Relation between the Gaia proper time and TCB

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Abstract

This note investigates the relationship between an ideal clock on-board of Gaia and the independent time coordinate to be used thoroughly for the data processing and the ephemerides of the solar system, including the representation of the motion of Gaia about L2. The whole method is first validated on the similar relationship for TCG before it is applied to Gaia. A practical formula is given from a quasi-periodic expansion of the time ephemeris which meets the accuracy needed for Gaia.

1 Introduction

Gaia will carry out observations from a spacecraft in continuous motion around the L2 (second co-linear Lagrange point of the Sun-Earth system). This point itself is in motion with respect to the Barycentric Celestial Reference System (BCRS), as shown in Fig. 1. Because of this motion and non-zero gravitational potential at the spacecraft location the clock on-board the spacecraft will deliver a time signal τ different from the BCRS coordinate time TCB that will be used as time parameter in the data processing. In particular all the ephemerides of the solar system bodies and of Gaia will be available with this time scale as independent parameter. In this note we address the question of the theoretical definition of the on-board timescale, as it would be realized by a perfect clock, and the more practical aspect of providing a time ephemeris relating the coordinate time t and the on-board time τ.

In a first section we recall the assumptions on the Gaia orbit which the subsequent derivations are based on. Then follows the theoretical derivation up to the PN (post-Newtonian)

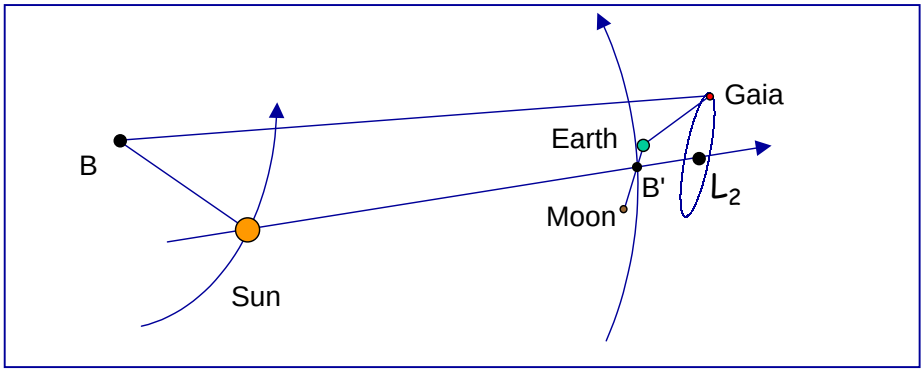


Figure 1: *Reference axes for the rotating frame with origin at L2.*

terms of the formula employed in the computation. As a check of the procedure, we apply the method to recompute the relationship between TCG and TCB at the geocenter and apply the frequency analysis to the time ephemeris to the same accuracy as needed for Gaia. Finally the same is carried out over the Gaia motion. In the last section we discuss the actual requirement and show that this is easily met with a single and simple quasi-periodic representation covering the mission lifetime.

2 Parameters of the orbit of Gaia

An approximate solution of the Lissajous orbit, sufficient for all the simulation purposes, has been given in a dedicated note (FM_Gaia_011) and implemented as the basis of the Ephemerides used in GDAAS and various groups. The linear solution of the elliptical three body problem was used, taking the Earth-Moon system as one body and referring to L2 as being along the line joining the Sun to the Earth-Moon barycentre. A Lissajous orbit was so selected as to avoid crossing the shadow of the Earth during a period of at least six years. Given the linear Lissajous orbit defined by,

$$\begin{aligned} X &= -\beta\, a\, \cos(2\pi\sigma t) \\ Y &= a\, \sin(2\pi\sigma t) \\ Z &= b\, \sin(2\pi\omega t) \end{aligned} \tag{1}$$

where the amplitudes are free parameters and $\beta = 0.313752136734$. The current version of the reference orbit used in the simulation has been adjusted to meet the injection possibilities of a Soyuz rocket combined with the constraint set by the radio link, yielding a maximum angular shift of Gaia from L2 to 15 degrees. Thus we have taken $a = 363\,750$ km, $b = 100\,000$ km. The practical definition of L2 and the values of the two frequencies ω and σ are given in [7] for the theoretical approach and in [8] for the implementation.

The above solution assumes that at $t = 0$ (in the following $t = 0$ means J2010.0 = 1.0 Jan 2010 at 0h TCB) one has $Y = Z = 0$.

3 Theoretical relationship between Gaia proper time and TCB

Nominally $TCB = t$ is the time used in the astrometric modelling. On the other hand the observations on Gaia are referred to the proper time τ on board. As a consequence, one needs to determine the relationship between the two timescales.

