1994SoPh..149..213I

SOLAR DIAMETER MEASUREMENTS WITH CALERN OBSERVATORY ASTROLABE AND ATMOSPHERIC TURBULENCE EFFECTS

A. IRBAH
Observatoire d'Alger, C.R.A.A.G., BP 63 Bouzareah, Alger, Algérie

F. LACLARE
Observatoire de la Côte d'Azur, Observatoire de Calern, Plateau de Caussols, 06460 St. Vallier de Thiey, France

J. BORGNINO
Département d'Astrophysique, URA 709 CNRS, UFR Faculté des Sciences, Université de Nice-Sophia Antipolis, Parc Valrose, 06108 Nice Cedex 2, France

and

G. MERLIN
Observatoire de la Côte d'Azur, Observatoire de Calern, Plateau de Caussols, 06460 St. Vallier de Thiey, France

(Received 3 May, 1993; in revised form 22 July, 1993)

Abstract. In order to deduce significant astrophysical results from solar diameter measurements it is necessary to take an accurate account of instrumental and atmospheric effects. This paper presents a comparison between visual and CCD camera measurements performed by means of the Calern Observatory solar astrolabe during the last 4 years; this allows us to evaluate visual measurements done previously (from 1975 to 1989). Then, a study of atmospheric effects is developed. From CCD measurements, the image quality, expressed by Fried's seeing parameter, r_0, is estimated and related to the errors occurring in solar diameter measurements. A statistical analysis gives about 0.26 arc sec (or 0.13 arc sec for the semi-diameter) as the lowest value that this error may reach at Calern Observatory. One conclusion of this work is that it is important in the future to have image quality observations, obtained using a dedicated monitor, in order to evaluate and classify the measurements. A survey of the seeing might so lead to improve the precision of the results by weighting each diameter estimation and eventually to schedule the observations.

1. Introduction

Several methods are used to measure the solar diameter. The first methods used were related to the solar transit observed with meridian instruments. The analysis of the Greenwich observations by Eddy and Boornazian (1979) showed a significant decrease of the solar diameter (2 arc sec per century). The measurements used in Washington and Greenwich were made over a long period of time but with many discontinuities (changes of observers and in instrumental devices). Thus, the result of Eddy and Boornazian was soon refuted by Parkinson, Morrison, and Stephenson (1980). Indeed, using the observations of solar eclipses and Mercury transits across the Sun, Parkinson, Morrison, and Stephenson did not observe significant secular variations. However, the result of Eddy and Boornazian added greater interest to

Solar Physics **149:** 213–230, 1994.
© 1994 *Kluwer Academic Publishers. Printed in Belgium.*

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

the solar diameter measurements. Furthermore, the measurements of the oblateness of the Sun (Dicke and Goldenberg, 1967; Hill, Stebbins, and Oleson, 1975) and the beginning of helioseismology have also contributed to the development of programs for the measurement and the surveillance of the solar diameter. For more than one solar cycle, visual or photoelectric measurements have been made using instruments dedicated to the study of solar geometry (Rösch and Yerle, 1982; Brown, 1991), or for measurement of the solar diameter (Wittmann, 1977; Wittmann, Bonet, and Wöhl, 1981; Brown, 1991).

One method for solar diameter measurement consists of the use of a Danjon's astrolabe (Laclare, 1987). Indeed, the knowledge of the solar diameter was needed in a program using an astrolabe for the determination of Earth orbit characteristics. The accuracy of the first solar diameter measurements was of the same order as these characteristics. Thus, since a long period of time is necessary to obtain good accuracy of the orbit characteristics, a continuous record of solar diameter measurement was also obtained. This record presented variations (Laclare, 1987) as already stressed by others (Gilliland, 1981). However, until 1989, diameter measurements with the astrolabe were made visually. Thus, the origin of the observed variations could be attributed to solar physics phenomena provided that the effects due to the observer and to atmospheric turbulence are taken into account. For this purpose, in 1989, a data acquisition system using a CCD camera was installed behind the astrolabe in order to avoid observer effects (Laclare and Merlin, 1991). However, the system is configured so as to permit both CCD and visual observations. Thus a comparison between the two sets of observations could be performed and the previous observations validated. If it is not easy to observe the atmospheric effects from the visual observations, the numerical data acquisition system offers this possibility. Thus, a value of a seeing parameter can be attributed to each solar diameter measurement, making a simultaneous study of atmospheric turbulence effects possible.

The aim of this paper is to present the first results obtained with the solar astrolabe using a numerical data acquisition system, and a study of atmospheric effects. The principle and the instrumentation of the diameter measurement with the astrolabe are first presented. The observations made with the CCD camera since 1989 are then compared with the visual observations made during the same period. For CCD observations, methods are introduced for evaluation of Fried's parameter which gives the image quality and for calculation of the diameter measurement error. The statistical evolution of the seeing at Calern Observatory during the past four years is deduced from the observing campaign. A theoretical expression of the diameter measurement error is established and related to the seeing parameter. A comparison with experimental data is made. Finally, the general results of the diameter measurement and the corresponding error are presented and related to the seeing observation conditions. A method is then proposed to improve the quality of the results.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

2. Principle of the Measurement and Instrumentation

The first measurements with the astrolabe were made visually but for more objectivity and in order to avoid observer effects, a numerical data acquisition system using a CCD camera was installed behind the instrument in 1989 (Laclare and Merlin, 1991). However, the actual system is still configured to permit both visual and CCD observations and so a comparison between the results is possible from the two sets of observations. The principle of the diameter measurement using the solar astrolabe consists in the determination of the time at which the solar edge crosses the small circle at an altitude defined by the angle of the prism (Laclare, 1987). Indeed, the astrolabe gives two images of the same solar edge: a direct and a reflected image. Each of them moves in the astrolabe focal plane due to the diurnal motion. The intersection of the two trajectory images corresponds to the instant when the solar edge crosses the small circle. In the case of visual observations, the intersection of the two images is estimated by the observer and noted by a time recording system. For CCD observations, direct and reflected solar images are alternatively recorded with a numerical acquisition system using a CCD camera (Figure 1). An analysis of the solar image is done by the acquisition system which extracts the solar edge constituted by the inflexion points of the solar limb function given by each CCD line. The inflexion points are determined in an analog mode as the zero of the second derivative of the solar limb function. The recorded images are then the solar edge associated with the corresponding time acquisition. The data consists of a sequence of about 80 images during a time of about 20 s. The spatial resolution of the CCD camera is 0.5 by 1 arc sec and its integration time is 0.020 s.

The image of the solar edge is reconstructed using a parabolic fit through the inflexion points and the maximum (or minimum) of the parabola gives one point of the solar trajectory associated with the corresponding time. The solar trajectory is so determined from the data for both direct and reflected images. The trajectories are then approximated by a straight line using a least-square fit and the point of intersection gives the time when the solar edge crosses the small circle. Figure 2 represents some acquisition data of the solar edge during the transit of the Sun by the small circle. The parabolic fit of the data is also represented as the reconstructed trajectory of both direct and reflected images. The solar diameter is deduced from the time interval between the two solar limb transists by the same small circle.

