h, one follows the grazing ray
- First assumption : vacuum and propagation in straight lines
  - elementary geometry of right triangles

$$d = \sqrt{2Rh} \qquad \varepsilon = \sqrt{\frac{2h}{R}}$$

h (m)	d (km)	ε (')	dε/dh ('' /m)
0	0	0	∞
2	5	2	40
5	8	4.3	25
10	11	6	18
50	25	14	8
100	36	19	5.5
250	56	30	3.6
500	80	43	2.5