The interval of proper time of a test (massless) observer between two events x^μ and $x^\mu + dx^\mu$ both lying on the worldline of the observer is well known [10, 9] to be defined as

$$\frac{d\tau}{dt} = \left(-g_{00} - \frac{1}{c}\, g_{0i}\, v^i - \frac{1}{c^2}\, g_{ij}\, v^i\, v^j \right)^{1/2}, \tag{2}$$

where $g_{\alpha\beta}$ are the metric tensor components in some selected reference system, and $v^i = \frac{dx^i}{dt} = c\, u^i/u^0$ is the coordinate velocity of the observer in that reference system, u^α being the 4-velocity of the observer. This formula follows directly from the basic idea of general relativity that the infinitesimal interval of proper time $d\tau$ between two events on the worldline of an observer coincides (up to a constant factor) with the 4-dimensional interval ds between those two events, ds^2 being defined as $ds^2 = g_{\alpha\beta}\, dx^\alpha\, dx^\beta$. For the signature of the metric tensor $(-+++)$ the relation reads

$$d\tau^2 = -\frac{1}{c^2}\, ds^2 = -\frac{1}{c^2}\, g_{\alpha\beta}\, dx^\alpha\, dx^\beta. \tag{3}$$

Eq. (2) immediately follows from that latter formula. By using (2) one neglects

1. the influence of the mass of the observer on the space-time metric, and
2. the spatial extension of the observer (e.g. the difference in the position between the center of mass of the satellite and the clock, and the spatial extension of the clock).

In case of GAIA the first constrain is certainly acceptable: since the mass of the satellite (a few thousand kilograms) is certainly too small to produce any conceivable gravitational field which could influence the motion of other bodies in the Solar system or change the local physical processes within the satellite ($U/c^2 \sim 10^{-24}$, U being the gravitational potential). Numerical estimates show again that at the level of 10^{-17} in the relative rate of τ and t, and at the level of 5 ps in absolute values of $\tau(t)$ the second constrain is also acceptable in case of GAIA. If higher accuracy is required, a local proper reference system should be used. There are several ways to construct such a reference system (tetrad formalism and related techniques [3] or explicit construction of the metric tensor in the post-Newtonian approximation [6]).

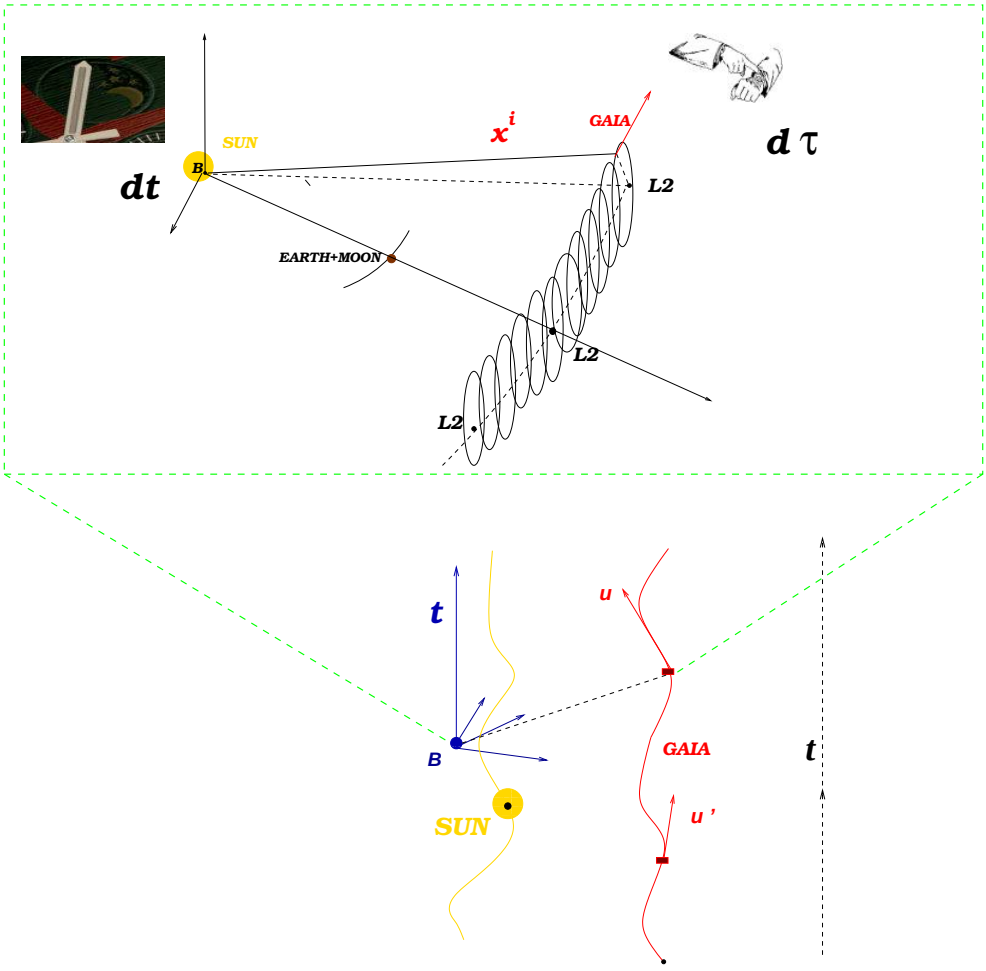


Figure 2: *World line of Gaia with respect the barycenter of the Solar System B.*