3. Comparison between Visual and CCD Observations

The visual observations of the diameter measurement made with the solar astrolabe since 1975 by Laclare (1987) show variations whose origin could be attributed to solar physics phenomena provided that effects due to atmospheric turbulence, observer or instrument are taken into account. It was for this purpose that in 1989, a numerical data acquisition system was installed on the astrolabe. Thus, we have

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

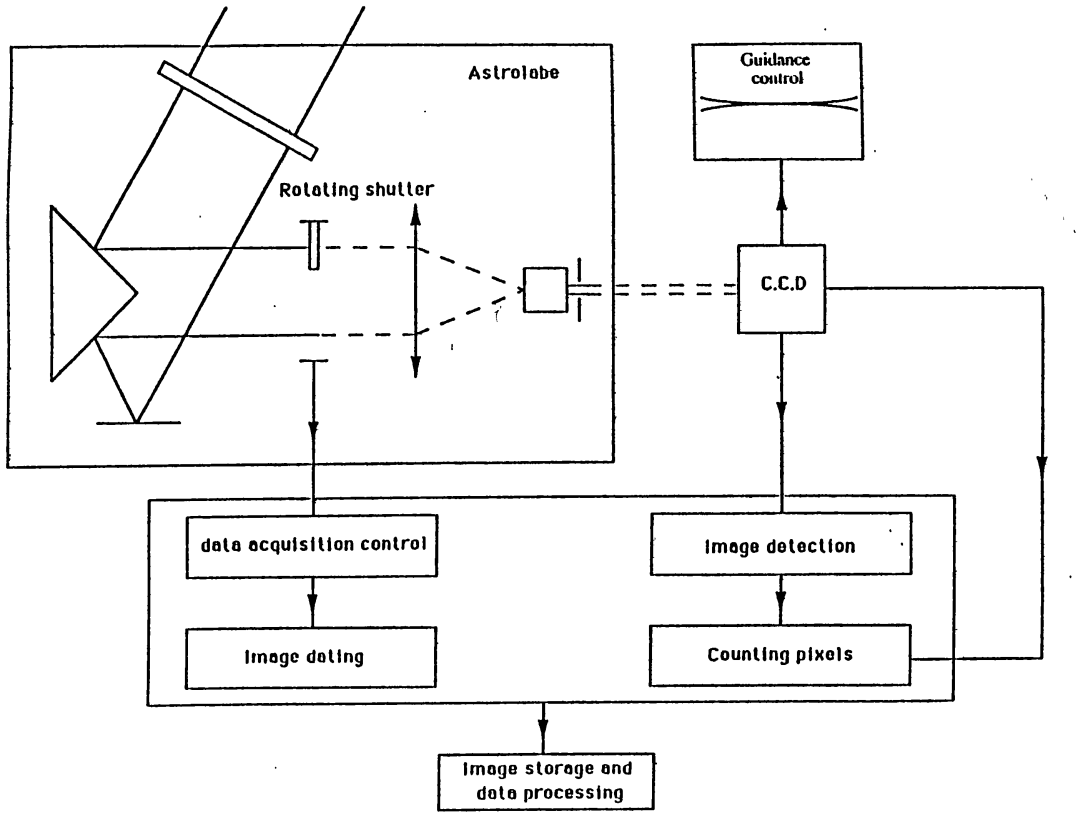


Fig. 1. The data acquisition device.

now four years of numerical data which can be analyzed and compared with visual observations made also during this period by the same observer. The atmospheric effects are not easy to observe from visual observations and so, the comparison is limited to the monthly diameter value and its scatter. A good agreement between the results for these observation campaigns is observed (Figure 3). In the figure, each diameter value corresponds to an average of diameter measurements over the month. The mean values of diameter measurements for visual and CCD observations during this period are, respectively, 959.37 and 959.40 arc sec with a standard deviation of their difference equal to 0.12 arc sec. The scatter of the data has also been calculated for each month and shown in Figure 4. The mean value during the period is 0.31 arc sec for visual observations and 0.26 arc sec for CCD observations, with a standard deviation of their difference equal to 0.08 arc sec. The statistical study is made over more than 1400 diameter values for visual observations while for CCD observations there are 400. All observations made with different zenith distances are included in the calculation of the mean diameter and the standard deviation. However, during the first and last months of the year, only observations corresponding to large zenith distances are possible. In order to observe effects of the zenith distance on the results, an average of diameter measurements has been performed and the corresponding standard deviations computed for observations

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

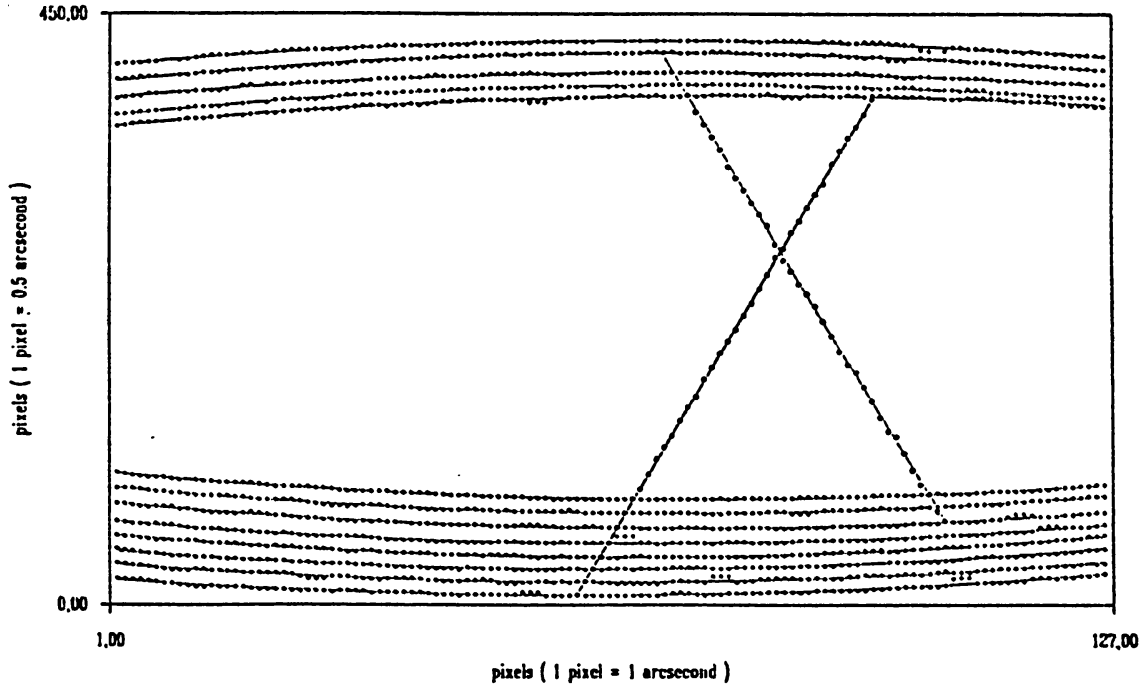


Fig. 2. Extraction of solar edges and reconstructed solar trajectory images. Each solar edge (♦) is approximated using a parabolic fit (solid lines). Minimum (or maximum) position of the parabolic fit (•) permits one to reconstruct solar trajectories for both direct and reflected images. Solar trajectory images are approximated by a straight line using a least-square fit (solid lines).

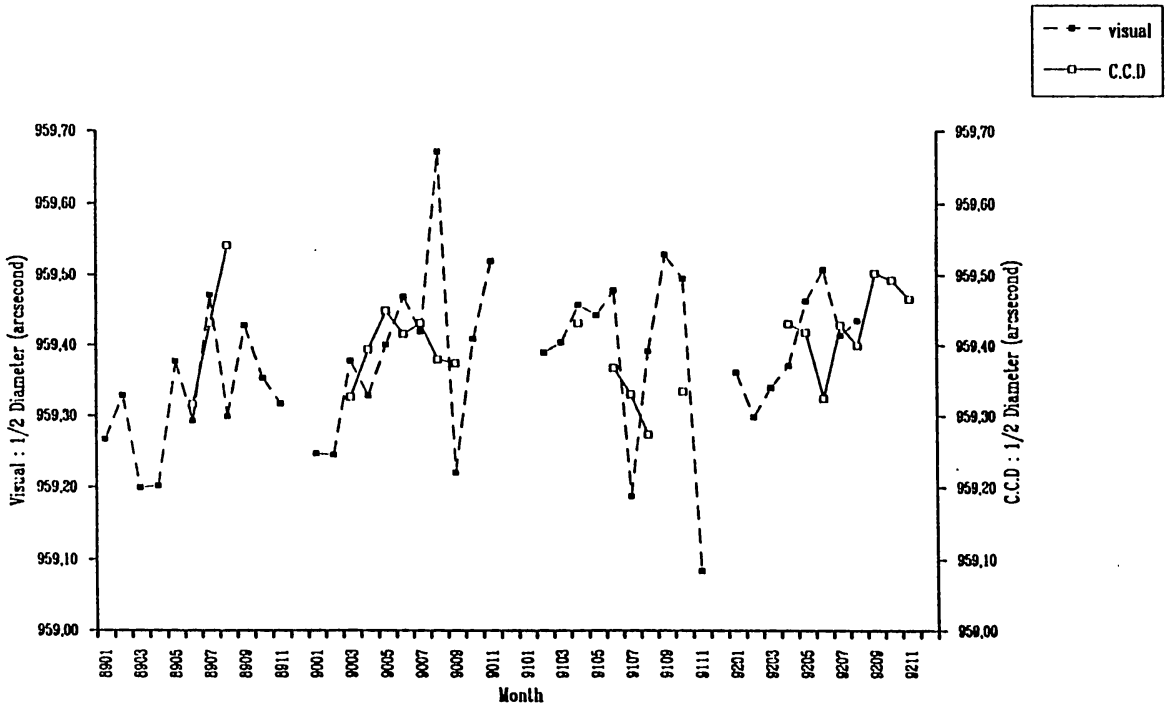


Fig. 3. Evolution of the monthly solar diameter deduced from CCD and visual observations for the period from 1989 to 1992.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