In the case of Gaia the BCRS of the IAU will be used as the relativistic reference system. The metric tensor of the BCRS is defined in the IAU 2000 resolution B1.4 to be (e.g., [12])

$$g_{00} \approx -\left[1 - \frac{2}{c^2}w(t,\mathbf{x}) + \frac{2}{c^4}w^2(t,\mathbf{x})\right],$$

$$g_{0i} \approx -\frac{4}{c^3}w^i(t,\mathbf{x}),$$

$$g_{ij} \approx \delta_{ij}\left(1 + \frac{2}{c^2}w(t,\mathbf{x})\right),$$

where the potentials w and w^i are determined by

$$w_A(t,\mathbf{x}) = G\int_A \frac{\sigma(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x' + \frac{G}{2c^2}\frac{\partial^2}{\partial t^2}\int_A \sigma(t,\mathbf{x}')|\mathbf{x}-\mathbf{x}'|d^3x'$$

$$w^i(t,\mathbf{x}) = G\int_A \frac{\sigma^i(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d^3x'.$$

Since the orbit around L2 is far enough from the gravitating bodies one can show that within an precision of $\tau(t)$ of 10^{-18} the gravitational field of each body can be characterized by its mass only (i.e. each body is a mass monopole), the above potentials w and w^i can reduced to

$$w = w_0 - \frac{\Delta}{c^2}, \tag{4}$$

and

$$w^i(t,\mathbf{x}) = \sum_A \frac{GM_A}{r_A}v_A^i, \tag{5}$$

where

1. $\mathbf{r}_A = \mathbf{x} - \mathbf{x}_A$ ($\mathbf{x}_A$ and v_A^i represent respectively the barycentric coordinates and velocity of the center of mass of body A)
2. $w_0 = G\sum_A(M_A/r_A)$ (the summation includes all solar system bodies A), with $r_A = |\mathbf{r}_A|$,
3. the function $\Delta(t,\mathbf{x})$ is expressed with sufficient accuracy as (note 2 of the resolution B1.4, [12])

$$\Delta(t,x) = \sum_A \frac{GM_A}{r_A}\left[-2v_A^2 + \sum_{B\neq A}\frac{GM_B}{r_{BA}} + \frac{1}{2}\left(\frac{r_A^k v_A^k)^2}{r_A^2} + r_A^k a_A^k\right)\right], \tag{6}$$

with $\mathbf{r}_{BA} = \mathbf{x}_B - \mathbf{x}_A$ and $\mathbf{a}_A = d\mathbf{v}_A/dt$.

Substituting these potentials into (2) one gets

$$
\begin{aligned}
\frac{d\tau}{dt} &= 1 - \frac{1}{c^2}\left[\frac{v^2}{2} + \sum_A \frac{G\,M_A}{r_A}\right] \\
&\quad + \frac{1}{c^4}\left[-\frac{v^4}{8} + \frac{1}{2}\left(\sum_A \frac{G\,M_A}{r_A}\right)^2 + \sum_A \frac{G\,M_A}{r_A} \sum_{B\neq A} \frac{G\,M_B}{r_{AB}}\right. \\
&\qquad + \sum_A \frac{G\,M_A}{r_A}\left(4\,v_A^i\,v^i - \frac{3}{2}\,v^2 - 2\,v_A^2\right. \\
&\qquad\quad \left.\left. + \frac{1}{2}\,a_A^i\,r_A^i + \frac{1}{2}\left(v_A^i\,r_A^i/r_A\right)^2\right)\right]. \qquad (7)
\end{aligned}
$$

The integration of (7) over Gaia world line provides the time ephemeris $\tau(t)$.

Alternatively, one could also have started from equation (2), to relate directly Gaia proper time and TCG, by using the GCRS coordinates system and considering the tidal potential of the planets, the Moon and the full potential of the Earth, and then combine the result with the TCG-TCB transformation. At the theoretical level this has been explicitly checked to produce the same formulas as the direct use of (2) in the BCRS. For a practical implementation, however, the direct TCB-τ relation is simpler to implement if we use directly BCRS for the orbit of Gaia, which is also close to the expected orbital data. The GCRS solution could be implemented for consistency check, but this has not been carried out so far.

4 Validation of the implementation

4.1 Principles

The computation reduces now to a quadrature of Eq. 7 with the evaluation of the gravitational and vector potentials from a solar system ephemeris completed by the motion of Gaia. The velocity and position of Gaia are taken from the provisional orbit about L2 used in the general simulation. As the independent parameter of the ephemeris is TDT rather than TCB the actual integral becomes,

$$
\int \frac{d\,\tau}{dt}\,\frac{dt_{\rm eph}}{1-L_B} \qquad (8)
$$

where $L_B = 1.5505197\times10^{-8}$. The barycentric acceleration $\mathbf{a}_A$ of each planet is computed in the newtonian approximation as,

$$
\mathbf{a}_A = -\sum_{B\neq A} \frac{GM_B}{|\mathbf{x}_A - \mathbf{x}_B|^3}\,(\mathbf{x}_A - \mathbf{x}_B) \qquad (9)
$$

The interval of time over which the integration is performed is not specified and may depend on the objectives of the integration. To produce a time ephemeris applicable during

the Gaia mission, one needs only to specify an origin where the two scales have identical reading for a particular space time event, and then carry out the integration with a convenient step to interpolate the ephemeris. If a simple formula is aimed with a meaningful secular term, one has to integrate over a significant timespan, so that most of the periodic terms average out. In order to check the program and validate the computation, we have tested the numerical implementation to the geocenter instead of Gaia. This is important for Gaia as there is no alternative way to obtain the solution, whereas for the application to the geocenter numerical or analytical solutions are available. The ability to recover the known results for $TCB - TCG$ will give us confidence into the application to Gaia, in unchartered grounds. One must also admit that the software development benefited very much from this checking.

But beyond this simple checking this has a real scientific interest to provide an independent evaluation of the secular part as the IAU value rests upon a single integration carried out by Irwin and Fukushima [5] using the JPL numerical ephemerides. In addition the subsequent frequency analysis (planned to be used for Gaia) results can also be easily checked on the analytical solution computed by Fairhead and Bretagnon [4].

The formula for the geocenter is very similar to Eq. (7), but one must consider the barycentric velocity of the Earth geocenter instead of Gaia and w_0 should be the gravitational potential generated by the Sun, planet and moon. The main difference lies in the various summations which must exclude the Earth, except for its role in the barycentric acceleration of the planets and the sum over $B \neq A$. The integration has been performed with a numerical quadrature over 500 years and with a step of 8 days, small enough to see the lunar period in the frequency analysis (this is needed for TCG, but not for Gaia as the action of the Moon at L2 turns out to be negligible). Several adaptive integrators were tested and found to end up with identical results. The output is a time ephemeris giving the value $TCB - TCG$ in seconds every step of the integrator, namely every 8 days.

4.2 Frequency analysis of the time ephemeris

In order to facilitate the comparison to published result and also to analyze the relative contribution of the different motions, it is interesting to represent the time ephemeris as a Poisson series with periodic terms whose frequencies should be easily interpreted with few base frequency of the planetary motions. The integral is then analyzed as a frequency of a signal $S(t)$ containing linear and periodic terms,

$$S(t) = a + bt + \sum (\alpha_i + \beta_i t) \cos(2\pi\nu_i t + \phi_i). \tag{10}$$

where the unknowns are the amplitudes and the frequencies. A dedicated program developed by one of us (FM) allows to estimate a secular trend originating from the linear part of (10) plus the detrended signal representing all the periodic contributions. The program is given an input text file in terms of two columns with the time and the signal to be analyses, few additional settings about the power of the time for each components, the typical range of frequencies expected and the number of lines. The search is based on

an orthogonal decomposition over the best frequencies. Not FFT is used in the program which has been designed to work on irregular time-sampling. The output is a list of frequencies, the amplitude and phase of each line and a statistical significance. The origin of the time and that of the secular term will be 2010 for Gaia but is as J2000 for TCG for the comparison with IAU values.

Table 1: *Standard parameters used in this work (TDB values)*

Parameter	value	unit
$GM_\odot$	$1.327\,124\,400\,18 \times 10^{20}$	$\mathrm{m^3\,s^{-2}}$
AU	149 597 870 691	m
c	299 792 458	$\mathrm{m\,s^{-1}}$
$M_{\mathrm{Earth}} / M_{\mathrm{Moon}}$	81.30058	-
$M_{\mathrm{Sun}} / M_{\mathrm{Mercury}}$	6 023 600	-
$M_{\mathrm{Sun}} / M_{\mathrm{Venus}}$	408 523.71	-
$M_{\mathrm{Sun}} / M_{\mathrm{Earth}}$	332 946.048	-
$M_{\mathrm{Sun}} / M_{\mathrm{Mars}}$	3 098 708	-
$M_{\mathrm{Sun}} / M_{\mathrm{Jupiter}}$	1 047.3486	-
$M_{\mathrm{Sun}} / M_{\mathrm{Saturn}}$	3 497.898	-
$M_{\mathrm{Sun}} / M_{\mathrm{Uranus}}$	22 902.94	-
$M_{\mathrm{Sun}} / M_{\mathrm{Neptune}}$	19 412.24	-
Lissajous Y amplitude	363 750	km
Lissajous Z amplitude	100 000	km

5 Results for TCG

5.1 The secular term

A straightforward analytical representation of the Keplerian orbit of the Earth indicates that the main contribution to b is,

$$\frac{V^2}{2c^2} + \frac{GM_\odot}{c^2 a} \sim \frac{3}{2}\frac{GM_\odot}{c^2 a} \tag{11}$$

where V is the circular velocity of the Earth about the Sun. With the date of Table 1 one finds that $b \simeq 1.4806 \times 10^{-8}$. The next largest contribution comes from Jupiter, not

directly from its gravitational potential at the geocenter, but from the motion of the Sun about the barycentre of the Solar system.

The IAU value based on Irwin and Fukushima [5] is

$$L_c = 1.480\,826\,867\,410 \times 10^{-8} \pm 2 \times 10^{-17}$$

and the modelling includes the post-Newtonian terms in $1/c^4$. There is a sign error in one of theses terms, but its effect is of 3×10^{-20}. The integration has been performed over 600 years with the ephemeris DE405, but the estimate of the mean rate between the two timescales has been evaluated primarily from analytical planetary theories.