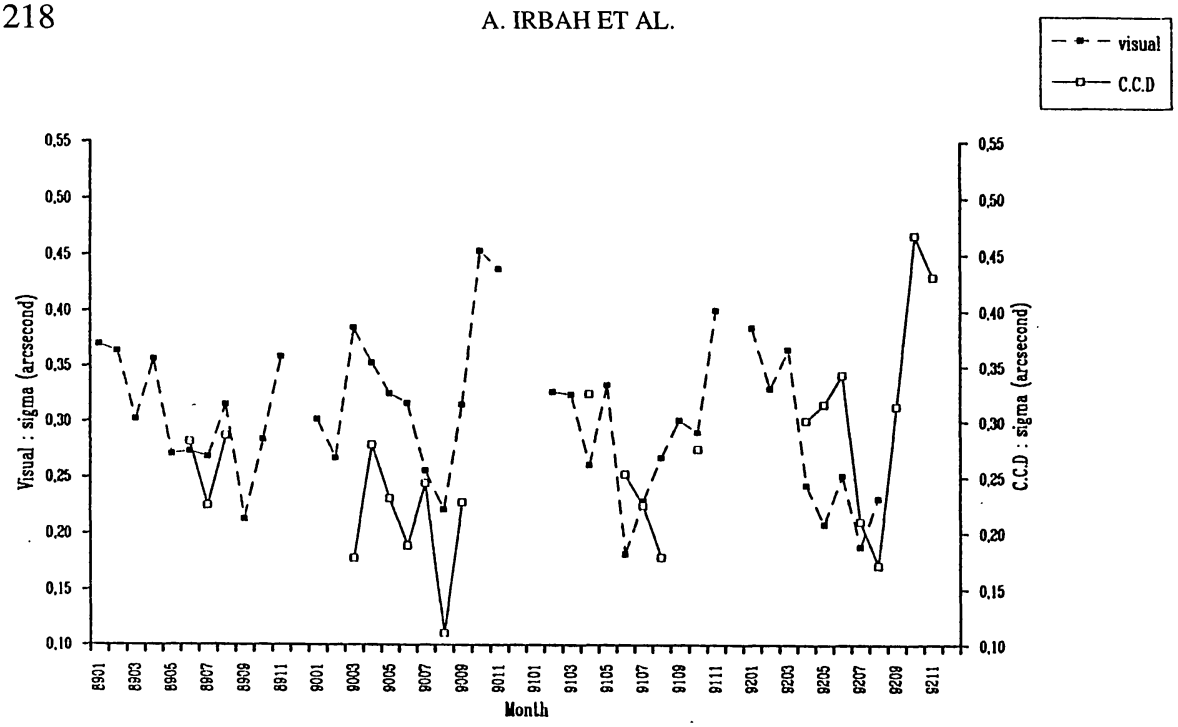


Fig. 4. Evolution of the scatter of solar diameter measurements calculated over each month.

made with the same zenith distances (Figures 5(a) and 5(b)). For CCD observations, the first point of Figure 5(a) is not statistically well defined and so there is no significant effect of the zenith distance on the mean diameter value. However, for visual observations there is a linear decrease of the mean diameter value with the zenith distance. This effect is probably due to the observer's eye which is sensitive to object contrast. Indeed, because of the absorption of the solar flux by the Earth's atmosphere, there is a decrease of the solar limb contrast with zenith distance. Assuming that the detection level of the eye is constant, the observed point on the solar limb will be more inside the solar disk (higher luminance) at low contrast, corresponding to large zenith distance than at high contrast. Consequently, for visual observations the diameter will decrease with the zenith distance. However, it can be shown from Figure 5(a) that the observed point corresponds to the inflexion point of the solar limb for a zenith distance which is equal to 45°. Thus, since the variation law is linear, a correction for zenith distance effect can be made to each diameter measurement. The scatter of diameter measurements with the zenith distance has also been analyzed for the two sets of observations. For CCD observations, there is no significant effect while it increases with the zenith distance for visual observations (Figure 5(b)).

Finally, a monthly evolution of the solar diameter with the associated statistical error is represented for the two sets of observations (Figure 6). The good agreement of the CCD observations with the visual observations is confirmed. In fact, for the two sets of observations, only few diameter measurements with the associated error bars are separated from the others. These diameter measurements correspond globally to months in which a few measurements were made. Thus, except

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

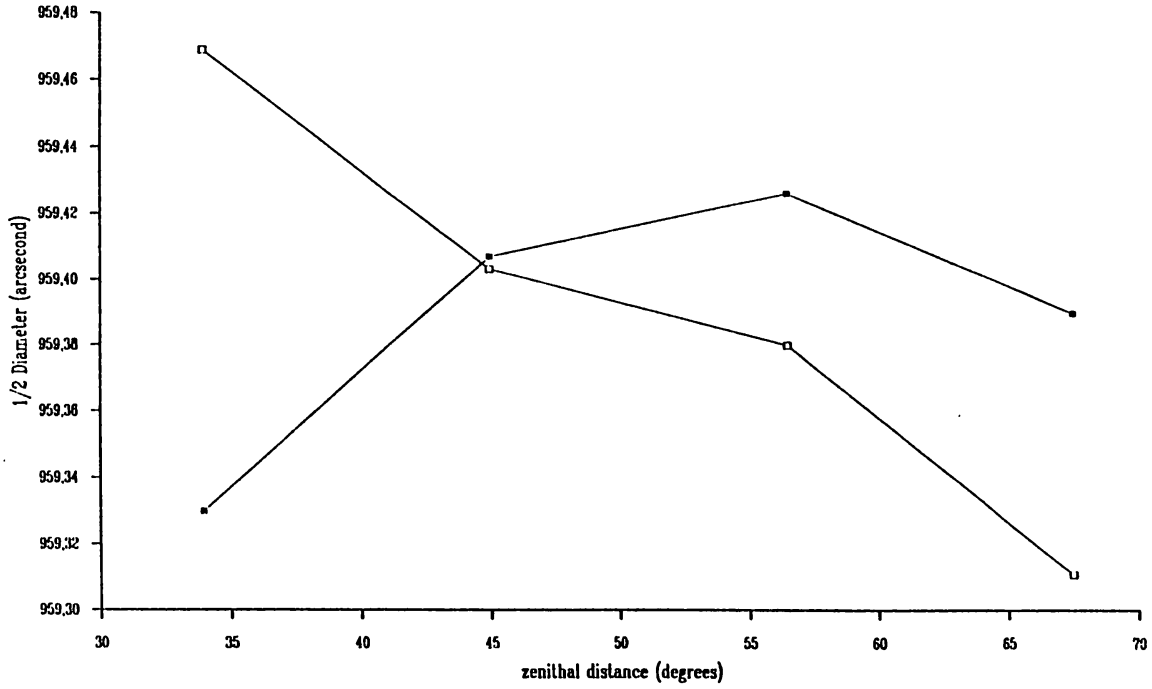


Fig. 5a.