However the number is not well defined as it depends on the reference ephemeris, the duration of the integration and to a less extend on the period covered by the integration. Changing from 600 years to 300 years decreases the secular trend by 10×10^{-17} and the latest value based on the analytical theory VSOP gives [1],

$$L_C = 1.480\,826\,855\,667 \times 10^{-8}.$$

More important the value of the secular term depends also on the method applied to extract it from the times series. Each algorithm has the same objective, that is to say compute some average, but in practice every method contains implicitly a definition of what the secular term is, and at the level of $\sim 10^{-8}$ they may produce different results. Formally the mean rate is defined as,

$$\left\langle \frac{dTCG}{dTCB} \right\rangle = 1 - L_C \tag{12}$$

In principle this can be obtained by averaging (7) over a sufficiently long period. One can also fit (7) to a simple model with a constant term and a sum of periodic terms, like $a + P$. In general the two methods would not be identical as $< \cos(2\pi\nu t) >$ over a length T is typically of the order of $\approx 1/(\nu T)$. Even with relatively short periods of ~ 1 year, the average over 500 years will be of the order of 10^{-3}. In addition the periodic terms arising from the planetary motion are more complex and include mixed terms like $t\cos(2\pi\nu t)$, which do not average out to zero. For example with a direct straight average over 500 years of (7) we have found $L_C = 1.480\,824\,500 \times 10^{-8}$ and by a fitting on to a model with three periodic lines, $L_C = 1.480\,826\,930 \times 10^{-8}$.

A second class of method starts from the integrated function, that is to say directly on $TCB - TCG$, from which one searches a linear trend with the time. Again, for an infinite duration and purely periodical terms in (7), this should lead to results identical to the direct averaging. But again the combination of the finite duration of the integration and the presence of mixed terms leads to results slightly different from the direct evaluation of the mean rate. By fitting a straight line $a + L_C t$ to obtain the trend we have found $L_C = 1.480\,826\,847 \times 10^{-8}$ and by adding three periodic terms in the fit, $L_C = 1.480\,826\,887 \times 10^{-8}$. Finally the analytic method (not implemented by us, in particular because it is

not applicable to Gaia) provides a third answer, and as mentioned earlier there are also disagreement (at the level of 10^{-16}) between groups using the same analytical formulation!

Therefore at this point it is wiser to consider that the uncertainty in L_c due essentially to the lack of perfect definition of the averaging process, of the duration of the integration, is of the order of 10×10^{-17} and a rounded value should be,

$$L_C = 1.480\,826\,87 \times 10^{-8} \pm 2 \times 10^{-16}$$

This value seems rather robust to various accurate implementation of the solar system ephemeris and to any reasonable method to evaluate the secular trend.

In the course of the experiments reported in this note, the secular trend is defined as the coefficient b of bt in (10) resulting from the frequency analysis of the time ephemeris. The value of L_c is rather stable with the increase of the number of frequencies on which the signal is fitted. Essentially the method provides the best fit of a time series to a quasi-periodic signal, (where the frequencies are also fitted) and the secular drift is just one element of the model.

From the integration over our reference 500 years one gets,

$$L_C = 1.480\,826\,881 \times 10^{-8}$$

and respectively for 250 and 750 years,

$$L_C = 1.480\,826\,871 \times 10^{-8}$$

$$L_C = 1.480\,826\,883 \times 10^{-8}$$

In these values the post-Newtonian terms in $1/c^4$ account for $\Delta L_C = 1.0968 \times 10^{-16}$.

5.2 Periodic terms

The comparison of the frequencies and amplitudes of the periodic terms in $TCB - TCG$ provides an additional check to our implementation. One compares the best fitted periodic lines down to a certain amplitude to the analytical expansion carried out by Fairhead and Bretagnon [4] over nearly one thousand terms computed analytically from VSOP. Due to the presence of long period inequalities, that cannot be found on a 500 year integration, and the neglect of terms smaller than 0.1 μs, the best fit must be different from their solution truncated after 10 or 20 terms. However, the difference must be reasonably small, although it is difficult to say by how much. The results for the terms than 1 μs are shown in Table 2, together with the reference values obtained by Fairhead and Bretagnon with their analytical expansion (3nd column) and the physical source in planetary perturbations (V = Venus, E= Earth ...). One should note the presence of a line due to the Moon as a result of its small distance to the Earth Geocenter.

All the lines but the first one are recovered by our frequency analysis technique to better than 30 ns, which is completely sufficient for the Gaia needs. As mentioned, because of the

number of lines fitted, one could not expect a perfect match between the analytical and numerical method. As for the first line, the situation is more complex as it is coupled with a secular term ($\alpha + \beta\, t \cos(2\pi\nu t + \phi)$) and the actual value of β (and then that of α) is very sensitive to neglected terms with period close to one year, since two lines with nearly equal frequencies can be represented by a single line with a time dependent amplitude. We have also found that β depends also very much on the implemented solar system ephemeris.

A final test aimed at evaluating the discrepancy in the lines found between our analysis and the theoretical values originating purely from the truncation to about 20 lines in the frequency analysis. We have used the fortran implementation of all the periodic terms of Fairhead and Bretagnon available in the Sofa repository to generate the time sequence over 500 years with 8 days step, as in our numerical quadrature. Then this time series has been subjected to the same frequency analysis as our signal, with exactly the same settings. Results are shown in the 4th column of Table 2. A comparison between the last two central columns, give a perfect idea of the distortion produces by the best fit in amplitude and frequency, in comparison to the quasi-periodic signal used as input, comprising more than 700 lines with mixed power of time. Now one sees that the 2nd and 4th columns are very alike, confirming our guess that most of the difference is well accounted for by the fitting and the timespan, and not by the quadrature or the solar system ephemeris used.

Regarding the Gaia need, one should not worry at all about the remaining differences and one can reasonably admit that the procedure has been validated by the analysis of $TCB - TCG$.

Table 2: *Solution for TCB-TCG : Periodic terms* (F. & B. = Fairhead and Bretagnon)

P(yr)	This work μs	F. & B. Theory μs	F. & B. Freq. Anal.	
1.00	1656.695	1656.675	1656.620	E
1.09	22.422	22.417	22.424	E-J
0.5	13.840	13.840	13.838	2E
11.9	4.772	4.770	4.766	J
1.04	4.674	4.677	4.675	E-S
29.5	2.216	2.256	2.217	S
0.08	1.550	1.555	1.555	Moon
0.80	1.277	1.277	1.277	2E-2V
1.20	1.195	1.193	1.191	E-2J
1.60	1.117	1.115	1.116	E-V

6 Results for Gaia

Now the integration along the Gaia world line must include all the planets as source of gravitational potential around L2. The motion of Gaia is the reference orbit used in the current simulation (see Sect.(2)). The quadrature has been carried out exactly in the same condition as for the geocenter, and various experiment were conducted to evaluate the effect of the timespan. The reference time in the frequency analysis had been taken as J2010 instead of J2000 (this is important for the phase and the term with the power of time, not listed here in the tables).

6.1 The secular term

By the same procedure as for TCG we have fitted the time ephemeris to the model (10) with 20 or 25 lines. The secular trend, similar to L_C in TCG, has been found equal to,

$$L_A = 1.481\,259\,940 \times 10^{-8}$$

with again an uncertainty estimated at 2×10^{-16}, that is to say about 100 times larger than the formal error in the fit. The uncertainty is based on the comparisons of runs covering different timespan and also by changing the number of lines in the quasi-periodic model. We have called the coefficient L_A (where A stands for *astrometry*) to maintain the same kind of notation, and avoid the more natural L_G which is already a constant of the astronomical system.

The difference with L_C is just 4.4×10^{-12}, needs some explanation as it differs from what a quick estimate would yield. One expects that the main source of the difference comes from the gravitational potential at L2 instead of its value at the geocenter. With the distance Earth-L2 $\approx \delta r = 1.5 \times 10^6$ km one has for the change in potential,

$$\delta\left(\frac{GM_\odot}{c^2 r}\right) = -\frac{GM_\odot}{c^2 r}\,\frac{\delta r}{r} \simeq -1 \times 10^{-10} \tag{13}$$

or 50 times larger that the expected difference. The point is the fact that if the orbital period of Gaia at L2 in its motion about the Sun is still one year, its linear barycentric velocity has increased with the distance to the Sun, with

$$\delta\, v \sim n\delta\, r = v\delta r/r \sim 0.3\mathrm{km\,s^{-1}} \tag{14}$$

where n is the orbital mean motion of the Earth. Therefore,

$$\delta\left(\frac{v^2}{2c^2}\right) = \frac{v^2}{c^2}\,\frac{\delta r}{r} \sim 1 \times 10^{-10} \tag{15}$$

Hence the two contributions are equal in magnitude and opposite in sign and cancel out. This equality holds true even with the elliptical motion and is a consequence of the energy theorem. The actual contributor to the difference $L_A - L_C$ are the gravitational potential of the Earth (not included at the geocenter) and the velocity of Gaia about L2 (combined with the rotation of the local frame), equal respectively to $GM_E/c^2/\delta r \sim 3 \times 10^{-12}$ and $< w^2 > /2c^2 \sim 3 \times 10^{-13}$ for the implemented orbit.