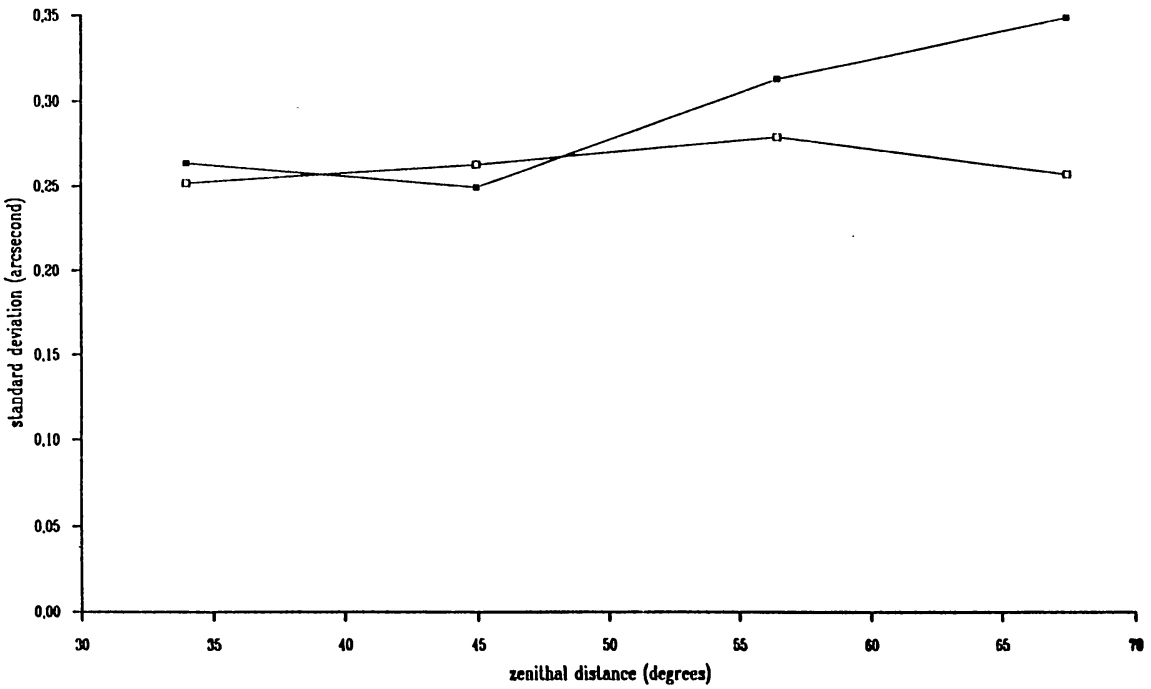


Fig. 5b.

Fig. 5. Effect of the zenith distance on solar diameter measurements. (a) Variation with zenith distance of mean solar diameter deduced from both CCD (■) and visual (□) observations and calculated over four years. (b) Same representation but for the scatter.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

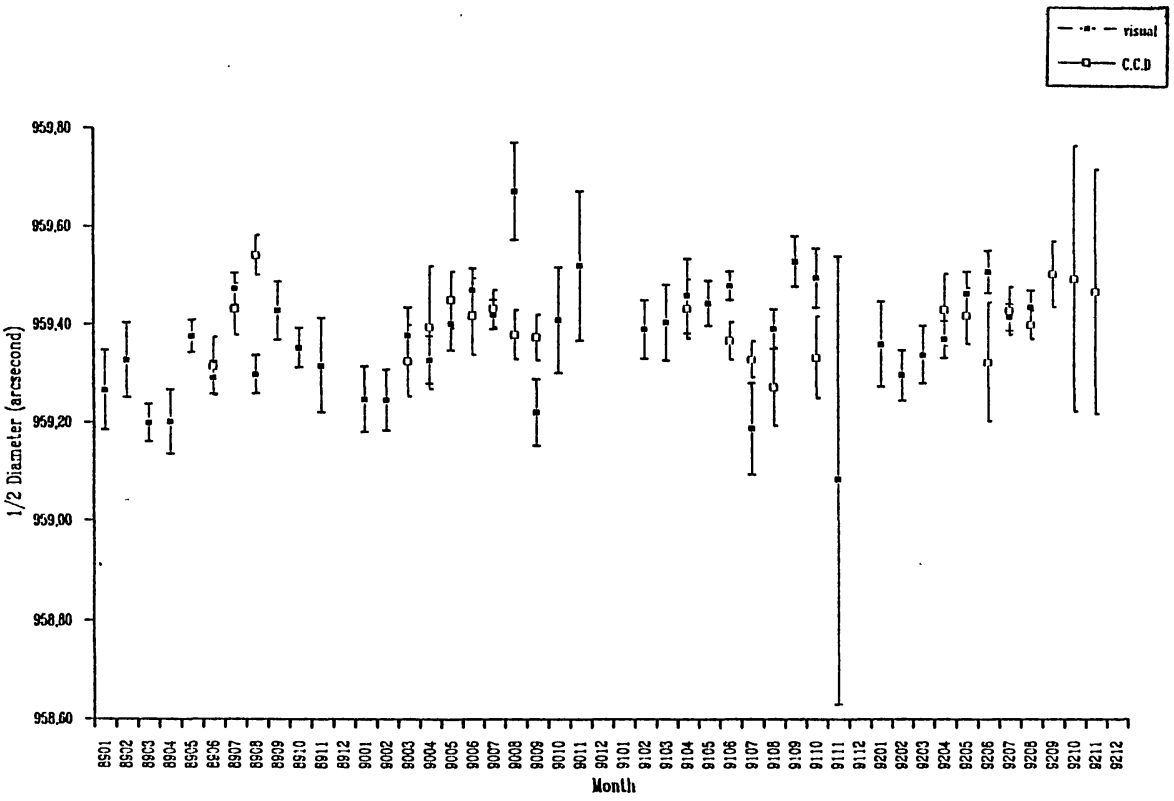


Fig. 6. Evolution of the monthly solar diameter with the associated statistical error deduced from CCD and visual observations (period from 1989 to 1992).

for the zenith distance effect which can be corrected, the solar diameter record obtained since 1975 seems to be not affected by personal bias due to the observer.

4. Measurement Quality – Relation to Atmospheric Seeing Conditions

The previous section shows that the observations are not affected by personal bias due to the observer but what about the atmospheric turbulence effects? If it is not easy to observe the atmospheric effects from the visual observations, it is now possible from the numerical data. This section will discuss these aspects and the relation of the diameter measurement error with the atmospheric seeing.

4.1. ATMOSPHERIC EFFECTS: r_0 MEASUREMENT

The parameter which describes the atmospheric seeing quality is Fried's parameter, noted generally by r_0 (Fried, 1966). This parameter is related to the size of the coherence areas of the disturbed wavefront arriving at the entrance of the instrument pupil. Here, since there is not a seeing monitor which permits us to have Fried's

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

parameter estimations simultaneous with the observations, r_0 is deduced from the data. In fact, the solar trajectory images present spatial fluctuations, σ_s, which can be interpreted as angle-of-arrival fluctuations averaged over the astrolabe pupil, D. Fried's parameter is directly related to the angle-of-arrival fluctuations, σ_s, by the expression (Borgnino *et al.*, 1982; Ricort, Borgnino, and Aime, 1982; Brandt, Mauter, and Smartt, 1987)

$$r_0 = 8.25 \times 10^5 D^{-1/5} \lambda^{6/5} (\sigma_s^2)^{-3/5} = K_1 (\sigma_s^2)^{-3/5} , \tag{1}$$

where λ is the wavelength. This relation supposes that r_0, D, and λ are expressed in meter and σ_s in arc sec.

Thus, the seeing parameter, r_0, can be deduced from the observations and associated with each solar diameter measurement. However, because of the duration of one solar diameter determination, a mean value of r_0, corresponding to each solar transit by the same small circle, will be taken.

The solar diameter record with the numerical data system covers now four years. A statistical study of the seeing quality at the Calern Observatory is given below. Using the principle developed above, a histogram of the daily Fried parameter occurrence is deduced from the observations (Figure 7). The figure shows that the best seeing for daytime observations is during the summer. The maximum of occurrence for the r_0 value is 4 cm. A knowledge of the seeing parameter during the observations is important to improve the quality of the result as will be shown in the next section.

4.2. Error on Diameter Measurements with the Astrolabe

The diameter as measured with the solar astrolabe depends critically on a good determination of the trajectory intersections for the two transits of the solar edge by the small circle. Because of the temporal fluctuations observed on the observed trajectories, the point of intersection is not well defined. It is the error in this time determination which will cause the diameter measurement error. Since the trajectories are approximated by a straight line using a least-square fit, the error in the point of intersection is given by (Bendat and Piersol, 1971)

$$\Delta(t_0) = \sigma_\tau T_{N-2;\epsilon/2} \left[\frac{1}{N} + \frac{(t_0 - \bar{t})^2}{\sum_{i=1}^{N} (t_i - \bar{t})^2} \right]^{1/2} = K(t_0) \sigma_\tau , \tag{2}$$

where σ_τ is the standard deviation of the temporal fluctuations observed on the solar trajectory, $T_{N-2;\epsilon/2}$ the Student's distribution, $1 - \epsilon$ the confidence interval, t_0 the intersection time, and N the number of images taken at the t_i instant relative to the mean time $\bar{t}$.

The standard deviation, σ_τ, is obtained from the scatter of the points around the adjusted trajectory during a time of about 20 s.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

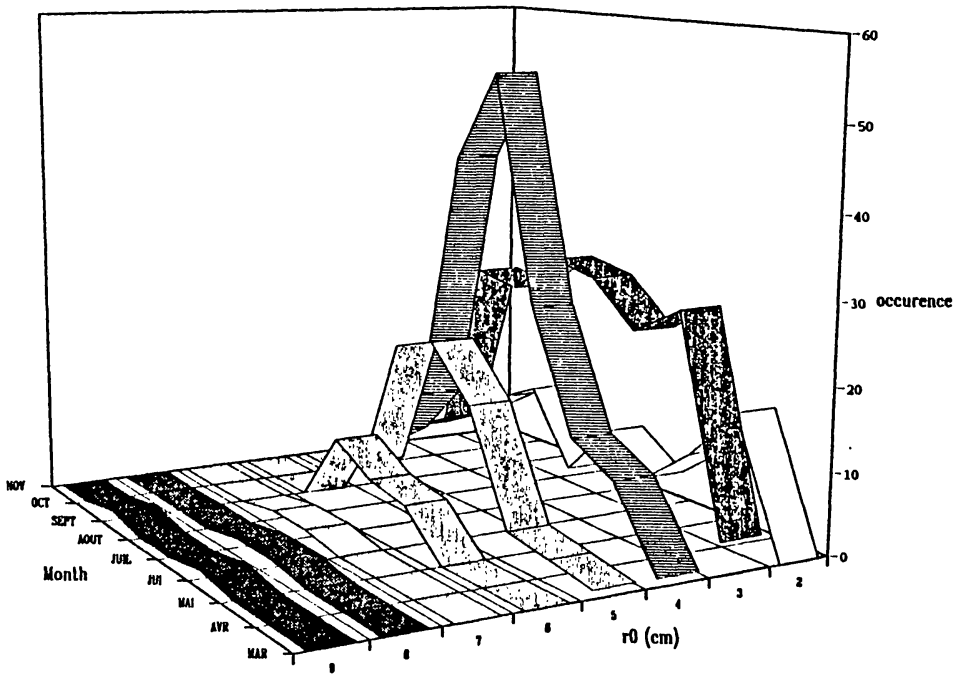


Fig. 7. Histogram of daily Fried parameter occurrence at Calern Observatory deduced from observations.

The diameter measurement error is deduced by considering the extreme values of the diameter corresponding to the extreme time intervals of the two solar edge transits. Using the expressions (1) and (2), the diameter measurement error, ΔD, can be related to the seeing parameter by the expression (see the detail of the calculation in Equation (A.7)

$$\Delta D = AK[2K_1^{5/6} r_0^{-5/6} + \kappa] \,, \tag{3}$$

where r_0 corresponds to the Fried parameter associated with the diameter measurement, and A is a constant related to the apparent angular velocity of the Sun around the Earth. The variable κ expresses the relative variations of the temporal fluctuations observed on the trajectories between the two solar edge transits by the small circle. Figure 8(a) shows the theoretical curves of the diameter measurement error with the seeing (expression (3)) and the error deduced from the observations. The theoretical curves are represented for different values of the variable κ with a constant K adjusted from the observations. A good agreement with the experimental data is observed in the figure. As expected, the diameter measurement error increases with bad seeing conditions. Thus, a knowledge of the seeing parameter will permit a classification of the observations and thus an improvement of the quality of the result. The statistical analysis of the seeing at Calern Observatory during four years has shown maximum quality in the summer. This result deserves

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

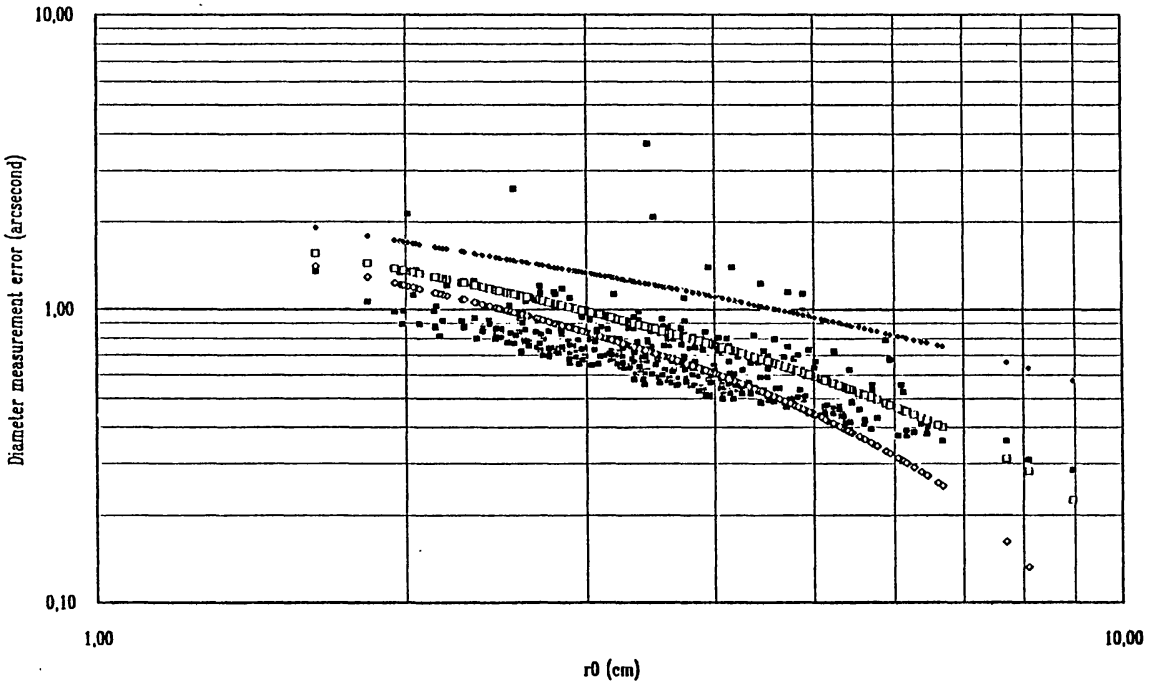


Fig. 8a.