Table 3: *Representation of the time ephemeris of $TCB - \tau$ fitted over a periodic lines. The result is based on an integration over 500 years from 1750 to 2250.*

line	frequency day^{-1}	period days	amplitude μs	phase deg	origin
1	$0.27377786 \times 10^{-02}$	365.2596	1664.955	267.432	sidereal year
2	$0.56316999 \times 10^{-02}$	177.5663	121.734	268.986	lissajous
3	$0.25070051 \times 10^{-02}$	398.8823	22.637	212.643	synodic Jupiter
4	$0.54755571 \times 10^{-02}$	182.6298	13.912	264.865	solar six months
5	$0.23078386 \times 10^{-03}$	4333.059	4.763	238.716	sidereal Jupiter
6	$0.26448227 \times 10^{-02}$	378.0972	4.621	18.094	synodic Saturn
7	$0.93002251 \times 10^{-04}$	10752.43	2.296	350.111	sidereal Saturn
8	$0.28939215 \times 10^{-02}$	345.5519	1.336	271.555	synodic lissajous
9	$0.34251747 \times 10^{-02}$	291.9559	1.287	76.804	2*synodic Venus
10	$0.17125042 \times 10^{-02}$	583.9402	1.121	83.200	synodic Venus
11	$0.22761895 \times 10^{-02}$	439.3307	1.011	249.649	Sun -2*Jupiter
12	$0.50139305 \times 10^{-02}$	199.4443	0.795	155.599	2 synodic Jupiter
13	$0.83694787 \times 10^{-02}$	119.4818	0.696	266.360	sun+lissajous
14	$0.68729122 \times 10^{-03}$	1454.987	0.603	247.213	3*Sun -2*Venus
15	$0.27050311 \times 10^{-02}$	369.6815	0.499	194.262	synodic Uranus
16	$0.27212869 \times 10^{-02}$	367.4732	0.481	224.027	synodic Neptune
17	$0.32521964 \times 10^{-04}$	30748.45	0.434	92.811	sidereal Uranus
18	$0.10700287 \times 10^{-04}$	93455.44	0.426	126.592	13*Sun -8*Venus[1]
19	$0.25517745 \times 10^{-02}$	391.8842	0.258	299.625	Sun -2*Saturn
20	$0.52447764 \times 10^{-02}$	190.6659	0.196	210.825	2*Sun - Jupiter
21	$0.82133357 \times 10^{-02}$	121.7532	0.174	262.348	3*Sun
22	$0.46156208 \times 10^{-03}$	2166.556	0.135	207.663	2*Jupiter
23	$0.11263400 \times 10^{-01}$	88.78314	0.133	267.959	2*Lissajous
24	$0.17129354 \times 10^{-04}$	58739.31	0.123	197.623	sidereal Neptune

[1] The actual period is 87000 days and is not properly recovered over 500 yrs, but perfectly over 750 yrs. The line is left here as it stands in the fit, with this identification, despite the incorrect period.

6.2 The periodic terms

The list of the periodic terms larger than 100 ns is shown in Table 3. The convention for the phase is such that each line should be read as $A * \cos(2\pi\nu(t - t_0) + \phi)$ where A is the

amplitude, ν the frequency, ϕ the phase and $t_0 = J2010$. As for the Earth, the frequencies are fitted to the times series and for the longer period, one should not be surprised to see a difference between the period of the physical component mentioned in the last column and the actual period found. For example the line 17, has a period of 238.92 years and while the fitted value is 255 years (over 750 years one finds 236.7 years). The solution is poor also with Neptune with a slightly smaller period but an amplitude four times smaller. We recover most of the lines already found for the Earth with slightly different amplitudes, with the following exceptions:

- The second largest periodic term with a magnitude of 122 μs is specific to the Lissajous motion and does not exist in TCG. Its period is that of the in-plane Lissajous motion and its amplitude is five times larger than Jupiter's.
- There are several other lines involving the Lissajous motion (lines 8, 13,23)
- There is no line coming from the Moon, because its distance to L2 is much larger than to the geocenter and its role is negligibly small at L2.

One must also add the semi-secular term associated to the main solar line as,

$$0.0419(t/36525)\cos(2\pi\,\nu_1 t + 87.557) \quad \mu\text{s} \tag{16}$$

where t is in days and $t = 0$ at J2010. The amplitude and phase of this mixed term are very sensitive to the duration covered in the integration.

6.3 Residuals

In Table 4 we show the improvement of the fit as more lines are entered into the model. The table gives the RMS of the residuals computed over the period of integration. The final residuals over 200 years are plotted in Fig. 3.

7 Requirements in accuracy

This point has been considered by J. De Bruijne [2] and discussed during the 5th meeting of the Relativity and Reference frame working group of June 2004. From a close examination of the requirements in absolute timing of time dependent events observable by Gaia, we found no serious accuracy constraints from the timing of variable astrophysical data, like variable stars, eclipsing or spectroscopic binaries, rotating asteroids. The call to solar system ephemerides during the mission, is as well no a cause of concern with time accuracy requirement no better than 0.1s, easily met. Regarding the barycentric motion of Gaia, and more specifically its velocity, with an acceleration of 6 mm s^{-2}, and a velocity requirement of 0.6 mm s^{-1}, this yields a timing accuracy of 0.1s. The astrometry of solar system objects to $\sim 10\mu$as, with motion as large as 200 mas s^{-1} gives a more stringent requirement of

Table 4: *Residuals RMS in μs after a model with k lines has been fitted over a period of 500 years.*

line	RMS μs	line	RMS μs
1	88.2	13	0.85
2	19.5	14	0.74
3	11.2	15	0.69
4	5.4	16	0.58
5	4.2	17	0.48
6	2.6	18	0.36
7	2.0	19	0.31
8	1.8	20	0.28
9	1.6	21	0.25
10	1.3	22	0.24
11	1.14	23	0.19
12	0.99	24	0.16

$\sim 10-100\mu$s. Finally, the a posteriori computation of the image location from the satellite attitude would need an absolute timing accuracy of $\sim 10\mu$s.