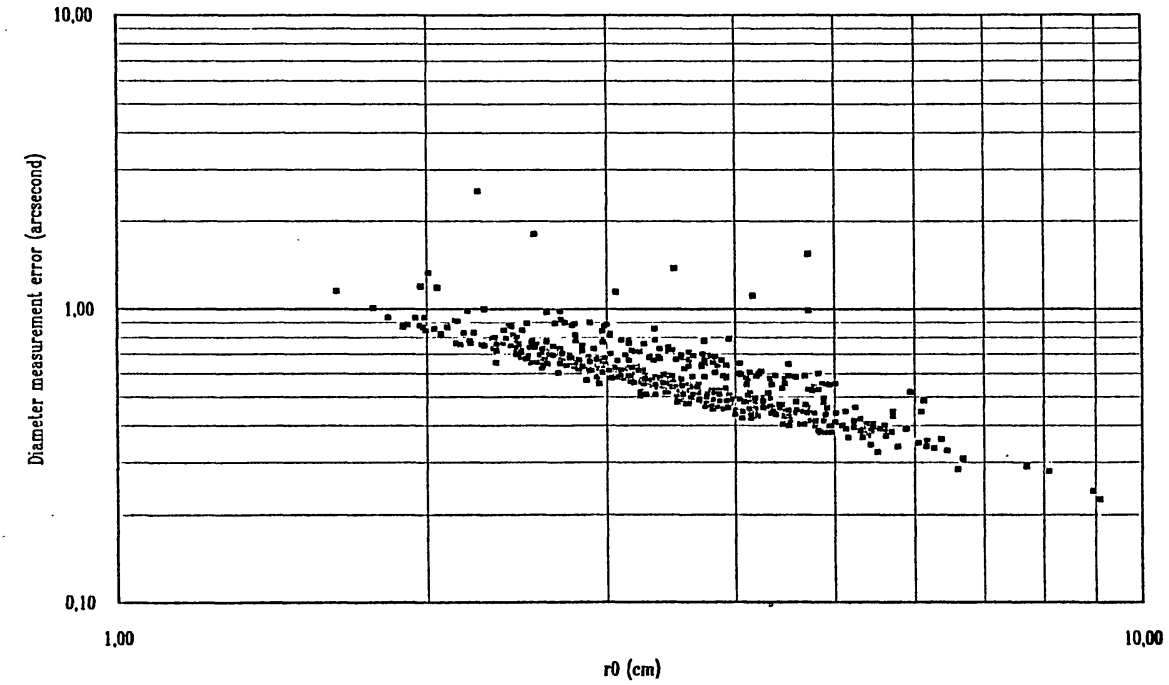


Fig. 8b.

Fig. 8. Variation of solar diameter error with the seeing. (a) Theoretical curves corresponding to different cases of atmospheric turbulence (♦, □, ◊) and diameter measurement error deduced from CCD observations where the two images are considered (■). (b) Diameter measurement error deduced from CCD observations where only the direct image is considered.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

to be confirmed in a general way. Indeed, a survey in time of Fried's parameter at an observatory may then give a statistical prediction to perform the best observations.

4.3. INSTRUMENTAL EFFECTS

The diameter measurement error is due in large part to fluctuations along the image trajectories. However, the astrolabe is responsible for several causes of image quality degradation. One of them is that the reflected image is more degraded than the direct image. Thus, the reconstructed trajectory is less well defined. For the determination of the time intersection error, the direct or the reflected trajectory image can be used. In Figure 8(a) were considered the two images in the determination of the diameter measurement error. In this case, the maximum value of the time intersection error deduced from the two trajectories is taken into account. However, Figure 8(b) shows the diameter measurement error where only the direct image is used. One can note that the scatter of the points is less important than in the other case: the difference comes in large part from the reflexion effects.

Another cause of the magnitude of the fluctuations is the small size of the astrolabe pupil. Indeed, the image motion is due to the angle formed by the pupil plane with the wavefront slope all over the pupil. The average angle of arrival will then cause a displacement of the image which can be more important when the pupil is small. Thus, according to the characteristic evolution time of the atmospheric turbulence (Aime *et al.*, 1981; Roddier, Gilli, and Lund, 1982) and the integration time of the CCD camera, these effects could be more notable for the astrolabe than for other instruments with a large pupil.

The last effect which can be noted is the displacement of the solar image during the acquisition of the data. In fact, the astrolabe does not compensate for the diurnal motion of the Earth and so, the solar image moves during the integration time of the CCD camera. Consequently, the solar limb function is smoothed and then, the inflexion point is not very well defined.

5. Results and Discussion

The previous section developed a method to estimate the seeing and the atmospheric effects on the solar diameter from observations made with the CCD camera. This section presents the results of the numerical observations which cover the period from 1989 to 1992. During this period, the observations were made with 11 different prisms which correspond to 11 zenith distances of the small circle. The diameter measurements are represented in Figure 9(a). Figure 9(b) shows the same representation with the diameter measurement error. In this case and for more convenience in the visualization, the values are distributed uniformly on the time axis in order of their occurrence. The diameter measurement error is determined from the two solar images (see Section 4.2). For each diameter measurement, the associated Fried parameter, r_0, is estimated. The evolution of the error with the seeing parameter (Figure 8(b)) shows a decrease of the diameter measurement

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

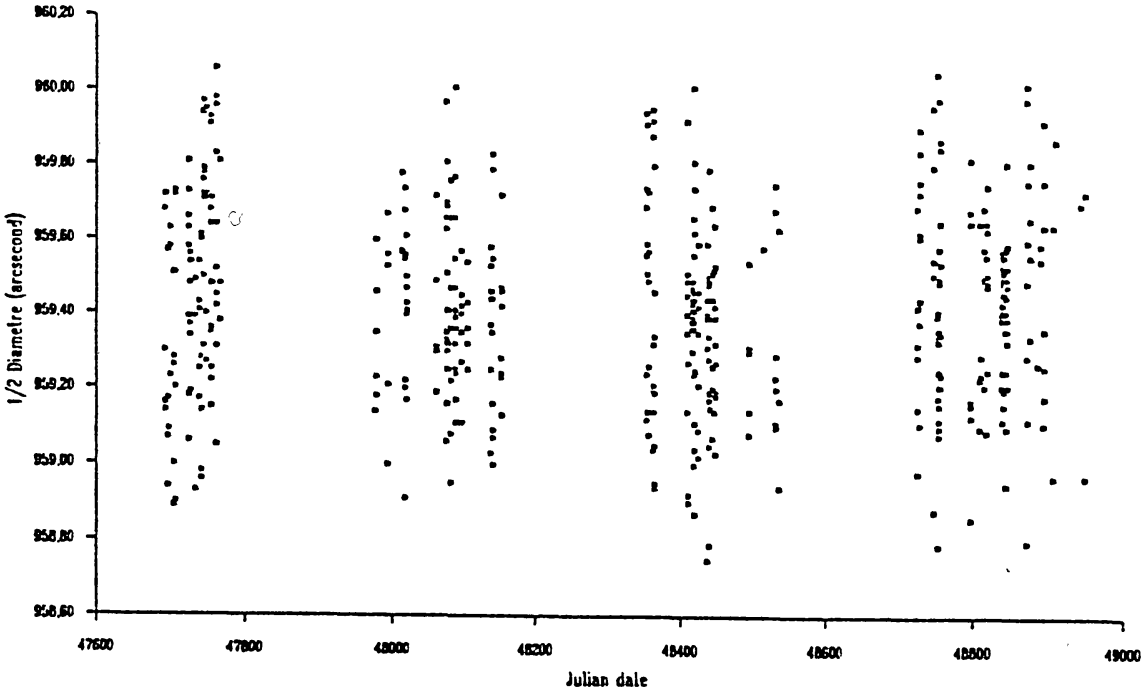


Fig. 9a.

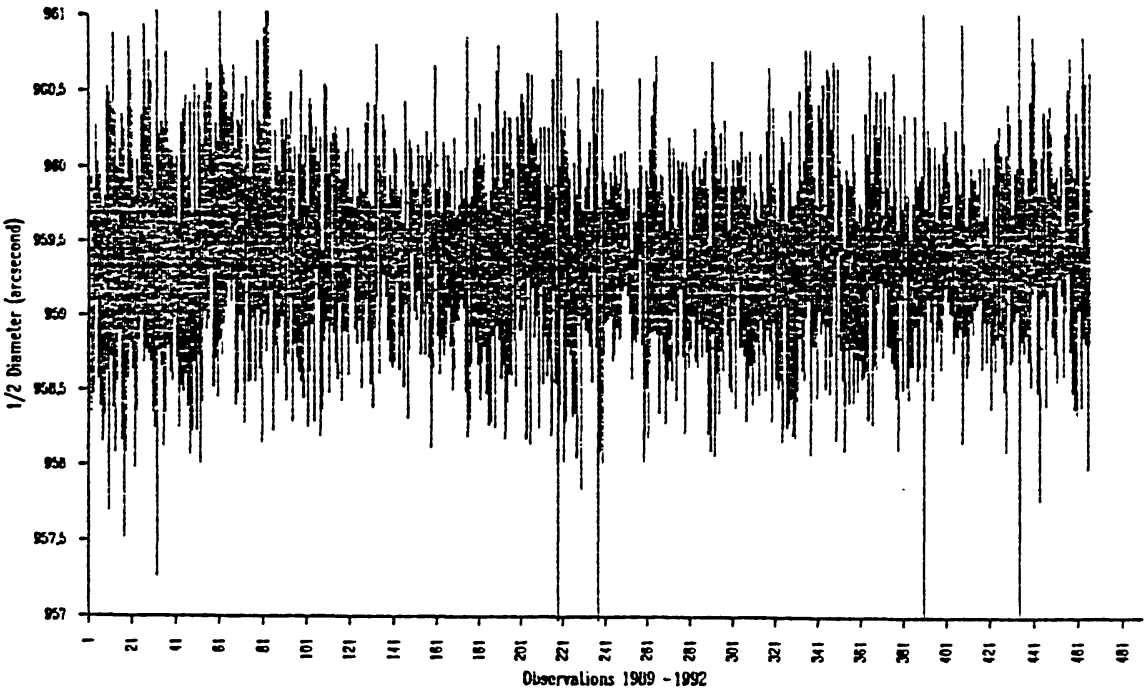


Fig. 9b.

Fig. 9. Diameter measurements deduced from CCD observations for the period from 1989 to 1992. (a) Evolution of diameter measurements with the corresponding time acquisition. (b) Evolution of diameter measurements with the associated error. In the figure, the values are uniformly distributed on the time axis in order of their occurrence.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

error with good seeing conditions as predicted by the theoretical curves (see Section 4.2). Indeed, the diameter measurement error is greater than 1 arc sec when the r_0 value is 2 cm and falls down to about 0.3 arc sec when r_0 equals 8 cm. With the intent of observing the atmospheric effects on the diameter value, solar diameter measurements are represented in Figure 10(a) according to the seeing parameter values. An increased scatter of the data with bad seeing conditions is observed. A classification of diameter values has been made with respect to the associated Fried parameter. The diameter values have been divided into 6 classes with equal width from $r_0 = 1$ cm to $r_0 = 7$ cm. A mean value of the solar diameter has been calculated for each r_0 class and is represented in Figure 10(b)). For the first and last classes, the calculation is made over a small number of diameter measurements, and so the corresponding points are not well defined. Figure 10(b) shows, however, a slight decrease of the solar diameter with a degradation of seeing conditions. The difference of solar diameter values corresponding to the worst and best seeing at Calern Observatory is of order of 0.08 arc sec. Although few diameter measurements were made during good seeing conditions ($r_0 > 7$ cm), it can be noted that the results are of the same order as the mean diameter value (these diameter measurements are not represented in Figure 10(b)). The standard deviation has been calculated for each r_0 class. As already observed with the diameter measurement error, the results improve with good seeing. In fact, the standard deviation of diameter values is about 0.3 arc sec when the r_0 value is 2 cm and falls down linearly to 0.15 arc sec when r_0 is equal to 7 cm (Figure 10(b)). Except for the insignificant decrease of the mean solar diameter with bad seeing, diameter measurements do not have systematic small or large values with bad seeing conditions: atmospheric turbulence causes a random displacement of the inflexion point with bad seeing around a mean position.

Consequently, it can be noted that variations less than 0.08 arc sec could not be observed in the mean solar diameter. Since the maximum occurrence of Fried's parameter during the observations at Calern Observatory is 4 cm, variations less than 0.26 arc sec of the solar diameter can not currently be observed.

However, the results presented lead us to consider a classification of diameter measurements with r_0 which can be taken into account in processing the data. Thus, each diameter measurement may be weighted by a coefficient related to the variation law of the scatter of diameter values with the seeing. Another way to improve the precision of the results is to make the observations only in good seeing conditions. To do that it is necessary to choose the data acquisition time. This is possible if a variable angle prism and a short integration-time camera can be used on the astrolabe. The variable angle prism may permit continuous daily observations while the short integration-time camera allows one to estimate the atmospheric seeing and to reduce the displacement effect of the camera during the integration time.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

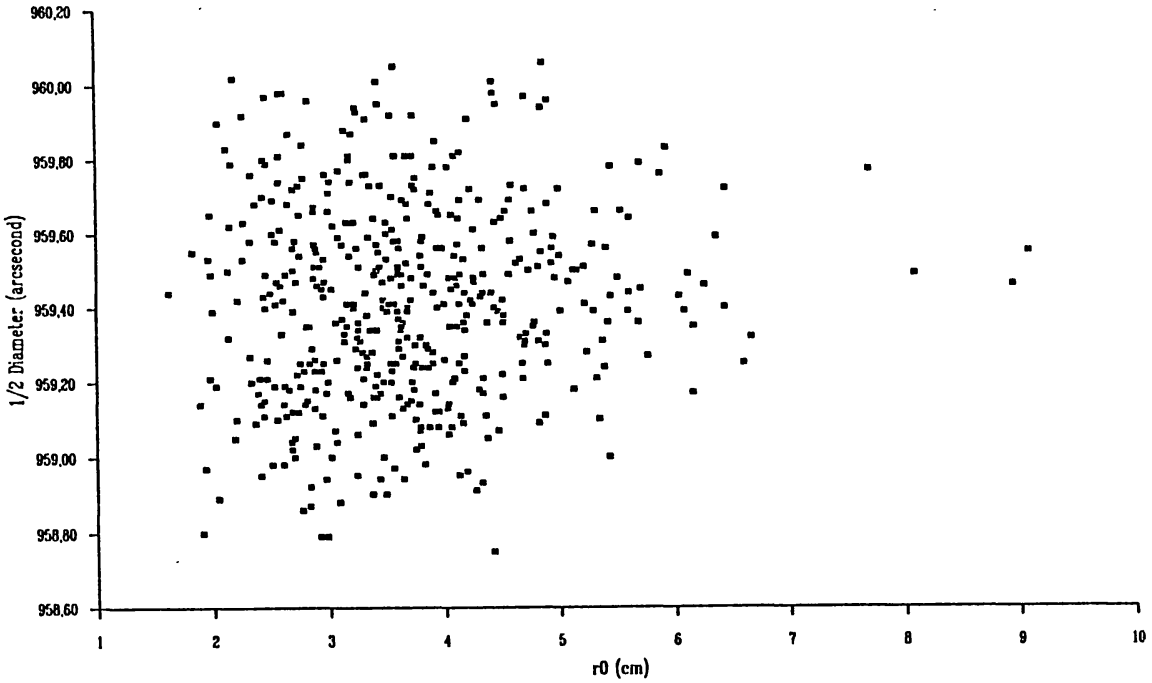


Fig. 10a.