It was agreed that the final representation accurate to 0.2 μs, would be fine with a safety margin. This could be achieved with $\sim$ 25 periodic and mixed terms, allowing to build an efficient function easier to construct than a Chebyshev representation. It was also agreed that the origin of the time argument and that of the secular term will be 2010, with $\tau - TCB = 0$ at this date.

8 Further works and conclusion

The above work is not yet the end of the story. At this stage we have (i) a good understanding of the problem and its intricacies to produce a time ephemeris meeting the Gaia requirements and (ii) everything in hand to produce a small piece of software to be used in the simulation and data processing chains, as long as one maintains the consistency with the reference orbit for Gaia. Regarding the actual mission, one can already identify

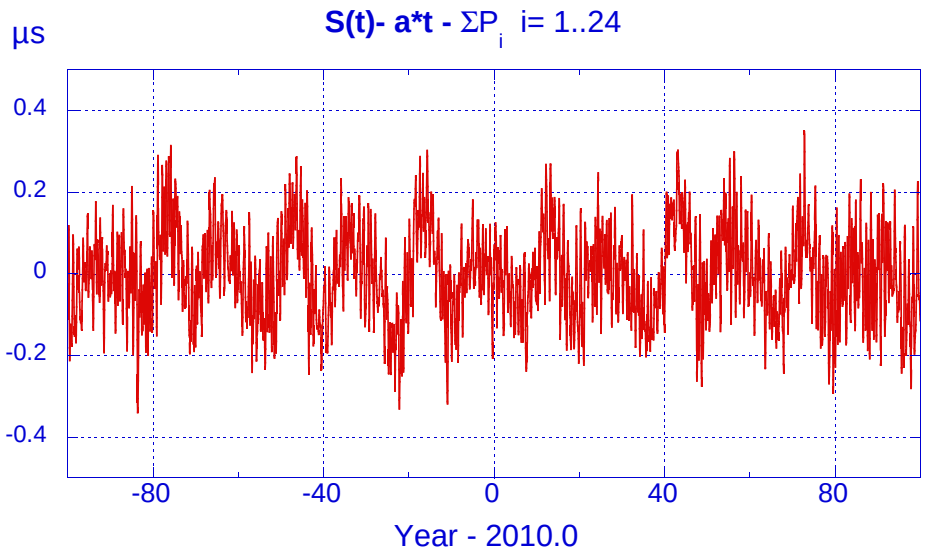


Figure 3: *Residual in the time ephemeris between the barycentric coordinate time and the Gaia proper time after a fit of the ephemeris over 24 periodic lines.*

several areas for further work.

The orbit will not be known in advance, and one will have to work on an initial approximation to supply a starting software. Then during the iteration, this will be replaced by a more advanced one based on the actual orbit up to a certain time. This orbit will be available on a form very likely different from the simple analytical representation used in this work. This should not be a problem to make the required adaptation. Because of orbit manoeuvres it is likely that the orbit will not be not as smooth as the simulated orbit, and in some case may cause numerical problems in the integration, because of the lack of differentiability at high order. So experiments would be needed on a mock-file that could be prepared by ESOC.

In order to assess the sensitivity to the detail of the actual orbit, we have made a specific run with a Lissajous orbit 10% larger than the reference orbit used in the Gaia simulator. The mean rate is increased in relative value by 2×10^{-8} and the four lines related to the Lissajous motion show an amplitude 10% larger. So starting from the largest amplitude of 122 μs, and from the accuracy requirement of 0.2 μs, this implies that the orbit must be known to 1.6×10^{-3}, or ~ 500 km to produce the final software applicable to the real mission. It remains to discuss with ESOC by how much the actual orbit may depart from an initial target orbit. If it is within this 500 km, one may limit the iterative versions of the software, to one initial version and one for the final iteration. Otherwise, updates will be required during the mission.

Then the representation of the time ephemeris should also be revisited, because the orbit will be known over a relatively small period of time. From experiments made with the

same algorithms over 10, 20 or even 50 years, we have found that the results are not so good as over a longer timespan, as a result of the poor representation of the longer periods from the outer planets. A straight Chebyshev analysis over intervals of $\sim$ 60 days (to allow for the 88 days periods of the first harmonics of the Lissajous motion), may prove preferable at the end.

One more question to be investigated is if the velocity of the clock (due to rotation of the satellite) relative to the center of mass of the satellite should be taken into account while relating their readings with TCB. Indeed, 1 mm/s of relative velocity change $d\tau/dt$ by $\sim 3 \cdot 10^{-16}$.

Finally one could also think about an operational time link [11] between the on-board clock and the ground that could ideally allow to monitor in real time the reading of the clock with respect to UTC. No solution have been yet proposed, but on paper this is feasible.

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