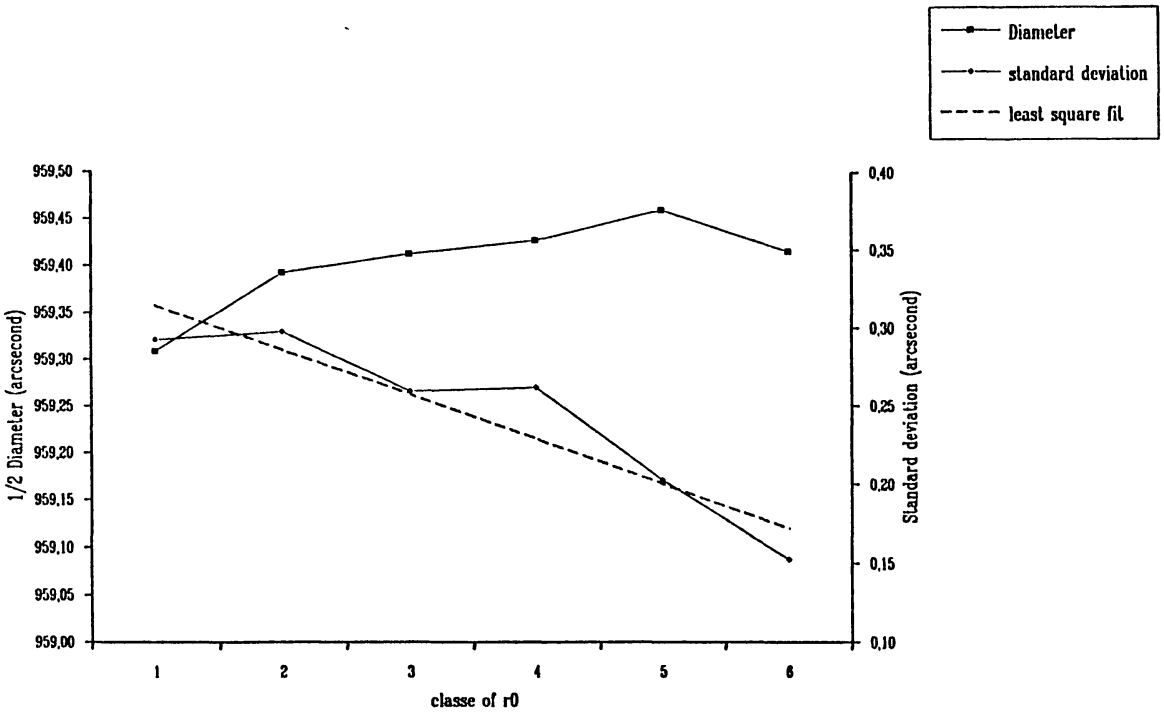


Fig. 10b.

Fig. 10. (a) Evolution of solar diameter measurements with the seeing. (b) Variation of the mean value of diameter measurements and the scatter according to different classes for r_0.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

6. Conclusion

Visual observations of the solar diameter with the solar astrolabe since 1975 show variations. It was in order to avoid personal bias and confirm the results that in 1989 a digital data acquisition system was installed on the instrument. Since this date, several observations were performed with the system together with visual observations. The comparison between the two sets of observations has shown a good agreement of the results. Thus, except for the effect of zenith distance, which can be corrected, it seems that the visual observations are not affected by personal bias. Because of this result, more credibility can now be attributed to the past observations. In addition, the system permits one also to deduce the atmospheric seeing from the CCD observations. Thus, a study of the atmospheric effects on the solar diameter measurements has been done for the CCD observations. A correlation of the diameter measurement error with Fried's parameter, r_0, was observed. The error decreases with good seeing conditions. The analysis has shown also a linear increase in the scatter of the diameter measurements with bad seeing conditions but with no significant correlation. Thus, a systematic estimate of the seeing, if possible independently of the observations, could improve the precision of the results, if the data acquisition time is chosen. This is possible if a variable angle prism and a short integration-time camera are used on the astrolabe. The variable prism is to allow continuous daily observations and the short integration-time camera is to freeze the atmospheric turbulence and reduce the displacement of the solar image during data acquisition. Finally, statistics of the seeing at Calern Observatory were deduced from the observations. The seeing parameter for daytime observations increases continuously and has its maximum occurrence during the summer. If the result is confirmed in a general way, a survey of this parameter in an observatory may give a statistical prediction of the seeing.

Appendix

Expression of the diameter measurement error derived from the relations (1) and (2) giving, respectively, Fried's parameter, r_0, and the error of the time intersection of the two images.

These relations (written here (A.1) and (A.2)) are:

$$\Delta(t_0) = K(t_0)\sigma_\tau \,, \tag{A.1}$$

where

$$K(t_0) = T_{N-2;\epsilon/2}\left[\frac{1}{N} + \frac{(t_0 - \bar{t})^2}{\sum_{i=1}^{N}(t_i - \bar{t})^2}\right]^{1/2}$$

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

and

$$r_0 = K_1(\sigma_m)^{-6/5} , \tag{A.2}$$

where

$$K_1 = 8.25 \times 10^5 \, D^{-1/5} \lambda^{6/5} .$$

Assuming a linear relation between the solar diameter, D, and the interval of time corresponding to the two solar edge transits by the small circle,

$$D = A(t_0 - t_0') . \tag{A.3}$$

The error in the solar diameter measurement is

$$\Delta D = A(\Delta t_0 + \Delta t_0') . \tag{A.4}$$

Since Fried's parameter associated with the solar diameter measurement, D, corresponds to a standard deviation, σ_m (relation (A.2)), it can be written for each solar transit:

$$\sigma_\tau = \sigma_m + \kappa_1 \quad \text{and} \quad \sigma_\tau' = \sigma_m + \kappa_2 . \tag{A.5}$$

Using the relations (A.1), (A.4), and (A.5) leads to the expression of the diameter measurement error:

$$\Delta D = A[(K(t_0) + K(t_0'))\sigma_m + \kappa_1 K(t_0) + \kappa_2 K(t_0')] . \tag{A.6}$$

Since

$$K(t_0) \cong K(t_0') = K ,$$

and using (A.2), the error on the solar diameter measurement takes the final form

$$\Delta D = AK[2K_1^{5/6} r_0^{-5/6} + \kappa] , \quad \text{where } \kappa = \kappa_1 + \kappa_2 . \tag{A.7}$$

Acknowledgement

This work has been performed with support of the French Foreign Affair Ministry in the framework of the contract No. 88 MER 80 of scientific cooperation between France and Algeria.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System

1994SoPh..149..213I

References

Aime, C., Kadiri, S., Martin, F., and Ricort, G.: 1981, *Optics Comm.* **39**(5), 287.
Bendat, J. S. and Piersol, A. G.: 1971, *Random Data: Analysis and Measurement Procedures*, Wiley-Interscience, New York, p; 133.
Borgnino, J., Ceppatelli, G., Ricort, G., and Righini, A.: 1982, *Astron. Astrophys.* **107**, 333.
Brandt, P. N., Mauter, H. A., and Smartt, R.: 1987, *Astron. Astrophys.* **188**, 163.
Brown, T. M.: 1991, in C. P. Sonett *et al.* (eds.), *The Sun in Time*, University of Arizona, Tucson.
Eddy, J. A. and Boornazian, A. A.: 1979, *Bull. Am. Astron. Soc.* **11**, 437.
Fried, D. L.: 1966, *J. Opt. Soc. Am.* **56**, 1372.
Gilliland, R. L.: 1981, *Astrophys. J.* **248**, 1144.
Hill, H. A., Stebbins, R. T., and Oleson, J. R.: 1975, *Astrophys. J.* **200**, 471.
Laclare, F.: 1987, *Compt. Rend. Acad. Sci. Paris* **305**, Série II, 451.
Laclare, F. and Merlin, G.: 1991, *Compt. Rend. Acad. Sci. Paris* **313**, Série II, 323.
Parkinson, J. H., Morrison, L. V., and Stephenson, F. R.: 1980, *Nature* **288**, 548.
Ricort, G., Borgnino, J., and Aime, C.: 1982, *Solar Phys.* **75**, 377.
Roddier, F., Gilli, J. M., and Lund, G.: 1982, *J. Optics* **13**(5), 263.
Rösch, J. and Yerle, R.: 1983, *Solar Phys.* **82**, 139.
Wittmann, A.: 1977, *Astron. Astrophys.* **61**, 225.
Wittmann, A., Bonet, J. A., and Wöhl, H.: 1981, in L. Cram and J. H. Thomas (eds.), *The Physics of Sunspots*, Sacramento Peak National Solar Observatory, p. 424.

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System